Review of Optimization Strategies for System-Level Design in Hybrid Electric Vehicles

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Abstract—The optimal design of a hybrid electric vehicle can be formulated as a multi-objective optimization problem that spreads over multiple levels (technology, topology, size and control). In the last decade, studies have shown that, by integrating these optimization levels fuel benefits are obtained, which go beyond the results achieved with solely optimal control for a given topology. Due to the large number of variables for optimization, their diversity, the nonlinear and multi-objective nature of the problem, a variety of methodologies has been developed. This review article presents a comprehensive analysis of the various methodologies developed and identifies challenges for future research. Starting from a general description of the problem, with examples found in the literature, we categorize the types of optimization problems and methods used. To offer a complete analysis, we broaden the scope of the search to several sectors of transport, such as naval or ground.

Index Terms—Multi-level optimal design, hybrid electric vehicles, optimization methods, powertrain design, coordination methods, driving cycle.

I. INTRODUCTION

CURRENT challenges for newly developed vehicles, as strict legislations on CO2 or the foreseen future-lack of oil, are addressed in various transportation sectors, with hybrid power trains, as viable solutions. Having more than one source of power, hybrid power trains give birth to a large design space for the physical system and increase the complexity of the control algorithm. The coupling (dependency) between the parameters of the physical system (e.g., topology) and the parameters of the control algorithm transforms the problem into a multi-level problem (as depicted in Fig. 1) that, if solved sequentially, is by definition sub-optimal [1]. Therefore, the physical system and the control algorithm should be designed in an integrated manner to obtain an optimal system design.

Because of the large dimensions of the design space, computer simulations of dynamical systems, e.g., for different architectures and component sizes, have become more important as a preliminary step to building prototypes [2]. Computer simulations significantly speedup the control synthesis of a given design and topology. However, even with computer systems, the problem of finding the optimal vehicle design that provides the best control performance is typically intractable. Obviously it is not feasible (cost or time-wise), given a design space, to build all possible vehicles and evaluate which configuration and parameters provide the best performance for control. Moreover, even when designing the control algorithm, due to the nonlinear, mixed-integer and multi-dimensional (several states) characteristics of hybrid electric vehicles (HEV) control problem, the simulations require large computational times. Ergo, it is not time-wise feasible to simulate all combinations (i.e., brute force searches) of the design variables [3]. Instead, optimization-based frameworks for plant and control synthesis of HEVs are being developed. Starting from the optimal control and continuing to the optimal sizing, different optimization algorithms were used to obtain the maximum power train energy efficiency and/or the minimum total cost of vehicle ownership.

Based on examples from recent literature, in this article we introduce the general problem of optimally designing a HEV. Then we summarize the common challenges in this design problem and present the different methods and frameworks that have been developed to improve the design of HEVs. The focus of this overview is on frameworks that include the co-design of HEVs, i.e. concurrent plant (as topology or size) and control optimization.

The remaining sections of this paper are organized as follows. After a description of HEV topologies is given in Section II, the system-wide optimization problem is described in Section III. Section IV discusses existent methodologies used for integrating the plant and the control optimization,
II. HYBRID ELECTRIC VEHICLES

Conventional vehicles run on internal combustion engines, consuming fuel to deliver the required power. Besides providing a useful work, conventional vehicles are encountered with dissipative energy, such as the braking energy, aerodynamic drag losses, tire friction losses, engine idling losses, etc. In this topology, emission reduction possibilities exist, such as lighter materials and more improved designs, but are limited. For instance, while reducing further the aerodynamic drag or the tire losses is possible, braking and idling losses will always be significant in conventional vehicles. Nevertheless, the sizing of the combustion engine will always be decided by the power it needs to provide. To circumvent this limitation, various hybrid architectures have been developed, where each architecture has its advantages and disadvantages.

Hybrid vehicles combine two, or more technology principles to produce, store and deliver power. Current market hybrid vehicles typically combine a combustion engine and an electric machine (EM), as power converters, and they are referred to as a hybrid electric vehicles (HEVs). This hybridization allows a wide variety of topologies for the configuration of the powertrain.

Three categories of topologies may be distinguished: series, parallel and series-parallel, as illustrated in Fig. 2. These topologies, and their applicability to various transportation sectors, have been researched intensively in recent years and are described in detail in survey articles such as [2]–[9], books [10]–[13]. In a HEV, depending on its topology and component technologies, an electric machine can function as tractor (delivering positive torque and speed to propel the vehicle) or as a generator (producing energy, from either the engine or from regenerative braking, to charge the battery).

Series HEVs, perform best in stop-and-go driving since there is no mechanical link between the combustion engine and the wheels. In this way the engine can be run at its most efficient point also in varying vehicle speeds. Moreover, because there is no mechanical connection between the combustion engine and the wheels, this configuration is rather flexible with regard to the physical location of the various components in the powertrain. This makes the series topology highly suitable for application with restricted (re)design space.

When a series HEV is used in highway or inter-urban driving, high powers need to be transmitted to the wheels from the EM. Hence, large electrical machines are needed to achieve high vehicle speeds. In addition, this topology requires a double energy conversion for delivering the required power, which induces efficiency losses. In this configuration the size of the traction EM is deduced from the vehicle’s required performance (such as the acceleration requirement). Thus, the sizing of the power train reduces to finding the optimal sizing of the battery and the power generating group (combustion engine/generator).

In parallel HEVs the combustion engine and the electric machine are both connected to a mechanical transmission and they can generate power independently of each other. The electric machine can be connected before or after the transmission as shown in Fig. 2 with (a) and (b). Moreover, the HEV can switch between the power sources given the driving conditions. In this configuration there is no separate generator. Whenever generating power is possible and needed (e.g., energy recuperated from braking) the electric machine functions as a generator.

