Switching data-processing methods for feedback control: 
Breaking the speed versus accuracy trade-off

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Abstract—In many digitally controlled applications, such as vision-based and data-intensive control, the choice of the data-processing method for distilling (control-relevant) state information is non-trivial and important. While accurate processing methods require significant processing time, limiting the closed-loop control rate, faster methods introduce larger inaccuracies in the distilled state information. This leads to a trade-off between speed and accuracy, which can be considered off-line or on-line. In this paper, we propose an on-line strategy to switch between different data-processing methods in real-time in order to improve closed-loop performance for linear systems.

I. INTRODUCTION

Several control applications require pre-processing of non-trivial amounts of measurement data to distill the information relevant for feedback. A prime example is vision-based control, where one or several images with millions of pixels must be translated into relevant information such as a robot position or attitude. The choice of processing method that is best for control purposes is then an important design feature.

Such data-intensive control applications require the data-processing method not only to provide accurate information, but also to be fast, in order to mitigate delays in the loop and enable a high closed-loop control rate. These two desired features are typically conflicting. For example, in the context of vision-based control, processing a large number of features leads in general to more accurate information, but requires more processing time.

A standard solution in data-intensive applications is to select a single data-processing method with a reasonable trade-off between the accuracy and speed, which maximizes the control performance. This is motivated by the fact that most current control design methods assume fixed sensor characteristics, modeled as sensor noise (accuracy) and a measurement delay which limits the fastest sampling period (speed). However, this design choice procedure is always limited by the mentioned speed-accuracy trade-off.

In this paper, we propose to tackle this trade-off by deciding on-line which data-processing method to use in the context of sampled-data control for linear continuous-time plants with stochastic state disturbances [1]. Control-relevant information is obtained through data-processing of measurement data by either a slow method with negligible measurement noise or a fast method with non-negligible measurement noise. The challenge is to decide on-line which data-processing method to use, and which control input to apply to the plant in order to optimize closed-loop performance (see Figure 1). Performance will be measured through quadratic discounted and average cost criteria (compare [2]–[4]). We take a suboptimal approach, since the co-design [2], [5], [6] problem is typically combinatorial and computationally intractable in its full generality. The control input is computed based on a combination of a standard Kalman estimator-predictor and an LQR controller for state estimation and actuation (e.g. [7]), respectively. The switching (i.e. the selection of the processing method) is obtained using approximate dynamic programming algorithms, and, in particular, exploits rollout ideas [2], [8]. The works of [9]–[14] consider related scheduling and control problems.

The main result shows that the proposed switching and control policy yields a better performance than using the optimal control policy corresponding to a single data-processing method. The switching condition manifests itself as an ellipsoidal separation of the state space and is therefore easy to implement on-line.

In Section II, the problem formulation is given. Section III explains the proposed methodology, which is formalized in Section IV and the main result is stated. A numerical example in Section V illustrates the benefits of the proposed method for a second-order system.

II. PROBLEM FORMULATION

This section describes the plant, cost criterion, and the measurement and actuation methods used, leading to the problem formulation.
A. Plant and performance criterion

Consider a continuous-time plant modeled by the following stochastic differential equation

\[ dx_C = (A_C x_C + B_C u_C)dt + B_d dw, \quad x_C(0) = x_0, \quad t \in \mathbb{R}_{\geq 0}, \]

where \( x_C(t) \in \mathbb{R}^{n_\omega} \) is the state and \( u_C(t) \in \mathbb{R}^{n_\omega} \) is the control input at time \( t \in \mathbb{R}_{\geq 0} \), and \( \omega \) is an \( n_\omega \)-dimensional Wiener process with incremental covariance \( I_{\omega \omega} dt \) (cf. [7]).

We assume that \( (A_C, B_C) \) is controllable and \( B_C \) has full rank. The initial condition \( x_0 \) is assumed to be known.

