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Design and Modeling Aspects in Multivariable Iterative Learning Control

Lennart Blanken, Sjirk Koekebakker, Tom Oomen

Abstract— **Iterative Learning Control (ILC) can significantly improve the performance of systems that perform repeating tasks. Typically, several decentralized ILC controllers are designed and implemented. Such ILC designs tacitly ignore interaction. The aim of this paper is to further analyze the consequences of interaction in ILC, and develop a solution framework, covering a spectrum of systematic decentralized designs to centralized designs. The proposed set of solutions differs in design, i.e., performance and robustness, and modeling requirements, which are investigated in detail. The benefits and differences are demonstrated through a simulation study.**

I. INTRODUCTION

Iterative Learning Control (ILC) [1] is widely used in control systems, since it can significantly improve the performance of systems that perform repeating tasks. Many successful applications have been reported, including additive manufacturing [2], industrial robots, microscopic imaging [3], printing systems [4], and wafer stages [5].

The observation that many ILC applications are multivariable has lead to the development of ILC theory for multivariable systems. Most of such algorithms have been developed in the so-called lifted or supervector framework [6], where the ILC controller follows from an optimization problem, see e.g., [7].

To deal with modeling errors, a key aspect of ILC algorithms includes robust convergence properties. Optimization-based algorithms have been further extended in, e.g., [5], [8], [9]. These approaches rely on a detailed specification of the nominal model and its uncertainty in some prescribed form. Despite being very systematic, this imposes a large burden on the model requirements, since the modeling of uncertainty is not straightforward [10]. An interesting fully data-driven alternative is presented in [11], but this comes at the requirement of a high experimental cost.

Although important developments have been made in the theory of robust multivariable ILC controllers, these approaches are often not employed due to high requirements on uncertainty modeling. Instead, in many applications frequency-domain based ILC controllers [1], [6], [12] are designed in a decentralized manner. Indeed, compared to a norm-optimal framework, frequency-domain ILC controllers impose a smaller modeling requirement, especially regarding the uncertainty, which can be based on accurate and inexpensive frequency response function (FRF) measurements [5], [13], [14]. However, since such approaches focus on multi-loop SISO systems, interaction is typically ignored, which potentially can lead to non-convergent algorithms.

The main contribution of this paper is a systematic framework for analysis and synthesis of multivariable ILC, while addressing modeling and robustness aspects arising in typical ILC applications, including mechatronic systems [15]. The proposed solutions range from decentralized to centralized designs with various levels of sophistication. A decentralized design is proposed that *a posteriori* robustifies for interaction, which is a direct extension of existing results. The contributions of this paper are: i) a decentralized design based on the structured singular value (SSV) [16] where the interaction is included in the independent decentralized designs, and ii) several centralized multivariable ILC controllers, including extensions towards multivariable zero-phase error tracking control (ZPETC) algorithms [17], which is a useful technique for non-minimum phase (NMP) systems, see, e.g., [18] for a NMP application. The present paper extends the results of [19], which focuses on implementation aspects rather than modeling and robustness aspects in the ILC design.

The proposed framework involves frequency-domain ILC designs [1], [15], and the multivariable aspect builds on the results in [20], [21]. The research in this paper extends and relates to the work presented in [22], [23], [24], [25], [26]. In [22], [23], multivariable Arimoto-type ILC is investigated; [24] addresses systems with unidirectional coupling; and the approach in [25], [26] considers strictly positive real systems.

In the next section, the frequency-domain multivariable ILC design problem is formulated. In Section III, the decentralized ILC control problem is analyzed. In Section IV, an approach to decentralized ILC is presented through a series of independent SISO designs. In Section V, approaches to centralized ILC design are proposed. In Section VI, benefits of the proposed approaches are demonstrated in a simulation study. Finally, conclusions are provided in Section VII.

Notation: $\mathbb{R}[z]$ denotes the polynomial ring in indeterminate z with coefficients in $\mathbb{R}$. $\mathcal{R}(z)$ denotes the field of real rational functions. The space consisting of all square summable sequences is denoted ℓ_2. Given $f(z), g(z) \in \mathbb{R}[z]$, $g(z)$ divides $f(z)$ if there exists a $h(z) \in \mathbb{R}[z]$ such that $f(z) = g(z)h(z)$. A polynomial matrix $U(z) \in \mathbb{R}^{n\times n}[z]$ is called unimodular if and only if $U^{-1}(z) \in \mathbb{R}^{n\times n}[z]$. A_{d} denotes the diagonal matrix containing the diagonal elements of A. Throughout, all systems are assumed to be discrete-time, multi-input multi-output (MIMO), and linear time-invariant. The complex indeterminate z is omitted when this does not lead to any confusion.