Parallel HEVs have a direct mechanical connection between the engine and the wheels. This leads to smaller energy losses (as they don’t require the dual energy conversion as the series topology) but also less flexibility in the mutual positioning of the power train components compared to the series HEV drivetrain.

Series-parallel HEVs have an extra direct mechanical connection between the generator and the traction motor via the transmission. These architectures combine the benefits from both series and parallel HEVs. They are usually constructed with one or more planetary gear sets (PGS), and require at least two electric machines. PGS are transmission elements with three connectivity points (ring, sun and carrier). These transmission elements, eliminate the need of a traditional stepped (manual or automatic) gearbox and other transmission components.

Due to their increased flexibility in operating the components (as in series HEVs) and the presence of mechanical
links (as in parallel HEVs), series-parallel HEVs can lead to a reduced fuel consumption for a wide variety of applications [14]. Yet, at the same time, they come at a higher price and require more complex control strategies.

Except these three HEVs categories, others can also be found in literature or practice, e.g., the dual mode hybrid and the four quadrant transducer. These mostly vary in the construction of the transmission components and will not be addressed here. The interested reader could refer to [15]–[19] for more information.

The efficiency of hybrid topologies varies according to the conditions under which they are driven. The design choice for one or other architecture depends on the (intended) mission of the vehicle and the trade-off between cost and performance. Given the pros and cons of the serial, parallel and series-parallel topologies, these are each predominantly used in certain transportation sectors. Serial topologies are currently most often found in buses [20]–[24], battery electric vehicles (BEV) [25] with range extenders, boats [26], heavy vehicles (military), locomotives [27]–[30] and other in-urban vehicles, such as taxis or passenger vehicles [31]–[33], while parallel topologies and series-parallel are very common in passenger vehicles [34]–[38].

Due to the high-cost and complexity of series-parallel topologies, the parallel topologies are, at the moment, the most commonly produced type of HEVs. Consequently, the parallel hybrids dominate the literature on supervisory control for HEVs [36], [39], [40].

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For different applications, dedicated research has been conducted on technologies for hybrid components and storage devices (as batteries, super-capacitors or flywheels). Overviews of electric motor drives and storage devices are well presented in [5], [41]–[46]. The requirements of each application determine the suitability of a certain technology, as well as the required dimensions of the respective hybrid component. In fact, determining the technology and dimension of a particular power train component represents also a discrete choice. This makes the optimal design of the power train of a hybrid electrical vehicle a discrete programming problem in terms of topological connectivity, technologies, and dimensions of the HEV power train components.

In the first research efforts on HEV development, the various options (topology, type, size) were investigated for a restricted set of design choices, (e.g., a battery versus fuel cells, or three dimensions for the same Li-ion battery). The limited search space already provided novel hybrid power train configurations with a lower fuel consumption than conventional vehicles. Recent research papers on HEV development increase the scale of the optimization problem, in an effort to further improve the HEV performance. Typically, one seeks to formulate and solve a system-wide optimization problem covering the various components and disciplinary aspects involved in the HEV power train design.

In the following section these approaches for design and control of HEVs will be presented and analysed, with their pros and cons. We address the design of hybrid electric vehicles alone, without considering their effect on infrastructures (charging, traffic/transport, communication). For details on co-optimization of both HEVs and infrastructure, interested readers are referred to [47]–[50].

III. PROBLEM STATEMENT FOR SYSTEM-WIDE OPTIMAL DESIGN

A hybrid vehicle contains multiple interconnected subsystems which, themselves, consist of several sub-systems. When a HEV is built, it is desired to minimize both operational and component/design cost.

A. Driving Cycle

To evaluate the fuel consumption of an HEV a drive cycle, \( \lambda \), is necessary. This is a series of data points,

\[
\lambda(t) = \begin{bmatrix} v(t) \\ s(t) \end{bmatrix}, \quad \text{with } t \in [t_0, t_f],
\]

with \( v(t) \) representing the speed of a vehicle over time, \( s(t) \) representing the slope (gradient) of the road and \( [t_0, t_f] \) representing the driving cycle length. The drive cycle represents the type of driving conditions in which the HEV is used. It is the main determinant for the fuel consumption and the design (such as dimensioning of components) of the vehicle.

Driving cycles, which can be either measured or artificially created, vary across applications, countries and organizations. Driving cycles are used to assess the performance of HEVs in different ways, as for example fuel consumption and pollution emissions [51]–[53]. In literature most driving cycles assume \( s(t) = 0 \). This is an important assumption for heavier vehicles, where the contribution in the total power demand, for \( s(t) \neq 0 \), becomes significant.

B. Plant and Control Optimization Problem

The HEV efficiency and cost is dependent on the components (their connections, technologies and sizes) but also on the control algorithm used. The varying parameters defining topology, sizing and control inputs constitute the design variables (denoted by \( x \)) in the optimal design problem, for both the plant and the control of a HEV,

\[
\min_{x_p, x_c(t)} J(x_p, x_c(t), \lambda)
\]

s.t.

\[
g_j(x_p, x_c(t)) \leq 0, \quad j = 1, 2, ..., m,
\]

\[
h_l(x_p, x_c(t)) = 0, \quad l = 1, 2, ..., e.
\]

\[
\dot{\xi}(t) = f(\xi(t), x_p, x_c(t), t),
\]

\[
\xi(t_0) = \xi_0,
\]

\[
\xi(t_f) = \xi_f.
\]

Here \( x_p \in \mathbb{R}^n \) and \( x_c(t) \in \mathbb{R}^z \) denote the design variable vectors with \( n \) independent plant variables and \( z \) independent control variables, \( m \) the number of inequality constraints, \( e \) the number of equality constraints, \( J \) is the cost function, and \( \xi \) the states of the dynamical system, e.g., the state of charge (SOC) of the electric buffer.