We measure the performance using the discounted cost given by

\[ J^d_C(x_0) := \int_0^\infty \mathbb{E}[e^{-\alpha t}g_C(x_C(t), u_C(t))]dt, \]

or the average cost

\[ J^v_C := \lim_{T \to \infty} \frac{1}{T} \int_0^T \mathbb{E}[g_C(x_C(t), u_C(t))]dt, \]

where \( g_C(x, u) := x^T Q_C x + u^T R_C u \), with positive definite matrices \( Q_C \) and \( R_C \), and \( \alpha_C \in \mathbb{R}_{\geq 0} \).

B. Model of the data processing methods

A digital control platform, which incorporates a data-processing unit and a control unit, is used to measure and control the plant. At sampling times \( t_\ell, \ell \in \mathbb{N} \), with \( t_0 = 0 \), a new sample of raw data pertaining to the plant is taken. At this time, a data-processing method must be selected to distill the information relevant for feedback. For simplicity, we consider that only two data-processing methods are available. Furthermore, we assume that the data-processing methods provide an estimate, in general noisy, of the full state \( x(t_\ell) \).

The two data-processing methods are modeled in terms of the processing time (speed) and the noise characteristics, i.e. the covariance of the state estimate (accuracy). Method 1 is called the ‘slow’ method, and requires a time \( T_1 \) to process data. When the slow method is used at time \( t_\ell \) it will make the data \( x(t_\ell + \nu_\ell) \) available for feedback at time \( t_\ell + T_1 \), where \( \nu_\ell \) is a zero-mean random variable with covariance \( \Phi^\nu_1 \). Method 2 is called the ‘fast’ method and requires a time \( T_2 \) to process data. It makes the data \( x(t_\ell + \nu_\ell) \) available for feedback at time \( t_\ell + T_2 \), where \( \nu_\ell \) is a zero-mean random variable with covariance \( \Phi^\nu_2 \). The fact that Method 1 requires more time to process the raw data but is more accurate than Method 2 is captured by assuming

\[ T_1 > T_2, \quad \Phi^\nu_1 < \Phi^\nu_2. \]

For simplicity, we assume that \( \Phi^\nu_1 = 0 \). Immediately after completion of the data-processing, new raw data can be obtained, and processed by selecting one of the two available methods. Let \( \sigma_\ell \in \{1, 2\}, \ell \in \mathbb{N} \), indicate the processing method selected at time \( t_\ell \). Assuming that other delays, e.g. due to the selection process or raw data acquisition, are negligible, we have

\[ t_{\ell+1} = t_\ell + T_{\sigma_\ell}, \]

We denote the processed measurement by

\[ y_{\ell+1} = x_C(t_\ell) + \nu_\ell, \quad \ell \in \mathbb{N}. \]

The noise variables \( \nu_\ell, \ell \in \mathbb{N} \), are assumed to be zero-mean random with covariance \( \Phi^\nu_{\sigma_\ell} \) and independent and identically distributed for all \( \ell \in \mathbb{N} \).

We will assume that Method 1 used alone (i.e. \( \sigma_\ell = 1 \), for all \( \ell \in \mathbb{N} \)) would lead to a better performance in terms of criteria (2) or (3) than applying Method 2 all the time (i.e. \( \sigma_\ell = 2 \), for all \( \ell \in \mathbb{N} \)). The case when using Method 2 all the time would lead to a better performance is briefly discussed at the end of Section IV in Remark 2.

C. Problem statement

Since measurements become available for feedback at times \( t_\ell \) (pertaining to raw data acquired at time \( t_{\ell-1} \)), it is reasonable to assume that control updates based on this new information also occur only at times \( t_\ell \). A zero-order hold (ZOH) actuation strategy is implemented such that the actuation signal is held constant between sampling times. Hence, \( u_C \) is a staircase signal. We have

\[ u_C(t) = u_C(t_\ell), \quad \ell \in [t_\ell, t_{\ell+1}). \]

Moreover, also at times \( t_\ell \), new raw data is acquired and a switching decision must be taken pertaining to which data processing method to use in the interval \([t_\ell, t_{\ell+1})\).

