Control of mechanical motion systems with non-collocation of actuation and set-valued friction: theory and experiments

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Abstract

The presence of friction in mechanical motion systems is a performance limiting factor as it induces stick-slip vibrations. To appropriately describe the stiction effect of friction, we adopt set-valued force laws. Then, the complete motion control system can be described by a Lur'e system with set-valued nonlinearities. In order to eliminate stick-slip vibrations for mechanical motion systems, a state-feedback control design is presented to stabilize the equilibrium. The proposed control design is based on an extension of a Popov-like criterion to systems with set-valued nonlinearities that guarantees input-to-state stability (ISS). The advantages of the presented controller is that it is robust to uncertainties in the friction and it is applicable to systems with non-collocation of actuation and friction where common control strategies such as direct friction compensation fail. Moreover, an observer-based output-feedback design is proposed for the case that not all the state variables are measured. The effectiveness of the proposed output-feedback control design is shown both in simulations and experiments for a typical motion control system.

1 Introduction

In many mechanical motion systems, the presence of friction gives rise to undesired behavior such as steady-state positioning errors, large settling times and stick-slip vibrations [Armstrong-Hélouvry, 1991, Armstrong-Hélouvry et al., 1994, Olsson et al., 1998, Canudas de Wit et al., 1995]. Especially, the friction-induced stick-slip vibrations lead to kinetic energy dissipation, noise, excessive wear, premature failure of machine parts and inferior positioning performance. Research on the presence of friction-induced stick-slip vibrations is conducted for different mechanical systems; e.g. drilling systems [Jansen and van den Steen, 1995, Navarro-López and Suárez, 2004], flexible rotor systems [Mihajlović et al., 2006], robots [Jeon and Tomizuka, 2005], servo systems [Olsson and Aström, 2001], turbine blade dampers [Pfeiffer and Hajek, 1992] etc. In this paper, a control strategy for mechanical motion systems is proposed in order to eliminate the undesired friction-induced stick-slip vibrations and to guarantee stability of the desired setpoint. The proposed control strategy is applied to an experimental setup and its effectiveness is shown in experiments. To properly describe the stiction effect in dry friction, set-valued friction models are commonly used [Brogliato, 2004, Glocker, 2001, Leine and Nijmeijer, 2004], which lead to dynamic models in terms of differential inclusions [Brézis, 1973, Aubin and Cellina, 1984, Filippov, 1988].

A common approach to tackle motion control problems for systems with (set-valued) friction is the application of direct friction compensation techniques, see e.g. [Armstrong-Hélouvry, 1991, Armstrong-Hélouvry et al., 1994, Olsson et al., 1998, Southward et al., 1991, Swevers et al., 2000] and many others. Since friction characteristics are known to be sensitive to temperature, humidity etc., it may be hard to obtain accurate friction models with limited complexity suitable for a compensation scheme. Furthermore, the absence of accurate friction models has been shown to be a performance limiting factor in employing friction compensation in practice [Canudas de Wit, 1993, Mallon et al., 2006], leading to limit cycles and steady-state errors. This indicates a first drawback of a typical compensation technique that it is not robust to friction uncertainties. Moreover, common friction compensation schemes are typically applied when the actuation and friction are collocated, meaning that the friction force and the actuation force act at the same place and the friction can be compensated directly (if an accurate friction model is available). However, this is not the case for...
many mechanical systems. Here, we are interested in a feedback control approach for motion systems with non-collocation of actuation and friction, that is inherently robust to uncertainties in the friction characteristic.

A robust compensation approach is discussed in [Taware et al., 2003], where they consider a motor-load system with, possibly discontinuous friction at the load and joint flexibility and damping (between the motor and the load). However, the model reference adaptive control scheme does not guarantee a zero tracking error. Many well-known control strategies are available for nonlinear systems, that can deal with the non-collocation of certain nonlinearities and the actuation, such as feedback linearization techniques or backstepping techniques [Nijmeijer and van der Schaft, 1990, Khalil, 2002]. These techniques rely upon the smoothness of the system to be controlled and such a property is not satisfied by motion systems with Coulomb friction. A variable structure control design is presented in [Kwatny et al., 2002] for systems with nonsmooth uncertainties, where the intended application is for systems in which the uncertain friction forces are relatively small. Since for motion systems exhibiting friction-induced vibrations the friction forces are relatively large, the latter approach is not directly applicable. Another approach for controller design can be based on the well-known absolute stability theory, using circle or Popov criteria (see e.g. [Khalil, 2002]). In particular, stabilization techniques based on absolute stability theory for locally Lipschitzian systems with slope-restricted nonlinearities are discussed in [Arcak and Kokotović, 2001, Arcak et al., 2003], which are not applicable to the systems under study since they contain set-valued nonlinearities to accurately describe the friction. A generalized circle criterion, that is suitable for systems with set-valued nonlinearities, is discussed in [Brogliato, 2004]. Unfortunately, the conditions of the circle criterion are rather restrictive for typical motion control applications as will be indicated in this paper.

In this paper, we present a generalization of the Popov-like criterion, in the sense that it is applicable to systems with set-valued nonlinearities. Moreover, we obtain input-to-state stability (ISS) (instead of only asymptotic stability) with respect to perturbations on the system (e.g. measurement noise). In analogy with the “absolute stability” property, obtained by satisfaction of the conventional Popov-criterion, one might call this property “absolute ISS”. The concept of absolute ISS is used for the design of a state-feedback controller for mechanical motion systems with set-valued nonlinearities that can be described by Lur’e-type systems, i.e. linear systems with continuous nonlinearities in the feedback loop. An advantage of the proposed control design is that it is applicable to systems with non-collocation of actuation and set-valued friction laws, a situation that has not been studied in literature before, at least not in the generality as presented here. Moreover, the fact that the satisfaction of such an adapted Popov criterion guarantees absolute ISS implies robustness with respect to uncertainties in the friction and measurement errors. Next, the ISS property is used to construct an observer-based output-feedback controller c.f. [Doris et al., 2008]. Moreover, we provide a separation principle for the output-feedback controlled system, i.e. the controller and the observer can be designed separately.