Note. For ease of understanding vectors are marked in bold, i.e., \( x \) is a vector of design variables, where each variable is denoted by \( x \). Moreover, \( (\cdot)_p \), represents a plant related variable.
(such as battery sizing) while \(c\), represents a control related variable (such as engine torque).

In cases where \(\xi\) denotes the battery SOC, the final state conditions

\[
\xi_f = \xi_0, \\
\xi_f' = \xi_{\text{min}},
\]

constrain the charge sustaining, (3), or depleting, (4), behaviour of the energy storage pack at the end of the driving cycle. Thus, (3) is used for charge sustaining hybrids and (4) is used for plug-in HEVs. Constraints, \(g_j\) and \(h_l\) contain per-component operational boundaries, such as the engine torque, \(T_e\), subject to the speed-dependent constraint, \(T_{e,\text{min}}(\omega_e) \leq T_e(t) \leq T_{e,\text{max}}(\omega_e)\), component sizing boundaries, such as the engine power, \(P_e\), \(P_{e,\text{min}} \leq P_e \leq P_{e,\text{max}}\), or other boundaries related to the HEV topology (connectivity of components).

The inter-links between different levels of vehicle design are illustrated in Fig. 3. We distinguish three design levels: (a) determining the topology \(T_k\), (b) determining component dimensions, and, (c) designing the control algorithm.

The coupling between the three design levels presents a multi-level optimization problem with discrete design variables (such as battery size, transmission gear, powertrain mode) as well as continuous design variables (such as engine torque, battery power). Furthermore, the component models and the optimization functions are generally nonlinear and non-convex [10].

![Fig. 3: System-level design (SLD) layers and interlinks in HEVs](image)

1) Design Space Selection: To illustrate the use of \(x_p\) and \(x_c\) in [2], consider the optimal sizing and control problem for a one-motor parallel HEV depicted in Fig. 4.

For the powertrain topology and components of Fig. 3 [combustion engine, electric machines, battery and transmission], \(x_p\) and \(x_c\) become

\[
x_p^{\text{FF}} = \begin{bmatrix} P_e & P_m & C \end{bmatrix}^T, \\
x_p^{\text{FF}}(t) = \begin{bmatrix} u_p(t) & \gamma(t) \end{bmatrix}^T.
\]

Herein \(P_e\) is the maximum power of the engine, \(P_m\) is the electric motor maximum peak power, \(C\) is the battery capacity, \(r_m\) is the maximum gear ratio, \(u_p\) is the power-split ratio that defines the portion of power delivered by the engine and electric machine, \(\gamma\) is the gear number and the superscript \(\gamma^{\text{FF}}\) indicates the parallel type of the topology. Next, \(\gamma^s\) indicates a series topology and \(\gamma^{\text{FF}}\) indicates a series-parallel topology.

For a series topology \(x_p\) and \(x_c\) become

\[
x_p^s = \begin{bmatrix} P_e & P_{m1} & C \end{bmatrix}^T, \\
x_c^s(t) = \begin{bmatrix} T_e(t) & \omega_k(t) \end{bmatrix}^T,
\]

with \(T_e\) and \(\omega_k\) the torque and speed of the combustion engine, for the input-split series-parallel topology \(x_p\) and \(x_c\) become

\[
x_p^{\text{PS}} = \begin{bmatrix} P_e & P_{m1} & P_{m2} & C & Z \end{bmatrix}^T, \\
x_c^{\text{PS}}(t) = \begin{bmatrix} \omega_k(t) & T_{m2}(t) \end{bmatrix}^T,
\]

with \(T_{m2}\) the torque of the second electric machine and \(Z\) the epicyclic gear ratio of the planetary gear set. For alternative topologies one may wish to include additional design variables related to clutches, more electric machines, more battery packs or alternative components.

When the topology or the technology are assumed variable too (besides the sizes of components), then more variables are included in the plant design variable vector, \(x_p\). Assume \(x_p\) consists of design variables from three plant design layers

\[
x_p = [x_p^{\text{top}}, x_p^{\text{tech}}, x_p^{\text{size}}],
\]

with \(x_p^{\text{top}}, x_p^{\text{tech}}\) and \(x_p^{\text{size}}\) the plant design variable representing the topology, technology and size layers. Each instance of \(x_p^{\text{top}}\) will influence the size of \(x_p^{\text{tech}}\) and \(x_p^{\text{size}}\), as well as their corresponding control variables, exemplified in [5], [8] and (7). Furthermore, the selection of components sizing will, partially, determine the constraints for the control algorithm.

Explicit derivations of the coupling between the sizing and the control layer, for different applications, and how they influence the overall design, are found in [1], [54].

Therefore, to find the vector \(x_p\) that minimizes the cost function \(J\), it is a challenge for the chosen multi-level optimization methods, and for the optimization algorithms used for each individual level.
2) Optimization Targets Selection: Given $J \in \mathbb{R}^k$ in (2), $k$ represents the vector of objective functions, that comprises the system-level design (SLD) objectives. As mentioned before, a HEV is generally built such that both operational and component/design cost are minimized. Nonetheless, other objectives, such as minimizing emissions or maximizing the payload of the vehicle, have been also used.