As a result, we can state the problem we are interested in as follows: Find a switching and control policy (co-design problem), i.e., a set of functions

\[ \pi = \{(\mu^0_0, \mu^0_1), (\mu^1_0, \mu^1_1), \ldots\}, \]

for \( \mu^0_\ell : (\mathbb{R}^{n_\omega})^{\ell} + 1 \times \{1, 2\}^\ell \to \{1, 2\} \) and \( \mu^1_\ell : (\mathbb{R}^{n_\omega})^{\ell} + 1 \times \{1, 2\}^\ell \to \mathbb{R}^{n_\omega} \), that determine the switching and actuation inputs at sampling times \( t_\ell \)

\[ (\sigma_\ell, u_\ell(t_\ell)) = (\mu^0_\ell(l_\ell), \mu^1_\ell(l_\ell)), \quad \ell \in \mathbb{N}, \]

based on the information in the ordered set

\[ l_\ell := \{(y_n, \sigma_n) \mid n \in [0, \ell]\} \cup \{y_\ell\}, \quad \ell \in \mathbb{N}, \]

available to the control platform at decision times \( t_\ell \).

III. Proposed switching and control policy

In this section, we start by providing an overview of the main idea behind the proposed switching and control policy. This will entail (i) designing a state estimator; (ii) specifying the control input; and (iii) specifying the switching policy. Each of these topics is addressed subsequently.

A. Overview of the method

From the problem formulation, we see that a given policy may result in decision times unknown at the initial time. Our policy is to fix a priori the switching decision times to be

\[ s_k = kT_1, \quad k \in \mathbb{N}, \]

and assuming that the processing times are related by an integer number, i.e.

\[ \frac{T_1}{T_2} = n_\tau, \quad n_\tau \in \mathbb{N}_{\geq 1}. \]
This can always be achieved by tuning the processing methods or adding small waiting times.

At each time \( s_k, k \in \mathbb{N} \), our policy decides either to use the slow method or use \( n_\tau \) times the fast method. Selecting the slow method means that the control input at time \( s_k \) is held constant in the interval \( [s_k, s_{k+1}] \), but an accurate measurement will become available at time \( s_{k+1} \). Selecting to use \( n_\tau \) times the fast method means that the control input is updated more frequently, at times \( s_k + k \sigma, h \in \{0, 1, \ldots, n_\tau - 1\} \), based on the \( n_\tau \) corresponding measurements. Note that the measurement available at time \( s_{k+1} \) is then inaccurate (noisy).

The switching decision at time \( s_k \) is conceptually defined as the one that would optimize the expected quadratic performance cost \( \mathbf{2} \) or \( \mathbf{3} \) while supposing that at times \( s_{k+1}, s_{k+2}, \ldots \) the slow method would be picked and the standard optimal LQG control policy would be used. This can be seen as a rollout method, in the context of dynamic programming algorithms [8] (for which the base policy is to select the slow method after the optimization horizon), and is repeated in a receding horizon fashion, i.e., the same procedure is repeated at time \( s_{k+1}, s_{k+2}, \ldots \).

**B. State-estimation**

The state (and delayed) measurements at discrete times \( t_\ell, \ell \in \mathbb{N} \), are described by

\[
\begin{align*}
\xi_{\ell+1} &= A_\ell \xi_\ell + B_\ell u_\ell + \nu_\ell, \quad \ell \in \mathbb{N}, \quad (10) \\
y_{\ell+1} &= \xi_\ell + \nu_\ell, \quad (11)
\end{align*}
\]

where \( \xi_\ell := x_\mathbf{C}(t_\ell) \in \mathbb{R}^{n_x} \) and \( u_\ell := u_\mathbf{C}(t_\ell) \in \mathbb{R}^{n_u} \) are the state and control input, respectively, at discrete time \( \ell \in \mathbb{N} \). Moreover,

\[
A_m := e^{A_C \tau_m}, \quad B_m := \int_0^{\tau_m} e^{A_C s} B_C ds, \quad m \in \{1, 2\}.
\]