The notion of ISS [Sontag, 1989] is a useful property in the field of control, which ensures that the state of the system is bounded for a bounded input. In [Arcak and Teel, 2002], a proof of ISS for Lur’e-type systems is given, which is not applicable to systems with set-valued modeled friction laws. Next to the fact that the usually work on ISS considers continuous systems, they focus typically on the use of smooth ISS Lyapunov functions (see e.g. [Arcak and Teel, 2002, Sontag and Wang, 1995, Sontag, 1989] to mention just a few). In case of extending the Popov criterion to the discontinuous systems as considered here, one has to adopt non-smooth (ISS) Lyapunov functions. The reason of non-smoothness is that the Lyapunov function contains a term consisting of an integral of the nonlinearity. Despite some recent attempts, see [Cai and Teel, 2005, Vu et al., 2007, Heemels et al., 2007], to bring ISS concepts to the realm of discontinuous and switched systems, none of these papers can be used in the present context.

Much experimental work is performed on the control of systems with collocation of actuation and friction, e.g. in the field of friction compensation for an industrial hydraulic robot [Lischinsky et al., 1999], a KUKA robot [Swevers et al., 2000], a robotic gripper [Johnson and Lorenz, 1992] etc. However to the best of our knowledge, no experimental work is performed for stabilization of setpoints for systems with non-collocation of actuation and set-valued modeled friction. Here, we will apply the proposed output-feedback controller to an experimental rotor dynamic system, which represents a typical motion control example of a motor-load system with non-collocation of actuation and set-valued friction laws. Earlier work [Mihajlović et al., 2006] has shown that this system exhibits stick-slip limit cycling. The control proposed here will eliminate these limit cycles. Moreover, it will be shown that the circle criterion is not feasible for this system implying that the extension of the Popov criterion is indispensable within this context. The effectiveness of the designed output-feedback controller is shown in simulations and experiments.

The structure of this paper is as follows: we start with some notation in Section 2. In Section 3, we introduce models that include set-valued nonlinearities in their structure, representing a large class of mechanical motion systems with dry friction. The control designs are presented in Section 4, where we discuss the state-feedback control design with the generalization of the Popov criterion and the absolute ISS property, the observer design and the output-feedback control design. A rotor dynamic system is presented in Section 5 as an example of a mechanical system with non-collocated
actuation and set-valued friction laws and the results of the application of the output-feedback control design are shown in simulations and experiments. We finish this paper with conclusions in Section 6.

2 Notations and definitions

A function \( u : \mathbb{R}_+ \rightarrow \mathbb{R}^n \) is piecewise continuous, if on every bounded interval the function has only a finite number of points at which it is discontinuous. Without loss of generality we will assume that every piecewise continuous function \( u \) is right continuous, i.e. \( \lim_{t \downarrow s} u(t) = u(s) \) for all \( t \in \mathbb{R}_+ \). With \( \| \cdot \| \) we will denote the usual Euclidean norm for vectors in \( \mathbb{R}^n \), and \( \| \cdot \|_1 \) denotes the 1-norm. A function \( \gamma : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) is of class \( K \) if it is continuous, strictly increasing and \( \gamma(0) = 0 \). It is of class \( K_{\infty} \) if, in addition, it is unbounded, i.e. \( \gamma(s) \rightarrow \infty \) as \( s \rightarrow \infty \). A function \( \beta : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) is of class \( K \mathcal{L} \) if, for each fixed \( t \in \mathbb{R}_+ \), the function \( \beta(\cdot, t) \) is of class \( K \), and for each fixed \( s \in \mathbb{R}_+ \), the function \( \beta(s, \cdot) \) is decreasing and tends to zero at infinity. \( \lambda_{\text{min}}(A), \lambda_{\text{max}}(A) \) denote the minimal and maximal eigenvalue of the matrix \( A \), respectively. A differential inclusion is given by an expression of the form

\[
\dot{x}(t) \in F(x(t), e(t)),
\]

where \( F \) is a set-valued mapping and with the state \( x \in \mathbb{R}^n \) and the input \( e \in \mathbb{R}^m \). An absolutely continuous function \( x \) is considered to be a strong solution of the differential inclusion (1) if (1) is satisfied almost everywhere.

Definition 1 [Sontag, 1995] The system (1) is said to be input-to-state stable (ISS) if there exist a function \( \beta \) of class \( K \mathcal{L} \) and a function \( \gamma \) of class \( K \) such that for each initial condition \( x(0) = x_0 \) and each piecewise continuous bounded input function \( e \) defined on \( [0, \infty) \),

- all solutions \( x \) of the system (1) exist on \( [0, \infty) \) and
- all solutions satisfy

\[
\|x(t)\| \leq \beta(\|x_0\|, t) + \gamma \sup_{\tau \in [0,t]}(\|e(\tau)\|), \quad \forall t \geq 0.
\]

The system is called globally asymptotically stable (GAS) if the above holds for \( e = 0 \).

Consider the following linear system

\[
\begin{align*}
\dot{x} &= Ax + Gw \\
z &= Hx + Dw,
\end{align*}
\]

with the state \( x \in \mathbb{R}^n \), input and output \( w, z \in \mathbb{R}^p \).

Definition 2 The system (3) or the quadruple \((A, G, H, D)\) is said to be strictly passive if there exist an \( \varepsilon > 0 \) and a matrix \( P = P^T > 0 \) such that

\[
\begin{bmatrix}
A^T P + PA + \varepsilon I & PG - H^T \\
G^T P - H & -D - D^T
\end{bmatrix} \leq 0.
\]

3 Mechanical motion systems with set-valued friction laws

A large class of mechanical motion systems with dry friction can be described by the following second-order form [Glocker, 2001]:

\[
M\ddot{q} + D\dot{q} + Kq = Su + T\lambda_T,
\]

with the generalized coordinates \( q \in \mathbb{R}^{n/2} \), the control input \( u \in \mathbb{R}^n \), friction forces \( \lambda_T \in \mathbb{R}^p \) and the mass, damping and stiffness matrices \( M \in \mathbb{R}^{n/2 \times n/2}, D \in \mathbb{R}^{n/2 \times n/2} \) and \( K \in \mathbb{R}^{n/2 \times n/2} \), respectively. The matrices \( S \in \mathbb{R}^{n/2 \times m} \) and \( T \in \mathbb{R}^{n/2 \times p} \) represent the generalized force directions of the actuation and friction, respectively. We adopt the following set-valued friction law (cf. [Glocker, 2001]) for the \( i \)-th frictional contact \( \lambda_{T,i} \):

\[
\lambda_{T,i} \in -\mu_i\lambda_{N,i}\operatorname{Sign}(T_i^{\top}\dot{q}) + F_{S_i,i}(T_i^{\top}\dot{q})
\]

for \( i = 1, \ldots, p \).