The most commonly employed objective functions, $J_i(x): \mathbb{R}^k \rightarrow \mathbb{R}^1$, are

$$
J_1 = \int_0^{t_f} \dot{m}_c(t) dt, \quad J_4 = \int_0^{t_f} \text{NO}_x(t) dt,
$$

$$
J_2 = \Psi_m + \Psi_i + \Psi_b, \quad J_5 = \int_0^{t_f} \text{HC}(t) dt, \quad (9)
$$

$$
J_3 = -m_0 + m_b, \quad J_6 = \int_0^{t_f} \text{CO}(t) dt.
$$

Herein $J_1$ represents the CO$_2$ reduction, or the overall fuel consumption; $J_2$ is the hybridization costs, i.e., the summed cost of the motor, $\Psi_m$, the cost of the inverter, $\Psi_i$, and the cost of the battery, $\Psi_b$. $J_3$ is the payload weight of the vehicle (on-board passengers or cargo), $m_0$ plus the weight of the battery, $m_b$. $J_4$, $J_5$ and $J_6$ are the nitrogen oxides (NO), hydrocarbons (HC) and carbon monoxides (CO) emissions.

The multi-objective character of the HEV system-level design problem (fuel, costs, etc.) requires dedicated multi-objective (MO) optimization algorithms/solvers, or reformulation of the problem into a single objective formulation. The later, referred to as also as scalarization of the cost function, is often used and represents a choice of the designer.

There are multiple methods for objective function scalarization [55]. The weighted sum formulation equals

$$
f(J, w) = w_1 J_1 + w_2 J_2 + ... + w_k J_k,
$$

(10)

with $w$ a vector of weight parameters, with

$$
w_1 + w_2 + ... + w_k = 1.
$$

(11)

The weights are adjusted such that a certain preference for the optimization targets is imposed. This scalarization, is used for example in [56 Ch.3],

$$
f(J, w) = (w - 1)J_1 + wJ_2 \quad (12)
$$

is proposed (with $J$ representing the normalized value of $J$) or in [57] where

$$
f(J, w) = w_1 \hat{J}_1 + w_2 \hat{J}_5 + w_3 \hat{J}_6 + w_4 \hat{J}_4 \quad (13)
$$

is used.

As mentioned before, when a HEV is built, it is desired to minimize both operational and component/design cost. The system-level design (SLD) problem is a challenge given that different optimization functions depend of different system levels. For example, minimizing the cost of electrification, $J_2$, is typically used for power-train component sizing (since $J_2$ does not depend on the control algorithm). Or, $J_1$ is always used as objective for the control algorithm design but it depends also on the component sizing. What are the possible

The authors define a normalized value $\hat{J} = \frac{J}{J^N} \in [0, 1]$, where $J^N$ is estimated as the largest possible value of $J$ within the search space.

IV. Published HEV Design Frameworks

In the context of HEV prototyping, a design framework is a methodology that uses existing optimization algorithms combined on multi-levels, to find the best design for given targets and constraints. This describes how and in which order the coupled optimization problems at the various levels are solved in an effort to solve the overall system level design problem. This relates to coordination methods in distributed multidisciplinary optimization, see for instance [58], [59], where the coordination method defines how the coupled disciplinary subproblems are solved to arrive at the system optimal solution.

For the plant and control design problem, there are basically three coordination architectures, as shown in Fig. 5 (i) alternating plant and control design, i.e., the plant is optimally designed. Using this outcome the controller is optimally designed. Subsequently, the plant is optimized again, etcetera. The coordinator alternates between optimizing the plant and optimizing the control until the coupled variables have converged. (ii) control design nested within plant design, i.e., every evaluation of a plant, requires the full optimization of the controller design; and (iii) simultaneous plant and controller design (i.e. solving all-in-one).

Fig. 5: Coordination Architectures for System-Level Design (SLD) in HEVs.

In mid ’90, when the hybrid vehicle market emerged, the plant design problem and the control design problem were treated completely independently [60]. Nowadays, in most literature and practice, a clear distinction is made between the plant and the control design variables and objectives, where (2) becomes the following co-design problem

$$
\min_{x_p, x_c} J(x) = \{ J_p(x_p, x_c(t), \Lambda), J_c(x_p, x_c(t), \Lambda) \}
$$

s.t. constraints as in eq. (2).

The plant cost function, $J_p$, and the control cost function, $J_c$, may contain any combination of the objectives from (9).

For the plant design problem, in the literature also distinction is made between topology design and component sizing
optimization. Usually, the component sizing problem is solved for a fixed topology. The choice of topologies to be analysed has, so far, been mainly dictated by practical experience rather than by a topology optimization procedure. An computational tractable method for combined topology and component sizing optimization of the plant design is an open research question.

In the next subsection we give an overview of the currently employed methods for topology optimization of the HEV plant. Most of these methods aim at finding feasible topologies, not necessarily optimal topologies. Subsequently, in the forthcoming subsections we survey methods for alternating, nested, and simultaneous plant and control design of HEV vehicles.

A. HEV Topology Generation or Selection

In practice, a HEV topology is often selected on the basis of criteria that derive from expert knowledge. In this approach the set of rules forming the criteria can be derived from expert knowledge, availability of components on the market, other HEVs and so on. The selected topology is very likely not optimal. Recent studies show that very small changes in known topologies, such as the Toyota Prius or Chevrolet Volt, can lead to more efficient HEVs (w.r.t. cost or fuel) \[61\].

Another approach for arriving at a suitable topology is to evaluate at all possible topologies that can be constructed from a predefined fixed set of components. This is sometimes referred to as topology generation.