We assume that the sampling periods (processing times) \( \tau_1 \) and \( \tau_2 \) are non-pathological, which implies that \( (A_1, B_1) \) and \( (A_2, B_2) \) are controllable 1. The disturbances \( \omega_\ell \) and \( \nu_\ell \) are sequences of zero-mean independent random vectors, \( \omega_\ell \in \mathbb{R}^{n_\omega} \) and \( \nu_\ell \in \mathbb{R}^{n_\nu} \), respectively, with covariances \( \mathbb{E}[\omega_\ell(\omega_\ell)'] = \Phi_\omega \) and \( \mathbb{E}[\nu_\ell(\nu_\ell)'] = \Phi_\nu \), for all \( \ell \in \mathbb{N} \), with \( \Phi_\nu := \int_0^{\tau_m} e^{A_C s} B_C B_\omega' e^{A_\nu} ds, \quad m \in \{1, 2\} \).

As a result, a predicted estimate \( \hat{\xi}_\ell \) of the state \( \xi_\ell \) from the information available until and including time \( t_\ell \) can be given by

\[
\hat{\xi}_\ell = A_{\sigma(t-1)}\hat{\xi}_{\ell-1} + B_{\sigma(t-1)}u_{\ell-1} + L_{\ell-1} (y_\ell - \hat{\xi}_{\ell-1}), \quad (12)
\]

using the time-varying Kalman filter which initial condition chosen as \( \hat{\xi}_0 = \xi_0 = x_0 \). The gains \( L_\ell \), for \( \ell \in \mathbb{N} \), of the estimator-predictor can be computed [7] by

\[
\begin{align*}
L_{\ell} &= A_{\sigma_{\ell}} \Theta_\ell \Theta_\ell + \Phi_{\sigma_{\ell}}^{-1}
\Theta_{\ell+1} &= (A_{\sigma_{\ell}} - L_\ell \Theta_\ell (A_{\sigma_{\ell}} - L_\ell \Theta_\ell)') + L_\ell \Phi_{\sigma_{\ell}}^{-1} \Phi_{\sigma_{\ell}}
\end{align*}
\]

where \( \Theta_\ell \) is the covariance of the state estimate with initial condition \( \Theta_0 = 0 \). For a fixed switching policy, the infinite-horizon solution \( \Theta_m, m \in \{1, 2\} \), is the stationary solution to the Riccati equation (14) when \( \sigma_\ell = m \), for all \( \ell \in \mathbb{N} \), i.e. it is the solution to corresponding the discrete-time algebraic Riccati equation (DARE). The stationary gain \( \Theta_m, m \in \{1, 2\} \) is then given by (13).

**C. Policy for the control input**

We show in this subsection that the feedback policy for the proposed rollout method is given by

\[
u_\ell = -K\xi_\ell, \quad (15)\]

where the control gains \( K_\ell \) for the period \( k \) (i.e. for \( t_\ell \in [s_k, s_{k+1}] \)) are determined at the decision time \( s_k \), independent of the measurements. For the slow method this corresponds to one gain, whereas for the fast method this corresponds to multiple gains.

To provide the expressions for the gains \( K_\ell \), we start by discretizing the cost function (2) with a fixed sampling period \( \tau_m, m \in \{1, 2\} \), corresponding to either the slow or the fast method. We focus for now on the discounted cost (2) and discuss the average cost in Remark 1. It can be shown (using for example the arguments in [7]) that, for \( \xi_0 = x_0 \), this discounted cost (2), apart from an additive constant factor, is given by

\[
J_m^d(\xi_0) := \sum_{l=0}^{\infty} \mathbb{E}[\alpha_m g(\xi_l, u_l, m)], \quad m \in \{1, 2\}, \quad (16)
\]

where \( \alpha_1 := e^{-\alpha C \tau_1}, g(\xi, u, m) := \xi^T Q_m \xi + 2\xi^T S_m u + u^T R_m u \), and, where

\[
\begin{align*}
Q_m &:= \int_0^{\tau_m} e^{-\alpha C s} \sigma_s^T \sigma_s e^{-\alpha C s} ds, \quad R_m := \int_0^{\tau_m} e^{-\alpha C s} I_{n_u} \sigma_s^T \sigma_s e^{-\alpha C s} ds, \quad \sigma_s := [A_C \ B_C] e^{A_C s}, \\
&= \int_0^{\tau_m} e^{-\alpha C s} \sigma_s^T \sigma_s e^{-\alpha C s} ds.
\end{align*}
\]

In accordance with the proposed rollout algorithm (see Section III-A), the control input obtained in the interval \( [s_k, s_{k+1}] \) results from the choice \( O \) of one of two options \( O \in \{opt1, opt2\} \) at time \( s_k \).