Herein, \( T_i \) represents the \( i \)-th column of \( T \) and \( T_i^{\top}\dot{q} \) is the sliding velocity in contact \( i \). Note that the sliding velocity and the friction forces are aligned and, therefore, the friction laws are a function of \( T_i^{\top}\dot{q} \). The first part in (6) reflects a set-valued Coulomb friction law with

\[
\operatorname{Sign}(y) \triangleq \begin{cases}
\{-1\}, & y < 0 \\
[-1, 1], & y = 0 \\
\{1\}, & y > 0
\end{cases}
\]

Moreover, \( \mu_i \) and \( \lambda_{N,i} \) are the Coulomb friction coefficient and the normal force in contact \( i \), respectively. The second contribution to the friction law (6) is the smooth function \( F_{S_i,i}(T_i^{\top}\dot{q}) \), which models the velocity dependency of the friction. The equations (5) and (6) together constitute a differential inclusion which can be written in the following state-space form:

\[
\begin{align*}
\dot{x} &= Ax + Gw + Bu \\
z &= Hx \\
y &= Cx \\
w &\in -\varphi(z),
\end{align*}
\]

where \( x = [\dot{q}^{\top} \dot{\dot{q}}^{\top}]^{\top} \in \mathbb{R}^n \) is the state system, \( w \in \mathbb{R}^p \) is the output, \( z \in \mathbb{R}^p \), with \( z = T^{\top}\dot{q} \), is the input of a set-valued nonlinearity \( \varphi \in \mathbb{R}^m \) is the control input and \( y \in \mathbb{R}^n \) is the measured output. The matrices \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, G \in \mathbb{R}^{n \times p} \) and \( H \in \mathbb{R}^{p \times n} \) are given by

\[
A = \begin{bmatrix}
0 & I \\
-M^{-1}K - M^{-1}D
\end{bmatrix},
\quad
G = \begin{bmatrix}
0 \\
M^{-1}T
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
0 \\
M^{-1}S
\end{bmatrix},
\quad
H = \begin{bmatrix}
0 & T^{\top}
\end{bmatrix},
\]

and \( C \in \mathbb{R}^{n \times n} \) is indicating the measured output. Finally, the nonlinearity \( \varphi = [\varphi_1(z_1) \ldots \varphi_p(z_p)]^{\top} \) is defined by

\[
\varphi_i(z_i) = -F_{f,i}(z_i) \quad i = 1, \ldots, p.
\]
Note that if the image of $T$ (respectively $G$) is not contained in the image of $S$ (respectively $B$), then the actuation and the friction are non-collocated (at least partly) and direct friction compensation is impossible. The state-space equations (8) are in Lur'e-type form, which means that the system consists of a linear system (8a), (8b), (8c) with the set-valued nonlinearity (8d) in the feedback loop.

4 Control design for Lur'e-type systems

In this section, we design controllers for systems in the form (8) aiming at the stabilization of the origin $x = 0$. We first state the following assumptions on the properties of the set-valued nonlinearity $\varphi(z)$ in (8d).

**Assumption 3** The set-valued nonlinearity $\varphi : \mathbb{R}^p \to \mathbb{R}^p$ satisfies

- $0 \in \varphi(0)$;
- $\varphi$ is upper semicontinuous (see [Aubin and Cellina, 1984]) and locally integrable with respect to $z$;
- $\varphi$ is decomposed as $\varphi(z) = [\varphi_1(z_1), ..., \varphi_p(z_p)]^T$, $z = [z_1, ..., z_p]$ and $\varphi_i : \mathbb{R} \to \mathbb{R}$, for $i = 1, ..., p$;
- for all $z_i \in \mathbb{R}$ the set $\varphi_i(z_i) \subseteq \mathbb{R}$, $i = 1, ..., p$, is non-empty, convex, closed and bounded;
- each $\varphi_i$ satisfies the $[0, \infty]$ sector condition in the sense that $z_i w_i \leq 0$ for all $w_i \in -\varphi_i(z_i)$ for $i = 1, ..., p$; (11)
- there exist positive constants $\gamma_1$ and $\gamma_2$ such that for $w \in \varphi(z)$ and for any $z \in \mathbb{R}^p$ it holds that $\|w\| \leq \gamma_1 \|z\| + \gamma_2$. (12)

The input functions $u(\cdot)$ are assumed to be in the space of piecewise continuous bounded functions from $[0, \infty)$ to $\mathbb{R}^m$, denoted by $PC^\infty$. Clearly, the nonlinear function $(t, x) \mapsto Ax - G\varphi(Hx) + Bu(t)$ is upper semicontinuous on intervals, where $u$ is continuous and attains non-empty, convex, closed and bounded set-values. From [Aubin and Cellina, 1984, p. 98] or [Filippov, 1988, § 7], it follows that local existence of solutions is guaranteed given an initial state $x_0$ at initial time $0$. Due to the growth condition (12), finite escape times are prevented and thus any solution to (8) is globally defined on $[0, \infty)$. Hence, solutions $x(\cdot)$ and also $z(\cdot) = Hx(\cdot)$ are absolutely continuous functions. Note that $0 \in \varphi(0)$ implies that the origin $x = 0$ is an equilibrium of system (8) for input $u = 0$.

4.1 State-feedback control

To stabilize the origin $x = 0$ of system (8), we propose a linear static state-feedback law (assuming $C = I$ and, therefore, $y = x$ for this case) where we take the measurement error $e$ into account, which is piecewise continuous and bounded:

$$u = K(x - e). \quad (13)$$

Here, $K \in \mathbb{R}^{m \times n}$ is the control gain matrix. Consequently, the resulting closed-loop system is described by the following differential inclusion:

$$\dot{x} = (A + BK)x + Gw - BK\epsilon$$

$$\dot{z} = Hx$$

$$w \in -\varphi(z). \quad (14b)$$

The transfer function $G_c(s)$ from the input $w$ to the output $z$ of system (14) is given by

$$G_c(s) = H(sI - (A + BK))^{-1}G, \ s \in \mathbb{C}. \quad (15)$$

The intended control goal here is to render the closed-loop system (14) “absolute ISS” with respect to $e$, as formalized below, by means of a proper choice of the control gain $K$.