Usually topology generation means the search for all feasible topologies, \( T^f \), within a (large) set of possible topologies, \( T^p \), given design constraints, \( c \),

\[
\text{Find all } T^f \subseteq T^p, \quad \text{s.t. } c(T^f) \leq 0 \tag{15}
\]

A method to solve (15) was proposed in \[62\] where \( c \) consists of functionality (i.e., power delivery, hybrid functions, feasibility) and cost constraints. Each topology is modelled as an undirected connected finite graph, where each component is a node of the graph. Based on these nodes, a set of constraints are defined and \( 15 \) is solved as a constraint satisfaction problem over finite domains \[63\]. The authors of \[62\], apply this method on a set of 16 power-train components (including two PGS, two EMs, three clutches, etcetera) searching for feasible series, parallel, and series-parallel HEV topologies. They show that the initial search space of \( 5.7 \cdot 10^{45} \) possible topologies is reduced to 4779 feasible topologies.

Another recent method by \[64\] to solve (15) aims at developing series-parallel topologies with one or multiple PGS. This method models a topology as a bond graph and, similar to the previous method, uses constraints to arrive at feasible topologies. Using this method, in \[65\], the topology generation and optimization of a mid-size passenger car is discussed. When series-parallel topologies with double planetary gears is used, in \[66\] a method to automatically model and exhaustively search for optimal topologies is proposed. The authors show, using Toyota Prius as a study case, that improved configurations (offering reduced fuel consumption) are found.

These studies show how the initial set of candidate topologies can be reduced in a systematic and complete way. At the same time, they highlight new challenges in defining and solving this kind of problems.

Once a topology has been decided on, co-design problem (14) is to be solved. Next we distinguish sequential, alternating, nested, and simultaneous methods. Sequential is a special instance of the alternating coordination-strategy (plant and control subproblem are solved only once, sequentially) and is also referred to as a design-first-then-control methodology.

B. Design-First-Then-Control for HEV Design

The design-first-then-control is the simplest strategy one can envision; the coupling between the plant design and control design problem is neglected. Mainly due to its decentralized manner, this strategy has been a pioneer when approaching HEV design. The control problem is approached for a fixed plant, i.e., fixed (a), (b) and (c) layers in Fig. 3.

The development of the control algorithm, i.e., the energy management system (EMS) of a HEV powertrain, consists of finding the set-points of the power converters that can deliver the driver’s required power in an “optimal” way. Optimality is defined in terms of fuel consumption \( J_1 \) from \[9\], but may also include pollutant emissions \( J_4 \) and \( J_5 \) from \[9\], drivability, or performance criteria related to the battery (e.g., life degradation or charge). This optimal control problem, given by

\[
\min J_1(x_p, x_c(t), \Lambda) \quad \text{s.t. } \text{constraints as in eq. (2)} \tag{16}
\]

has been approached by two main categories of methods as depicted in Fig. 6 (i) optimization based methods and, (ii) rule based methods.

The strategies based on rules, either heuristics \[67\] or fuzzy logic \[68\], \[69\], are based on expert knowledge translated into boolean rules, to make the power sources work in their most efficient regions. These algorithms are easy to implement and they don’t require high computation times. Yet they can not offer any proof of optimality of the solution found. They may require significant tuning effort and may change significantly for each topology. This disadvantage has motivated the investigation and the applicability of rigorous optimization algorithms.

There exist a wide variety of optimization algorithms for controller design. Two categories may be distinguished: real-time implementable \[70\] or off-line algorithms \[71\]. Dynamic Programming (DP) is widely used for off-line optimization and DP typically serves as a benchmark for evaluating other (real-time) algorithms \[72\]–\[77\]. There exist also optimization-based algorithms that can be online implementable. These are mostly based on Equivalent Consumption Minimization Strategy (ECMS) \[40\], \[78\]–\[83\], Stochastic Dynamic Programming (SDP) Strategies \[84\]–\[88\], or Model Predictive Control (MPC) Strategies \[89\], \[90\]. Reviews of EMS can be found in review articles as \[91\]–\[97\]. Benchmark comparisons are given in \[98\] and \[99\], where several algorithms are implemented and compared for controlling the Plug-In Chevrolet
Volt HEV. Note again that all these energy/power control algorithms are derived for an a priori defined HEV. Therefore, the dependence between the system design and the control algorithm design is not taken into account. Yet, this coupling exists, e.g., the dimension of the battery will influence the optimal control problem. To overcome this limitation, attempts to design better systems have been developed using design-and-control methodologies (in either an alternating, nested or simultaneous fashion).

C. Alternating, Nested and Simultaneous Coordination Schemes

For each topology, to find the set of optimal \( \mathbf{x}_p^* \) with a nested coordination scheme, various authors [98–106], have used exhaustive search in the plant design optimization problem, combined with a rule based or DP for control design. With exhaustive search, also referred to as brute force search, the design space is gridded and for each grid point the cost function is evaluated [107]. This is depicted in Fig. 7 for the parallel topology from Fig. 6 where the hybridization potential is analysed in terms of fuel consumption for \( \mathbf{x}^{\text{pr}} = \left[ P_m \ C \right]^T \).

Using the values of the cost function at each point the shape of this function can be interpolated, and a design can be chosen. For the sake of clarity, we depict this for two plant design variables only. If more design variables are included the visualisation and interpretation of results will be difficult. Then, Latin Hypercube Sampling can be used to explore the cost function in all the feasible design spaces [108].