In the first option, \( O = opt1 \), the slow method is selected at time \( s_k \) supposing that it will also be selected at times \( s_{k+1}, s_{k+2}, \ldots \). The discounted cost in this case is simply given by (16) when \( m = 1 \), which is then to be optimized by the control inputs. Thus, the control inputs follow from the standard optimal control policy for the discrete-time LQG problem resulting from minimizing (16) for (10) when \( m = 1 \) and \( \sigma_\ell = 1 \) for every \( \ell \in \mathbb{N} \) (taken into account our assumption on controllability and positive definiteness for the cost). The control input of this policy is given by (see, e.g., [7], [8])

\[
u_\ell = -K\xi_\ell,
\]

where

\[
\begin{align*}
K &= G^{-1}(B_1^T \hat{P} A_1 + (S_1)^T) \\
\hat{P} &= \alpha_1 A_1^T \hat{P} A_1 + Q_1 - K^T G K \\
G &= (R_1 + \alpha_1 B_1^T \hat{P} B_1),
\end{align*}
\]

1 The sampling period \( \tau_m \) is non-pathological if \( A_C \) does not have two eigenvalues with equal real parts and imaginary parts that differ by an integer multiple of \( \frac{\pi}{\tau_m} \) (cf. [1, p. 45]).
In the second option, $O = \text{opt}2$, the fast method is selected at the $n_τ$ times $s_k, s_{k+1}, \ldots, s_k + (n_τ - 1)τ_2$ assuming that the slow method will be selected at times $s_{k+1}, s_{k+2}, \ldots$. The optimal control policy can still be derived from standard LQG arguments for time-varying systems, since the scheduling decisions are fixed.

For the first option, the cost-to-go (22) at decision time $t_{ℓ} = s_k$ is given by

$$J_{\text{opt}1}(\xi_{ℓ}, Θ_{ℓ}) = \xi_{ℓ}^T P_{ℓ} ξ_{ℓ} + Χ_{\text{opt}1}(Θ_{ℓ}),$$

where $Θ_{ℓ} = Θ_{0}$. For the second option, the cost-to-go (22) at decision time $t_{ℓ} = s_k$ is given by

$$J_{\text{opt}2}(\xi_{ℓ}, Θ_{ℓ}) = \xi_{ℓ}^T P_{ℓ} ξ_{ℓ} + Χ_{\text{opt}2}(Θ_{ℓ}),$$

where $Χ_{\text{opt}2}(Θ_{ℓ}) = tr(Θ_{0}^k (P_{0} + K_{0}^T G_{0} K_{0})) + \sum_{h=1}^{n_{τ} - 1} [α_h tr(Φ_{0}^h P_{h} + Θ_{h}^k K_{h}^T G_{h} K_{h})] + α_{τ} tr(Φ_{0}^{τ} P + Θ_{τ} K_{τ}^T G_{τ} K_{τ}),$ (26)

and where $Θ_{k}^h$ for $h \in \{0, \ldots, n_τ - 1\}$ follows from (14) with initial condition $Θ_{0}^0 = Θ_{0}$.

For $\text{opt}1$ and $\text{opt}2$ the costs are given by (23) and (25), respectively. The switching signal, in accordance with Section III-A, is then given by

$$σ_{ℓ} = Π_{k} \ \text{for all } t_{ℓ} \in [s_k, s_{k+1}),$$

where the proposed switching decisions $Π_k \in \{1, 2\}$ are decided at the switching times $t_{ℓ}$.