**Definition 4** We call a system (14) absolute ISS with respect to input $e$, if the system (14) is ISS with respect to input $e$, as in Definition 2, for any $\varphi$ satisfying Assumption 3.

To obtain sufficient conditions to guarantee that system (14) is absolutely ISS, we use, as in [Khalil, 2002] for smooth systems, a so-called dynamic multiplier with transfer function $M(s)$ given by

$$M(s) = I + \Gamma s, \ s \in \mathbb{C}, \quad (16)$$

where $\Gamma = \text{diag}(\eta_1, ..., \eta_p) \in \mathbb{R}^{p \times p}$, with $\eta_i > 0 \text{ for } i = 1, ..., p$. The inverse of $M(s)$ will be chosen to be passive, because (as we will explain later) the multiplication of the set-valued nonlinearity in (14) with the inverse of the dynamic multiplier must yield a passive system. A cascade that represents system (14) together with the multiplier $M(s)$ is shown in Figure 1(a). Using the dynamic multiplier $M(s)$ we aim to transform the original system into a feedback interconnection of two passive systems (with the perturbation input $e$), as is done in [Khalil, 2002] and [Arcak et al., 2003] for systems with Lipschitz continuous nonlinearities.

In state-space formulation, the interconnected system $\Sigma_1, \Sigma_2$ takes the following form:

$$\Sigma_1 = \left\{ \begin{array}{l} \dot{\tilde{x}} = (A + BK)x + Gw - BK\epsilon(t) \\ \tilde{z} = \tilde{H}x + Dw + \tilde{Z}\epsilon(t) \end{array} \right. \quad (17a)$$

$$\Sigma_2 = \left\{ \begin{array}{l} \dot{\tilde{z}} = -\Gamma^{-1}z + \Gamma^{-1}\tilde{z} \\ w \in -\varphi(z) \end{array} \right. \quad (17b)$$

See also Figure 1. Herein, $\tilde{z} \in \mathbb{R}^p$ and the matrices $\tilde{H} \in \mathbb{R}^{p \times n}, \ D \in \mathbb{R}^{p \times p}$ and $\tilde{Z} \in \mathbb{R}^{p \times n}$ can be derived from the fact that $\tilde{z} = z + \Gamma \tilde{z}$ (due to the choice of the multiplier $M(s)$ as in (16)) and hence,

$$\tilde{H} = H + \Gamma H(A + BK),$$

$$\tilde{D} = \Gamma HG, \ \tilde{Z} = -\Gamma HBK. \quad (18)$$

The following theorem states sufficient conditions under which system (14) is ISS with respect to input $e$ for any $\varphi \in [0, \infty)$, i.e. the system (14) is absolutely ISS.
Theorem 4.1 Consider system (14) and suppose there exists a diagonal matrix \( \Gamma = \text{diag}(\eta_1, ..., \eta_p) \in \mathbb{R}^{p \times p} \) with \( \eta_i > 0, i = 1, ..., p \), such that \( (A + BK, G, \bar{H}, \bar{D}) \) with \( \bar{G} \) and \( \bar{H} \) as in (18) is strictly passive, then system (14) is absolute ISS with respect to input \( e \) for any \( \varphi \) satisfying Assumption 3.

The proof of Theorem 4.1 is given in Appendix A. We also note that in [Yakubovich et al., 2004], frequency-domain conditions (including Popov-type conditions) guaranteeing a property close to GAS are stated for Lur’e-type systems with discontinuous nonlinearities, where the discontinuity points may not condense on a finite interval. Here, we provide a Popov-like criterion for systems with integrable set-valued nonlinearities (where discontinuity points may condense on a finite interval) that guarantees ISS with respect to input \( e \).

An advantage of achieving absolute stability is the robustness to uncertainties in the nonlinearity \( \varphi \) in the feedback loop, i.e. uncertainties in the friction models if one considers mechanical motion systems as in (5) and (6). Note that if the input \( e \) is zero, the origin \( x = 0 \) of system (14) is absolutely stable under the conditions of Theorem 4.1.

4.2 Observer design

Following [Doris et al., 2008], we propose the following observer for the system (8)

\[
\begin{align*}
\dot{\hat{x}} &= (A - LC)\hat{x} + Gw + Bu + Ly \\
\hat{w} &\in -\varphi(\hat{z}) \\
\hat{z} &= (H - NC)\hat{x} + Ny \\
\hat{y} &= C\hat{x}.
\end{align*}
\]

with the observer gains \( N \in \mathbb{R}^{p \times k} \) and \( L \in \mathbb{R}^{n \times k} \). At this point, we state an additional assumption on the set-valued nonlinearity \( \varphi(\cdot) \) of system (8):

Assumption 5 The set-valued nonlinearity \( \varphi : \mathbb{R}^p \to \mathbb{R}^p \) is such that \( \varphi \) is monotone, i.e. for all \( z_1 \in \mathbb{R}^p \) and \( z_2 \in \mathbb{R}^p \) with \( w_1 \in \varphi(z_1) \) and \( w_2 \in \varphi(z_2) \), it holds that \( \langle w_1 - w_2, z_1 - z_2 \rangle \geq 0 \), where \( \langle \cdot, \cdot \rangle \) denotes the inner product in \( \mathbb{R}^p \).

Since the right-hand side of (19a) is again upper semi-continuous in \((t, x)\) due to continuity of \( y \) and piecewise continuity of \( u \), using Assumptions 3 and 5 on \( \varphi \) it can be shown that there exist global solutions of (19) [Filippov, 1988, Aubin and Cellina, 1984]. Knowing that both the plant and the observer have global solutions, the dynamics for the observer error \( e := x - \hat{x} \) is given by

\[
\begin{align*}
\dot{e} &= (A - LC)e + G(w - \hat{w}) \\
\hat{w} &\in -\varphi(He) \\
\hat{z} &\in -\varphi(He + N(y(t) - C\hat{x})).
\end{align*}
\]

The problem of the observer design is finding the gains \( L \) and \( N \) such that all solutions to the observer error dynamics converge exponentially to the origin, which implies that \( \lim_{t \to \infty} |\hat{x}(t) - x(t)| = 0 \).