In [100], such a nested exhaustive search framework is used to compare four topologies (a conventional, a start-stop, a full parallel HEV and a power-split HEV), for a passenger car application given different driving cycles. Due to hybridization and engine downsizing, the authors present more than 33% CO\(_2\) decrease for the full-parallel and power-split (similar to Toyota Prius) HEVs. In [104], focusing on the transmission selection, three full-parallel hybrid electric drivetrain topologies are investigated. In [102], one-variable-at-a-time exhaustive search is used for the component sizing optimization loop and DP is used for the control algorithm. Considering a series-hybrid microbus, the authors define \( \mathbf{x}_p^* = \left[ P_{e+m1} \ C \right]^T \), with \( P_{e+m1} \) representing the generating group power (i.e., the combined generator motor and engine) and \( C \) representing the battery capacity. With a fixed battery pack, the generating group of the series architecture, \( P_{e+m1} \), is varied in size and the possibility of downsizing or upsizing the engine is analysed. Once a value was found for \( P_{e+m1} \), this is fixed, and the variation on the battery pack sizing is investigated. We refer to this as one-variable-at-a-time exhaustive search, since when mapping this problem to the previous example from Fig. 7 the authors vary one variable at a time, resulting in only one row/column, and repeat this process for all design variables.

The exhaustive search strategy is simple and insightful, but only works for a limited number of plant design variables. The computational burden quickly grows, for increasing number of plant variables. The computational time may be expressed as \( T = T_a \prod_{i=1}^{N} g_i \), with \( N \) the number of plant design variables, \( T_a \) the time in which the optimal control problem is solved and \( g_i \) the number of grid points for variable \( i \). The grid needs to be sufficiently dense to guarantee a reasonable accurate interpolation between the grid points. Alternatively, for increasing number of plant variables, one may consider to use a Latin hyper cube design exploration with a radial basis of Kriging type of surrogate model for the interpolation.

In recent years, the usage of optimization-base multi-level design, (introduced already for different applications [109–111]), has seen an increased interest. By using an optimization algorithm for the plant design problem, one seeks to reduce the number of cost function evaluations, compared to exhaustive search (see for example Fig. 7), with a better exploration of
TABLE I: Classification of several frameworks from existing literature, as a function of \textbf{coordination methods} and \textbf{algorithms} used for sizing and control design (ECMS = Equivalent Consumption Minimization Strategy, (S)DP = (Stochastic) Dynamic Programming, SQP = Sequential Quadratic Programming, SA = Simulated Annealing, PSO = Particle Swarm Optimization, RB = Rule Based, SADE = Self-Adaptive Differential Evolution, DS = Downhill Simplex Method)

<table>
<thead>
<tr>
<th>ALGORITHMS</th>
<th>Component Sizing</th>
<th>Control</th>
<th>COORDINATION METHODS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed</td>
<td>RB</td>
<td>Parallel HE Truck [112]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ECMS</td>
<td>Parallel Small HEV [40]</td>
<td>Through-the-road Midsize HEV [113]</td>
</tr>
<tr>
<td></td>
<td>SDP</td>
<td>Mid-size Series-Parallel HEV [87]</td>
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</tr>
<tr>
<td></td>
<td>DP</td>
<td>Parallel HE Truck [112]</td>
<td></td>
</tr>
<tr>
<td>Nested</td>
<td>RB</td>
<td>Large-size passenger parallel HEV [114], Medium-Duty Parallel HE Truck [101], Small passenger HEV with CVT [57], Torque-Assist Midsize HEV [115], Parallel HE Truck [3]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ECMS</td>
<td>Fuel Cell HE Truck with two in-wheel EMs [103]</td>
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<tr>
<td></td>
<td>DP</td>
<td>Passenger HEV (Parallel [36], [116], Torque-Assist [115], Large Parallel [104], Compact Parallel [98], Several vehicles [100]), Heavy-Duty HE [108], HE microbus [102]</td>
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</tr>
<tr>
<td></td>
<td>SQP</td>
<td>RB PNGV passenger HEV [117]</td>
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<tr>
<td></td>
<td>DP</td>
<td>Parallel HE Class 8 Truck [118]</td>
<td></td>
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<tr>
<td>Exhaustive Search</td>
<td>RB</td>
<td>Parallel passenger HEV [119], Parallel HEV [6], Mid-size HE SUV [120], Mid-size parallel HE SUV [127], Parallel passenger HEV [122]</td>
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</tr>
<tr>
<td></td>
<td>ECMS</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>DP</td>
<td>Hybird and Electric Submarine [128]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SQP</td>
<td>Hybrid and Electric Submarine [128]</td>
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</tr>
<tr>
<td></td>
<td>PSO</td>
<td>RB Parallel passenger HEV [6, 57, 119]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>DP</td>
<td>Midsize Parallel HEV [56, 130], Torque-assist and Parallel passenger HEV [131], Parallel Class 8 HE Truck [129], Series HEV [132]</td>
<td></td>
</tr>
<tr>
<td>Single / Multi-Objective GA</td>
<td>RB</td>
<td>Parallel passenger HEV [119], Parallel HEV transit bus [20], Fuel-cell HEV [126], Parallel HEV [6], Hybrid and Electric Submarine [127]</td>
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</tr>
<tr>
<td></td>
<td>SQP</td>
<td>Hybrid and Electric Submarine [128]</td>
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<tr>
<td></td>
<td>PSO</td>
<td>RB Parallel passenger HEV [6, 57, 119]</td>
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<td></td>
<td>DP</td>
<td>Midsize Parallel HEV [56, 130], Torque-assist and Parallel passenger HEV [131], Parallel Class 8 HE Truck [129], Series HEV [132]</td>
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<tr>
<td>Alternating</td>
<td>SQP</td>
<td>ECMS Mid-size passenger HEV [138]</td>
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</table>

The system level design problem is usually nonlinear and often also has mixed-integer characteristics. In the literature about multilevel optimization of HEV, a wide variety of algorithms has been selected for the plant optimum design. One may distinguish between derivative-free and gradient based algorithms. Examples of derivative free algorithms include: Dividing Rectangles (DIRECT) [122], [139], Particle Swarm Optimization (PSO) [56, 130], Genetic Algorithms (GA) [20, 51, 140, 142] and Simulated Annealing (SA) [123, 143]. Articles that use a gradient based algorithm include Sequential Quadratic Programming (SQP) or Convex Optimization (CO) [133, 136, 137, 144].