The optimal cost-to-go for fixed data processing methods $O_{ℓ} \in \{\text{opt}1, \text{opt}2\}$, denoted as

$$J_{O_{ℓ}}^{d} = \E[J_{O_{ℓ}}^{d}(ξ_{ℓ}) | I_{t_{ℓ}}],$$

at each decision time, i.e. when $t_{ℓ} = s_k$ for any $k \in \mathbb{N}$. This is then repeated for every time $s_k, k \in \mathbb{N}$ in a receding horizon fashion.

For the first option, the cost-to-go (22) is now given by

$$J_{\text{opt}1}(\xi_{ℓ}, Θ_{ℓ}) = ξ_{ℓ}^T P_{ℓ} ξ_{ℓ} + Χ_{\text{opt}1}(Θ_{ℓ}),$$

where $Χ_{\text{opt}1}(Θ_{ℓ}) = tr(Θ_{0}^k (P_{0} + K_{0}^T G_{0} K_{0})) + \sum_{h=1}^{n_{τ} - 1} [α_h tr(Φ_{0}^h P_{h} + Θ_{h}^k K_{h}^T G_{h} K_{h})] + α_{τ} tr(Φ_{0}^{τ} P + Θ_{τ} K_{τ}^T G_{τ} K_{τ}),$ (24)

with $Θ_{0} = Θ_{ℓ}$.

The discount up to time $t_{ℓ}$ does not affect the decision and is omitted.

For the second option, the cost-to-go (22) at decision time $t_{ℓ} = s_k$ is given by

$$J_{\text{opt}2}(\xi_{ℓ}, Θ_{ℓ}) = ξ_{ℓ}^T P_{ℓ} ξ_{ℓ} + Χ_{\text{opt}2}(Θ_{ℓ}),$$

where

$$Χ_{\text{opt}2}(Θ_{ℓ}) = tr(Θ_{0}^k (P_{0} + K_{0}^T G_{0} K_{0})) + \sum_{h=1}^{n_{τ} - 1} [α_h tr(Φ_{0}^h P_{h} + Θ_{h}^k K_{h}^T G_{h} K_{h})] + α_{τ} tr(Φ_{0}^{τ} P + Θ_{τ} K_{τ}^T G_{τ} K_{τ}),$$

and $Θ_{0} = Θ_{ℓ}$.

The discount up to time $t_{ℓ}$ does not affect the decision and is omitted.
IV. PERFORMANCE ANALYSIS

We are now in the position to present our main result. Let \( J_d^d(\xi_0) \), \( J_a^a(x_0) \) denote the discounted cost (2) and average cost (3), respectively, when the proposed policy, characterized by (12), (21), (28) is applied to the plant (1).

**Theorem 1:** Consider the plant (1) with two possible sensor data-processing methods providing measurements (5) at times (4), and a digital controller, which estimates the state at sampling times \( t_\ell, \ell \in \mathbb{N} \), according to (12), provides the control input according to (21), and selects on-line the data processing method at times \( s_k = k \tau_1, k \in \mathbb{N} \) according to (28). Then

\[
J_d^d(\xi_0) \leq J_1^d(\xi_0), \quad \text{for all } \xi_0 \in \mathbb{R}^n,
\]
and

\[
J_a^a(x) \leq J_1^a(x).
\]

Moreover,

\[
\Pi_2 \preceq \Pi_1,
\]
and therefore the switching policy (28), at switching decision time \( t_\ell = s_k \), can be written as

\[
\sigma_k = \begin{cases} 
1, & \text{if } \xi_k^T \Omega \xi_k \leq \eta_2(\Theta_k) - \eta_1(\Theta_k), \\
2, & \text{otherwise},
\end{cases}
\]

with constant positive semidefinite matrix \( \Omega := \Pi_1 - \Pi_2 \).

The theorem states that both the discounted and average costs of the proposed policy are not larger than the corresponding costs of applying the slow method at every time step. The proof is omitted for brevity.

The fact that \( \Pi_2 \preceq \Pi_1 \) implies that the fast data processing method is selected if the state estimate lies outside an ellipsoid in the state space, and the slow method is selected otherwise. Note that if \( \sigma_k = 1 \), then \( \Theta_\ell = \Phi_1^\ell \) at \( t_\ell = s_{k+1} \), i.e., the variance (and therefore the ellipsoid) ‘resets’.