Theorem 4.2 [Doris et al., 2008] Consider system (8) and the observer (19) with \((A - LC, G, H - NC, 0)\) strictly passive and the matrix \( G \) being of full column rank. Then, the point \( e = 0 \) is a globally exponentially stable equilibrium point of the observer error dynamics (20) for any \( \varphi(\cdot) \) satisfying Assumptions 3 and 5.

4.3 Output-feedback control

In this section, an observer-based output-feedback controller is presented, where we use the observer design, presented in the previous section, to estimate the system state. Next, the estimated state \( \hat{x} \) is fed back to the system (8) with \( u = K\hat{x} = K(x - e) \) as in (13), where \( e \) now represent the estimation error. Application of the observer-based output-feedback controller results in an interconnection of system (14) and system (20), which is depicted in Figure 2. We aim to prove global asymptotic stability (GAS) of the equilibrium \((x, e) = (0, 0)\) of the interconnected system (14),(20).

Theorem 4.3 Consider system (14) and observer (19).

Suppose there exists a matrix \( \Gamma = \text{diag}(\eta_1, ..., \eta_p) \in \mathbb{R}^{p \times p} \)
with \( \eta_i > 0 \), \( i = 1, \ldots, p \), such that \((A + BK, G, \tilde{H}, \tilde{D})\) is strictly passive with \( \tilde{G} \) and \( \tilde{H} \) as in (18). Moreover, suppose, \((A - LC, G, H - NC, 0)\) is strictly passive and \( G \) being full column rank. Then, \((x, e) = (0, 0)\) is a globally asymptotically stable equilibrium point of the interconnected system (14), (20) for any \( \varphi(\cdot) \) satisfying Assumption 3 and 5.

**Proof** According to Theorem 4.1, under the hypotheses of the current theorem, system (14) is ISS with respect to the observer error \( e(t) \). Moreover, according to Theorem 4.2, the observer error dynamics are globally exponentially stable. Using the proof of Lemma 4.7 in [Khalil, 2002], we can conclude that \((x, e) = (0, 0)\) is a globally asymptotically stable equilibrium point of system (14), (20). Note that Lemma 4.7 in [Khalil, 2002] is given for locally Lipschitzian systems. However, the proof is given on a trajectory level and, therefore, it can also be applied to differential inclusions. □

5 Application to a rotor dynamic system with friction

5.1 Experimental setup and modeling

The experimental setup consists of an upper disc actuated by a drive part (consisting of a power amplifier, DC-motor and a gear box), a steel string, a lower disc and a brake device, see Figure 3. The actuator input voltage of the drive part is limited to the range \([-5V, 5V]\). The upper disc is connected to the lower disc by a steel string, which is a low-stiffness connection between the discs. A brake disc is connected to the lower disc and a brake device exerts an oil layer between the disc and the brake device, resulting in a friction characteristic with a Stribeck effect [Olsson et al., 1998] (i.e. with a so-called negative damping characteristic). Two incremental encoders are used to measure the angular positions of the lower and the upper discs. The configuration of the experimental setup can be recognized in the structure of drilling systems and other rotor dynamic motion systems.

We define \( u \) as the input voltage to the drive part. The system has two degrees of freedom: the angular displacements of the upper and lower discs, \( \theta_u \) and \( \theta_l \), respectively. The equations of motion for the upper disc and the lower disc are given by

\[
J_u \ddot{\theta}_u + k_\theta (\theta_u - \dot{\theta}_l) + b(\dot{\theta}_u - \dot{\theta}_l) + T_{fu}(\dot{\theta}_u) - k_m u = 0 \tag{21}
\]

\[
J_l \ddot{\theta}_l - k_\theta (\theta_u - \dot{\theta}_l) - b(\dot{\theta}_u - \dot{\theta}_l) + T_{fl}(\dot{\theta}_l) = 0,
\]

Set-valued force laws are needed to model the friction acting on the upper and lower disc to account for the pronounced sticking effect in both characteristics. The friction torque acting on the upper disc is caused by the electromagnetic field in the drive part and the bearings that support the disc and is modeled by \( T_{fu} \), see Figure 4:

\[
T_{fu}(\dot{\theta}_u) \in \begin{cases} T_{cu}(\dot{\theta}_u) \text{sgn}(\dot{\theta}_u) & \text{for } \dot{\theta}_u \neq 0, \\ [-T_{su} + \Delta T_{su} \cdot T_{su} + \Delta T_{su}] & \text{for } \dot{\theta}_u = 0, \end{cases} \tag{22}
\]

\[
T_{cu}(\dot{\theta}_u) = T_{su} + \Delta T_{su} \text{sgn}(\dot{\theta}_u) + b_u |\dot{\theta}_u| + \Delta b_u \dot{\theta}_u. \tag{23}
\]

The friction torque \( T_{fl} \), see Figure 5, is caused by bearings that support the lower disc and, mainly, by the brake device and is represented by

\[
T_{fl}(\dot{\theta}_l) \in \begin{cases} T_{cl}(\dot{\theta}_l) \text{sgn}(\dot{\theta}_l) & \text{for } \dot{\theta}_l \neq 0, \\ [-T_{sl}, T_{sl}] & \text{for } \dot{\theta}_l = 0, \end{cases} \tag{24}
\]

\[
T_{cl}(\dot{\theta}_l) = T_{sl} + (T_{sl} - T_{cl}) e^{-\frac{\dot{\theta}_l}{\gamma T_{sl}}} + b_l |\dot{\theta}_l|. \tag{25}
\]