When in the system design process separate plant and controller optimization sub-problem are considered, a coordination method between these two optimization layers is needed. Based on the coordination schemes defined in Fig. 5, in Table I a classification of several frameworks from existing literature is shown. This table tabulates the type of algorithm for the plant design problem, the type of algorithm for the control design problem, and the coordination strategy to arrive at the system optimal solution. One may notice that recent studies use either nested, simultaneous or alternating coordination methods to reach an optimal design. The structure of Table I indicates also the evolution of the strategies used. Methods have evolved from sequential to mostly nested plant and controller design. Quite recently, also the simultaneous and alternating coordination schemes have been proposed for use in HEV frameworks, which may provide computational advantages compared to the nested scheme.

Vehicle simulation packages, as ADVISOR [145] or PSAT [146], containing rule based algorithms for HEV control, have facilitated the fast development and simulation of design frameworks. For instance, using a RB control algorithm nested within multi-objective GA (having UDDS as input driving cycle) [51], in [20] the sizing of a parallel hybrid bus is
discussed for multiple objectives, \( J_1, J_4, J_5 \), and \( J_6 \) from \((9)\). Besides the benefits for design, the authors highlight that: (a) the increase of population size of the algorithm will result in improved accuracy of results; (b) no user-supplied weights of each objective must be provided; and, (c) more driving cycles must be used to improve this methodology and the design. This is addressed in \((127) \) and \((128) \), where the same strategy is applied to find the optimal design of a hybrid submarine, investigating three different topologies for four different driving cycles. This study shows that multi-objective GA can handle a very large design problem, with 16 objective functions and a 9 dimensional design space, with both discrete and continuous design variables.

One clear drawback in these studies is the usage of rule based algorithms for controller design, which is sub-optimal. An alternative is to use for example an evolutionary evolutionary algorithm as Particle Swarm Optimization (PSO) in combination with DP for optimizing the control strategy, as used in \((56) \) and \((130) \). In this novel framework, Dynamic Programming ensures finding the optimal control policy for every population point candidate selected by PSO in the outer-loop. The authors use this framework to optimally size and control a parallel passenger HEV, and compare its results with previously developed frameworks, that use SQP in the outer loop (plant design) and RB algorithms in the inner loop (controller design). It is shown that RB algorithms are less fuel efficient (by 11% for this case) and lead to a more expensive system (by 14%) than optimal solutions obtained by PSO.

The frameworks that solve the plant design problem using stochastic algorithms such as PSO, GA, or SA, or using deterministic search algorithms such as DIRECT, can handle nonlinear cost function and constraints, searching the design space globally. Yet, when the cost function behaves smooth and has only few local minimizers, a derivative based algorithm will offer a faster solution to the optimization problem. Also, a larger number of plant variables can be addressed in that case.

The typically used \( J_1 \) cost function from \((9) \) is multimodal (with many local minima), and sometimes noisy and discontinuous \((122) \). To ensure the receivability of the global optimum, in \((133), (144), (147) \) and in \((22) \) the HEV design problem is formulated as a convex optimization problem, with proposed convex component models and integer control signals obtained by heuristics. Comparative studies of the gradient-based and the derivative-free algorithms for HEVs optimal design are presented in \((148) \). Further, comparisons between only the derivative-free algorithms for HEVs optimal design can be found in \((122) \) and \((119) \). Choosing one optimization algorithm, to find the optimal solution to each design layer, is is not straightforward, it depends strongly on the problem set-up and will briefly be described next.

V. TRENDS IN OPTIMAL SYSTEM LEVEL DESIGN FOR HEVs

An important driver for optimization approaches in HEV vehicle design is the legislative restrictions which have become increasingly tight during the last two decades. Emission regulations have evolved from Euro 1 in 1993 to Euro 6 in 2014 (changing both permissiveness (e.g., CO\(_2\) levels) and focus (e.g., from CO\(_2\) to NO\(_x\) or PM)). The number of yearly publications on HEV optimization approaches has steadily grown (see Fig. 8). Within the hybrid vehicle research publications area, the plant and control design areas have also grown in recent years.

![Fig. 8: Research trends in hybrid vehicles design and optimization algorithms used. The curve shows the number of papers in the Google Scholar database containing the key words hybrid vehicle and the keywords in the legends as parts of their title.](image)

When defining an optimization problem, its target is a formal transposition of vehicle manufacturer preferences on the constructed system. In turn, the manufacturer tries to meet all legislative restrictions and create a vehicle competitive on the market, appealing to customers and financially-beneficial. In this frame-up, the challenge to have a general problem definition is even bigger, since these dependencies are changing over time (e.g., emissions regulations). These challenges have led to constant development of control algorithms for HEVs (named either Supervisory Control or Energy Management Systems). In Fig. 8 one can see an ascending trend in the use of Dynamic Programming as a control algorithm. In fact, DP is used as a benchmark comparison for the development of other algorithms (real time implementable).