**Remark 2:** In this paper, we have assumed that the fixed policy for Method 1 is inherently better than the fixed policy for Method 2. Therefore, we selected Method 1 as the base policy. If Method 2 would be better, we would choose Method 2 as the base policy. The proposed methodology can be adapted to account for this.

V. SIMULATION

Here, we show simulation results for a second-order system. In particular, we consider the system (1) given by

\[
A_C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B_C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_\omega = \begin{bmatrix} 0.81 \\ 0.30 \end{bmatrix},
\]

and a cost given by (3) and

\[
Q_C = \begin{bmatrix} 0.1 & 0 \\ 0 & 10 \end{bmatrix}, \quad R_C = 1.
\]

We define the base sampling time to be \( \tau_s = 1 \), and \( n_\tau = 3 \) for the sampling time ratio in (9). Hence, the system is discretized with two sampling periods \( \tau_1 = \tau_s \) and \( \tau_2 = \tau_s/3 \) to get (10)-(11), which correspond to two data-processing algorithms \{Alg1, Alg2\}. The measurement noise on Alg2 is given by

\[
\Phi_2^\omega = \begin{bmatrix} 0 & 0 \\ 0 & 9 \end{bmatrix}.
\]  

The expected average costs of the fixed policies can be computed (from (30)) to be \( J_1^a = 3.1319 \cdot 10^3 \) and \( J_2^a = 3.1774 \cdot 10^3 \), i.e. Alg2 performs 1.45% worse. This shows that persistent use of Alg2 would perform worse than using only Alg1, which is consistent with the assumptions made in our problem set-up.

Figure 2 shows ellipsoids representing the switching condition, based on the average cost, for various values of the covariance \( \Theta_t \) of the state estimate. The outer black ellipsoid is for the case \( \Theta_\ell = \Phi_1^\ell \) at \( s_k = t_\ell \), i.e., the previous measurement was taken using Alg1. The inner black ellipsoids are further iterations if Alg2 is used multiple times consecutively. When Alg1 is used, the ellipsoid resets to the outer black ellipsoid. The cyan-colored ellipsoid is generated from \( \Theta_\ell = \Phi_2^\ell \), i.e. the non-switching solution for Alg2. For a given system, the initial ellipsoid results from \( \Theta_0 = 0 \).

We run 10 Monte Carlo simulations of the system. We use a small initial condition near the origin \( x_0 \approx [0, 0]^T \). We take \( k \in \mathbb{N}_{[0,10000]} \) decision moments, since we are interested in the average cost.

In Figure 3, the state (green), and the estimate of the state (red/blue) are shown for one simulation. The color indicates the switching decision, red is Alg2 and blue is Alg1. The black lines are part of the outer ellipsoid in Figure 2. For this case, from Figure 3, we see that outside the black ellipsoid Alg1 is never selected. Also, occasionally, it happens that Alg2 is chosen inside the ellipsoid due to the scaling of the ellipsoids, thereby explaining the occurrence of red circles inside the region with mainly blue markers. The green markers indicate the true state at the decision moments.

The disturbances occasionally push the state outside the region where using only Alg1 is optimal, i.e. outside the black ellipsoid. This happens even though we have an initial
In this paper, we proposed a switching and control policy that breaks the speed versus accuracy trade-off that is encountered in many data-intensive or vision-based control applications having both slow but accurate data-processing methods and fast but coarse data-processing methods to extract state information out of the (big) measurement data. We formally showed that a lower expected cost is achieved compared by our policy to when no switching is used. In fact, simulation results demonstrated that a profit of around 15% can already be achieved for a simple second-order system, and we expect that even larger gains in performance can be realized for more complex systems. Our results can be extended to more than two data processing methods in a straightforward manner.

The result (28) can readily be extended to the case \( m \in \{1, \ldots, M\} \) with \( M > 2 \). However, this situation does not yield an ellipsoidal separation of the state space.

### REFERENCES