We define the state vector \( x \) as

\[
x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \alpha \\ \omega_u \\ \omega_l \end{bmatrix} = \begin{bmatrix} \theta_u - \theta_l \\ \dot{\theta}_u \\ \dot{\theta}_l \end{bmatrix}. \tag{26}
\]
Note that the state $x_1 = \alpha = \theta_u - \theta_l$ represents the relative angular displacement of the lower disc with respect to the upper disc, which can be obtained via the encoder measurements of $\theta_u$ and $\theta_l$ ($y = x_1$). The desired solution for the rotor dynamic system is a constant (and identical) velocity for both discs, which corresponds to an equilibrium (note that in drilling systems such constant velocity solution corresponds to nominal operating conditions). The state-space equations of the rotor dynamic system in Lur'e-type form are given by (8), with state $x \in \mathbb{R}^3$, input $u \in \mathbb{R}$, measured output $y \in \mathbb{R}$, and $\varphi(z) = [\varphi_1(z_1) \varphi_2(z_2)]^\top = [T_{fu}(z_1) \ T_{fl}(z_2)]^\top$ with $\varphi_i : \mathbb{R} \to \mathbb{R}$ for $i = 1, 2$. The matrices and the non-linearity $\varphi(z)$ in (8) are given by

$$A = \begin{bmatrix} 0 & 1 & -1 \\ \frac{k_n}{\tau_n} & b & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{k_m}{\tau_w} \\ 0 \\ 0 \end{bmatrix}, \quad (27)$$

$$G = \begin{bmatrix} \frac{1}{\tau_c} & 0 \\ 0 & \frac{1}{\tau_f} \end{bmatrix}, \quad H = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C^\top = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad (28)$$

$$\varphi(z) = \begin{bmatrix} T_{fu}(z_1) \\ T_{fl}(z_2) \end{bmatrix}. \quad (29)$$

The parameters of the rotor dynamic model (8), (27), (28), (29) are estimated by dedicated parameter identification experiments, see Table 1, following a similar procedure as described in [Mihajlovic et al., 2006], and are validated by comparing the steady-state solutions of the simulations with those of the experiments.

For varying constant inputs $u_c$ (i.e. $u = u_c$ in (8)), we observe several bifurcations in simulations and experiments. Different (co-existing) steady-state solutions of the rotor dynamic system as result of the simulations and experiments, are depicted in a bifurcation diagram in Figure 6 for constant input voltages $u_c$, which is the bifurcation parameter. For the case where the steady-state response is a periodic solution (stick-slip limit cycle), we plot the maximum and minimum value of the state variable $\omega_l$ (velocity of the lower disc) in the bifurcation diagram. For the region with constant input voltages up to approximately $u_c = 2.7 \, V$, we observe only stable limit cycles. Figure 7 shows such a limit cycle response for $u_c = 2.7 \, V$. In the region from approximately $2.7 \, V - 4.5 \, V$, two stable steady-state solutions co-exist: an equilibrium point or a stick-slip limit cycle, depending on the initial conditions. For constant input voltages higher than $4.5 \, V$, only a stable equilibrium point occurs. The reader is referred to [Mihajlovic et al., 2006] for an extensive investigation of the analysis of the dynamic behaviour of the rotor dynamic system. As we remarked earlier, the equilibria of the rotor dynamic system correspond to both discs rotating with the same constant velocity, which are the desired operating conditions. As
such, the control goal is to stabilize the unstable equilibria up to $u_c = 2.7$ V and to eliminate the limit cycles up to $u_c = 4.5$ V.

5.2 Output-feedback controller

One could opt to design an output-feedback controller by using the circle criterion, instead of the more involved Popov-criterion-inspired approach with the dynamic multiplier $M(x)$. However, the control design based on the circle criterion is not feasible for the rotor dynamic system (8) according the presented feasibility conditions in [Arcak and Kokotović, 2001]. In order to satisfy the feasibility conditions in [Arcak and Kokotović, 2001], the damping coefficient $b$ should satisfy $\{b > \min_{\omega_l} \frac{1}{\omega_l} J_l f(u_l)(\omega_l) | \omega_l > 0\}$. This would imply that the negative damping in the friction model $T_{fl}$, which is, basically, the cause of the friction-induced stick-slip vibrations is dominated by such viscous damping $b$ in the string. However, the damping coefficient $b$ reflects only material damping in the string which is generally very low and will not satisfy the above condition ($b = 0$ see Table 1). As many mechanical motion systems consist of inertias coupled by a low-damping connection, there exists a large class of systems for which a circle-criterion based control design is not feasible. For these systems, the output-feedback control design presented in Section 4.3, can be a solution since the use of the dynamic multiplier relaxes the circle-criterion conditions. The control strategy presented in Section 4.3 is applied to the rotor dynamic system (8) and the output-feedback control law is given by

$$u = u_c + u_{\text{comp}} + K(\hat{x} - x_{eq}),$$

with $x_{eq} = [\alpha_{eq}, \omega_{eq}, \omega_{eq}^T]$ the desired equilibrium of the rotor dynamic system (8), the control gain $K \in \mathbb{R}^{1 \times 3}$ and

$$u_{\text{comp}} = \frac{1}{k_n}(T_{fl}(\hat{x}_3) - b_u \hat{x}_2).$$

The part $u_{\text{comp}}$ of the control law compensates partly the friction acting at the upper disc of the rotor dynamic system. The ‘effective’ friction after compensation acting at the upper disc is purely viscous. Note that such a friction compensation can not be employed to compensate for the friction at the lower disc (which is responsible for the stick-slip limit cycling), due to the non-collocation of actuation and friction. We can easily transform the closed-loop rotor dynamic system (8), (30) to a system in

Figure 7. Experimental limit cycle response of the rotor dynamic system for $u_c = 2.7$ V.

Lur’e-type form with the origin as equilibrium by choosing, for example, the new state $\xi = x - x_{eq}$. For the sake of brevity, we will omit this transformation here (see [Doris, 2007] for more details). Assumption 5 requires that the set-valued nonlinearity $\varphi$ is monotone. If we consider the friction model $T_{fl}$, see Figure 5, which is contained in $\varphi$, then it is clear that $T_{fl}$ is not monotone. We will render $\varphi$ monotone by applying a loop transformation, which will add ‘viscous’ damping to $T_{fl}$ and subtract it from the linear part of (8), see [Doris, 2007]. The following feedback and observer gains satisfy Theorem 4.3:

$$K^T = \begin{bmatrix} 15.9 \\ 1.57 \end{bmatrix}, \quad L = \begin{bmatrix} 195 \\ -312 \\ -9080 \end{bmatrix}, \quad N = \begin{bmatrix} -22.2 \\ -37.8 \end{bmatrix}. \quad (32)$$

with the multiplier matrix $\Gamma = 10I$. A solution for $P$ that satisfies the strict passivity condition for $(A + BK, G, \hat{H}, \hat{D})$ is

$$P = \begin{bmatrix} 3.639 & 0.431 & 6.382 \\ 0.431 & 0.070 & 0.740 \\ 6.382 & 0.740 & 11.627 \end{bmatrix}. \quad (33)$$

The above results are obtained by a linear matrix inequality (LMI) solving routine within the program MATLAB.