For solving the problem of optimal system design there is no universally accepted or widely used algorithm (as for example in control design DP). The trend in algorithms selection, for component sizing is to use evolutionary optimization algorithms. Among these, most commonly used optimization algorithms are GA and PSO, has Shown in Fig. 13. Furthermore, multiple research articles report the computational inefficiency of exhaustive search, that leads to its inapplicability for large multi-dimensional design spaces.
Another trend is the increased focus on the driving cycles used in the HEV optimization problem formulation. Each manufacturer will design a car suitable for certain road types (road (e.g., highway, in-city, inter-urban), off-road, ship, rail or air) and applications (e.g., heavy duty vehicle, passages, bus), that will use a specific driving cycle. These range from high speed highway driving on flat road, to city driving with altitude variations, and all the variations in-between [154]. The ideal HEV should be fuel efficient in all situations in which it is used. In most cases, designers/researchers choose to vary the driving cycle in the design step of the hybrid vehicle to have a more efficient vehicle (in terms of energy) [103].

Also, synthetic cycles can be constructed to be shorter (enabling thus faster simulations or larger design space explorations) but more representative of the actual driving cycles [153]. In this direction, the methods based on Markov Chain theory show promising results, as presented in [52], [154], [155].

Depending on the shape of the optimization function, and the types of constraints, an optimization algorithm may prove to be better than others. Typically, the road types and applications dictate a choice of topology, eliminating layers (a) and (b) in Fig. 3.

In [122] and [129] different optimization algorithms, for the sizing loop (plant design), are compared to find the optimal design for one topology. For the control, one algorithm is used in all cases. At the expense of larger batteries, GA reaches a design with 7% reduced fuel consumption. Next, a design that doesn’t require engine downsizing is reached with PSO algorithm, where the 5% fuel consumption is achieved with a smaller electric machine. Without continuing with this analysis, one must be aware that these results are sensitive to how the algorithms are tuned (such as maximum number of function evaluations and, to what supervisory control algorithm is used).

In the case of a strong nonlinear optimization function, the algorithms that use the gradient of the function, as SQP, often converge to a local minimum. To avoid premature convergence and local optima, one can start from different initial points, $x_{p0}$ or use a global optimization algorithm, as GA, PSO or another. Population-based evolutionary algorithms, as GA, PSO and SA, will have overall more function evaluations then gradient-based algorithms, since at each iteration (generation) they will evaluate $J$ for multiple starting points $x_{p0}$ (often named population).

Summarizing, different tricks must be made when one desires to use a certain kind of optimization algorithm for sub-problem solving: (i) when convex optimization is used, the convexification of the optimization problem is required to guarantee finding the global optimum; (ii) when SQP is used, for the original problem (non-convex), the initial point $x_{p0}$ can be varied to test the reach of local or global minimum; and (iii) when evolutionary algorithms are used various parameters have to be tuned (e.g., population size). Also, as stated earlier, it is important what coordination strategy is used, and which decomposition paradigm (overviews of such paradigms are found in [157] or [158]).

Designing a HEV with explicitly considering the coupling between the plant and its control has proved more promising than sequential design. These novel design approaches (nested or simultaneous) were investigated for the main components of the propulsion, i.e. electric motor, battery and combustion engine. Following this trend of combining the plant and control design, in the future more components can be considered as variables in the design process. Examples can include auxiliary units, e.g., air conditioning system or the power steering system, as considered in [3]. With the the inclusion of more components as variables, the design problem becomes more difficult to define and handle.

VI. Conclusions

This paper reviewed the current state of design of hybrid vehicles, including architecture, sizing components, control algorithm and methods of finding the optimal system level design. Although, at first glance, there seem to be three major classes of HEV topologies to chose from (serial, parallel and serial-parallel), current market vehicles prove that minor design changes can lead to significant improvements in fuel consumption, costs of electrification, performance and generated emissions. These small changes, like the addition of a clutch or resizing the battery, cause many changes in different design levels (both at the subsystem level as well as at the system level). Thus, the interaction between components is becoming increasingly important and, neglecting it in the design step leads to loss of potential after hybridization.

Starting with sequential designs, usually made in a top-down manner, a transition to coupled plant and control designs commenced in the last decade. The most popular variant being controller design nested within plant design. These approaches prove clear advantages but also introduce several challenges in solving this optimization problem. Sequential design is simple and intuitive, but neglects the influence of the plant design on the controller design. The plant is designed without taking the controller into account. Subsequently, the controller is designed using the given design as is.

Bi-level optimization frameworks take the coupling between plant and controller design into account. One may distinguish a nested and an alternating formulation. Often used, nested optimization poses more challenges on finding a global optimal solution at the system level and creates a shift towards multi-disciplinary design. Even so, recent studies have shown that, HEV designs with significantly lower fuel consumption and emissions can be found. These are opportunities to be further investigated.

By analyzing existing publications, we can conclude that using optimization algorithms, to solve different optimization layers, have proven beneficial for design. These could be further used, in more extended coordination methods to include the selection of topologies and technologies. For instance, these extended coordination methods might include: (i) simultaneous topology and sizing design) alternating with controller design; (ii) controller design nested with respect to simultaneous topology and sizing; (iii) topology alternating with sizing alternating with control; or (iv) simultaneous topology, sizing, and control design.
To substantially reduce the computational burden one can introduce approximations of the original problem (e.g., the convexification of the problem or metaheuristic models), can shorten the driving cycle used for design or can use parallel computing. Driving cycles used as input for the control algorithm (energy management strategy) should be build short, more realistic and more representative of realistic driving types.

How to address, in an (more) automatic way, multiple topologies with a large variety in the components types and numbers remains an open question. Further, the topology automatic construction and optimization problems create challenges in the control algorithm development, that has to handle various topologies in an automatic way. To solve the system level design problem and find a HEV that can be market competitive, one may define the optimization targets to include besides fuel, also costs, emissions or performance aspects. Easy-to-use methodologies must be developed, to help developers, and industry in general, to reach better designs in early steps of HEV development process.

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