5.3 Simulations and experiments

The presented output-feedback controller is applied to the rotor dynamic system to stabilize the equilibria of the rotor dynamic setup for a large range of constant inputs. We show the experimental closed-loop transient response for the constant input voltage $u_c = 2.5$ V and $u_c = 4.0$ V, respectively, in Figure 8. The only stable open-loop solution for $u_c = 2.5$ V is a stick-slip limit cycle, see Figure 6. For an input of $u_c = 4.0$ V, two stable open-loop solutions exist: an equilibrium and a stick-slip limit cycle. The output-feedback controller is switched on at $t = 5$ s for $u_c = 2.5$ V and the closed-loop system converges to the equilibrium state ($\omega_{eq} = 4.40$ rad/s). Also for $u_c = 4.0$ V, the closed-loop system converges to the equilibrium state ($\omega_{eq} = 7.06$ rad/s) where the initial open-loop solution is a stick-slip limit cycle. Both experimental and model-based bifurcation diagrams for the closed-loop rotor dynamic system are depicted in Figure 9. The simulated bifurcation diagram shows that for all constant $u_c$ the desired equilibrium is globally asymptotically stabilized. In experiments, the output-feedback controller is able to eliminate the stable limit
cycles and to stabilize the unstable equilibria for a large range of constant inputs $u_c$. However, for a small range of low voltages, the output-feedback controller cannot stabilize the equilibria of the experimental rotor dynamic setup. The remaining closed-loop limit cycles up to $u_c = 1.5$ V differ from the open-loop limit cycles. A cause for this lack of stability of the equilibria at these low input voltages may be some unmodeled position-dependent friction acting on the lower disc.

6 Conclusions

In this paper, we considered the feedback control of mechanical motion systems with set-valued frictional nonlinearities. The concept of absolute ISS was presented, together with a generalization of a Popov-like criterion that guarantees ISS for systems with set-valued nonlinearities. The latter concept is used to design a state-feedback controller and the related absolute stability conditions are less restrictive than those related to a control design based on the circle criterion. Since the presented controller induces absolute ISS, an advantage is that the closed-loop system is robust to uncertainties in the friction, a crucial property in practice. A second advantage is that the state-feedback controller is applicable to systems with non-collocation of actuation and friction for which well-known strategies such as direct friction compensation techniques fail. Furthermore, an output-feedback control design is constructed by exploiting the ISS property, where a model-based observer, for which stability of the error dynamics is proven, is used to estimate the system state. We provided a separation principle for the considered class of Lur’e-type systems. The effectiveness of the proposed output-feedback control strategy is shown both in simulations and experiments for a typical motion system with dry friction.

References


A Proof of Theorem 4.1

Proof Consider the following ISS Lyapunov function candidate (see [1, 2]: consider the following ISS Lyapunov function candidate [1, 2], which will be continuous, but not necessarily differentiable:

\[ V(x) = V_1(x) + V_2(z), \]

with \( z = Hx \) and

\[ V_1(x) = \frac{1}{2} x^T P x, \quad P = P^T > 0, \]

\[ V_2(z) = \sum_{i=1}^{p} V_{2,i}(z_i), \]

where for \( i = 1, \ldots, p \)

\[ V_{2,i}(z_i) = \eta \int_0^{z_i} \varphi_i(\sigma) d\sigma, \]

where \( d\sigma \) denotes the ordinary Lebesgue integration on the real line. We will show that the function \( V \) satisfies the following bounds

\[ \psi_1(\|x\|) \leq V(x) \leq \psi_2(\|x\|), \]

where \( \psi_1 \) and \( \psi_2 \) are class \( \mathcal{K}_\infty \)-functions. To do so, note that \( V_{2,i}(z_i) \geq 0 \), since \( \varphi_i \) belongs to \( [0, \infty) \), see (11). Consequently, a lower bound of the ISS Lyapunov function is obtained by

\[ \psi_1(\|x\|) = \frac{1}{2} \lambda_{\text{min}}(P) \|x\|^2. \]

Using the growth condition (12), an upper bound for \( V_2 \) can be derived:

\[ V_2 \leq \sum_{i=1}^{p} \eta \int_0^{z_i} |\varphi_i(\sigma)| d\sigma \leq \sum_{i=1}^{p} \eta \int_0^{z_i} (\gamma_1 \sigma + \gamma_2) d\sigma = \sum_{i=1}^{p} \frac{1}{2} \eta \gamma_1 z_i^2 + \sum_{i=1}^{p} \gamma_2 \eta z_i \|e_i\| \leq \frac{1}{2} \eta \|e\|^2 \|z\|^2 + \gamma_2 \eta \sqrt{P} \|z\| \leq \psi_{2,1}(\|x\|), \]

where \( p \) is the dimension of \( z \), i.e. \( z \in \mathbb{R}^p \). Using (A.6), an upper bound of the ISS Lyapunov function \( V \) is given by

\[ \psi_2(\|x\|) = \frac{1}{2} \lambda_{\text{max}}(P) \|x\|^2 + \psi_{2,1}(\|x\|). \]

We are now going to consider the derivative of \( V \) along trajectories of the system. To do so, let \( x \) be a solution trajectory of the system (14) given a piecewise-continuous external (measurement noise) signal \( e \). First, observe that \( t \mapsto V(x(t)) \) (locally) Lipschitz continuous and \( t \mapsto x(t) \) is absolutely continuous. This implies that \( t \mapsto V(x(t)) \) is absolutely continuous as well. Hence, the time derivatives of \( t \mapsto V(x(t)) \) and \( t \mapsto x(t) \) exist almost everywhere. The time derivative of \( V_1 \) can be written as follows:

\[ \dot{V}_1 = \frac{1}{2} [(A + BK)x + Gw - BK e]^T P x + \frac{1}{2} x^T P [(A + BK)x + Gw - BK e]. \]
At this point we add and subtract $\frac{1}{2}\{z^\top w + w^\top z\}$ to the right-hand side of (A.8) and use (17a):

$$\begin{align*}
V_1 &= V_1 - \frac{1}{2}(Hw + \bar{D}w + \tilde{Z}e)^\top w - \frac{1}{2}w^\top (Hw + \bar{D}w + \tilde{Z}e) + \tilde{Z}^\top w \\
&= \frac{1}{2}x^\top F(P,K) \left[ \begin{array}{c} x(w) \\ w(s) \end{array} \right] - \frac{1}{2}e^\top K^\top B^\top Px - \frac{1}{2}w^\top P BK e - \frac{1}{2}e^\top \tilde{Z}^\top w - \frac{1}{2}w^\top \tilde{Z} e + \tilde{Z}^\top w,
\end{align*}$$

with $F(P,K)$ defined as

$$F(P,K) := \left[ (A + BK)^\top P + P(A + BK) - PG - B^\top (G^\top P - \bar{H} - \bar{D}) \right].$$

For the $V_2$ contribution, with $V_2$ as in (A.2b), (A.3) and $z(t) = Hx(t)$, we have that:

$$V_2(z(t_2)) - V_2(z(t_1)) = \sum_{i=1}^p [V_{2,i}(z(t_2)) - V_{2,i}(z(t_1))],$$

for $i = 1, \ldots, p$.

$$V_{2,i}(z(t_2)) - V_{2,i}(z(t_1)) = \eta_i \int_{z_i(t_1)}^{z_i(t_2)} \varphi_i(z_1(s))dz_1(s).$$

The integral in (A.12) can be expressed as

$$\int_{z_i(t_1)}^{z_i(t_2)} \varphi_i(z_1(s))dz_1(s) = \int_{t_1}^{t_2} \varphi_i \circ z_i(s)\frac{dz_i(s)}{ds}ds,$$

where $dz_i$ denotes the usual Lebesgue measure on the real line and the latter integral denotes the Lebesgue-Stieltjes integral with respect to the Lebesgue-Stieltjes measure $dz_i(s)$ corresponding to the generating function $z_i$, see [Kolmogorov and Fomin, 1970]. For this Lebesgue-Stieltjes integral we can use the following result in [Kolmogorov and Fomin, 1970],

$$\int_{t_1}^{t_2} \varphi_i(z_i(s))dz_i(s) = \int_{t_1}^{t_2} \varphi_i \circ z_i(s)\frac{dz_i(s)}{ds}ds + \int_{t_1}^{t_2} w_i(s) (z_i(s) - z_i(s))ds,$$

for $i = 1, \ldots, p$. Combining (A.1), (A.9) and (A.15) we can derive that for all $t_1, t_2 \in \mathbb{R}$, with $t_2 \geq t_1$,

$$V(x(t_2)) - V(x(t_1)) = \frac{1}{2} \int_{t_1}^{t_2} \left[ x(s)^{\top} F(P,K) x(s) + 2w(s)^{\top}(s)w(s) - e(s)^{\top} K^\top B^\top Px(s) - x(s)^{\top} PB K e(s) - e(s)^{\top} \tilde{Z}^\top w(s) - w(s)^{\top} \tilde{Z} e(s) \right]ds.$$

Hence, the derivative of $V$ with respect to time, for all $x \in \mathbb{R}^n$ and $z_i \in \mathbb{R}$, $i = 1, \ldots, p$, can be, almost everywhere, written as

$$\dot{V} = \frac{1}{2} x^\top F(P,K) \left[ \begin{array}{c} x \\ w \end{array} \right] + z^\top w,$$

for some $\varepsilon > 0$. In the sequel, we will use the following inequality

$$y_1^\top y_2 \leq \frac{1}{2}y_1^\top R y_1 + \frac{1}{2}y_2^\top R^{-1} y_2,$$

for any $R = R^\top > 0$ and for all vectors $y_1, y_2$. More specifically, using (A.19) with $R = I$ and the definitions $\tilde{X} := K^\top B^\top P$ and $E := \tilde{X}X$ we have for the term $-e^\top K^\top B^\top Px$ in (A.17) that

$$-e^\top K^\top B^\top Px \leq \frac{1}{2} e^\top E e + \frac{1}{2} e^\top x^\top x \leq \frac{1}{2}\lambda_{\max}(E)\|e\|^2 + \frac{1}{2}\|x\|^2.$$

Next, we use the growth condition (12), inequality (A.19) and we dominate the term $-e^\top \tilde{Z}w$ in (A.17) as:

$$\begin{align*}
-e^\top \tilde{Z}w &\leq \|e^\top \tilde{Z}w\| \\
&\leq \|\tilde{Z}\||e||\gamma_1||H||\|x\| + \gamma_2\|e\| \\
&= \gamma_1\|e\||x|| + \gamma_2\|e\| \\
&\leq \frac{\varepsilon}{2}\|x\|^2 + \frac{\varepsilon}{2}\|e\|^2 + \gamma_2\|e\|,
\end{align*}$$

with $\gamma_1 := \gamma_2 := \frac{1}{2}\|H||x\|$. Based on (A.18), (A.20) and (A.21), (A.17) yields almost everywhere

$$\dot{V} \leq -\frac{\varepsilon}{4}\|x\|^2 + \frac{1}{2}\lambda_{\max}(E) + \frac{1}{2}\|e\|^2 + \gamma_2\|e\||e\|.$$ (A.22)

From (A.22) it follows that almost everywhere

$$\dot{V} < -\alpha(\|x\|) := -\frac{\varepsilon}{8}\|x\|^2,$$

where $\alpha$ is a positive definite function and with the following definition for the class $K$-function $\chi$:

$$\chi(\|e\|) := \frac{\varepsilon}{4}\left(\lambda_{\max}(E) + \frac{1}{2}\|e\|^2 + 2\gamma_2\|e\|\right).$$ (A.24)

Consequently, we have proven (A.4) and $V(x(t)) \leq -\alpha(\psi^{-1}(V(x)))$ when $\|x(t)\| \geq \chi(\|e(t)\|)$. Using now the reasoning as in [Sontag and Wang, 1995, Lemma 2.14] with the necessary adaptations due to the non-smoothness of $V$ (see proof of Theorem 2 in [Heemels et al., 2007] for details), this proves ISS.