L. Blanken and T. Oomen are with the Eindhoven University of Technology, Dep. of Mechanical Engineering, Control Systems Technology Group, Eindhoven, The Netherlands. S. Koekebakker is with Océ Technologies B.V., Venlo, The Netherlands. Email: l.l.g.blanken@tue.nl.

II. PROBLEM FORMULATION

In this section, the multivariable ILC design problem is formulated, and challenges are indicated motivating the analysis and development of (de)centralized ILC design techniques in Sections III to V.

A. ILC Setup

Consider the control configuration depicted in Figure 1, consisting of a true system $P(z) \in \mathcal{R}^{p \times q}(z)$, and a stabilizing feedback controller $C(z) \in \mathcal{R}^{q \times p}(z)$. Each repetition j of the reference signal r is called a task. The feedforward is denoted f_j, and y_j is the output. The reference is iteration-invariant, i.e., $r_j = r$. The error e_j in task j is given by

$$e_j = Sr - S_P f_j, \tag{1}$$

with $S = (I + PC)^{-1}$ and $S_P = SP$.

The goal of ILC is to minimize, or achieve convergence of, e_j in terms of an appropriate norm. To this purpose, e_j is measured in task j and used to construct f_{j+1} for task $j+1$. Typically, the following general ILC algorithm is invoked:

$$f_{j+1} = Q(f_j + Le_j), \tag{2}$$

with $L \in \mathcal{R}^{q \times p}(z)$, and $Q \in \mathcal{R}^{q \times q}(z)$.

B. Convergence Analysis

Combining (1) and (2) yields the following expression for the closed-loop feedforward propagation

$$f_{j+1} = Q(I - LS_P)f_j + QLSr. \tag{3}$$

A condition for convergence of (3) is given next.

Theorem 1. *Consider the control configuration in Figure 1, and suppose $f_j, e_j \in \ell_2$. Then, the ILC algorithm (2) converges to a fixed point f^* if*

$$\rho\left(Q(e^{j\omega})(I - L(e^{j\omega})S_P(e^{j\omega}))\right) < 1,\ \forall \omega \in [0, 2\pi], \tag{4}$$

where $\rho(H) = \max_i |\lambda_i(H)|$ denotes the spectral radius.

For a proof of Theorem 1, see [27, Theorem 6]. Although this result guarantees convergence of f, it does not guarantee good learning transients. The following theorem provides a condition for monotonic convergence of (3).

Theorem 2. *Consider the control configuration in Figure 1, and suppose $f_j, e_j \in \ell_2$. Then, the ILC algorithm (2) converges monotonically in $\|f\|_2$ to a fixed point f^* if*

$$\|Q(I - LS_P)\|_\infty < 1 \tag{5}$$

where $\|H\|_\infty = \sup_{\omega \in [0,\pi]} \bar{\sigma}(H(e^{j\omega}))$ is the $\mathcal{L}_\infty$-norm, and $\bar{\sigma}$ denotes the maximum singular value.

A proof of Theorem 2 is omitted, since it follows along similar lines as [6, Theorem 3.1] for the $\mathcal{H}_\infty$-norm, and can be appropriately extended to the $\mathcal{L}_\infty$-norm to account for noncausal L, Q using [28, Theorem 2.1.10].

If (4) or (5) is satisfied, the fixed point f^* is given by

$$f^* = (I - Q(I - LS_P))^{-1} QLSr,$$

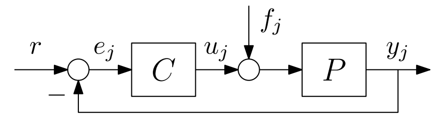

Fig. 1. ILC control configuration.

and the resulting fixed point of the error is given by

$$e^* = \left(I - S_P\left(I - Q(I - LS_P)\right)^{-1} QL\right) Sr. \tag{6}$$

Theorems 1, 2 and (6) lead to the observation that $LS_P = I$ and $Q = I$ yield $e^* = 0$. In decentralized ILC, L is typically designed as $(S_P)_{\mathrm{d}}^{-1}$. However, this does not imply $LS_P = I$, which reveals potential issues due to neglected interaction.

C. Problem Formulation

The problem addressed in the paper is to propose and analyze ILC designs with respect to the following requirements:

R1) Robust convergence, i.e., Theorems 1 and 2;
R2) High performance, i.e., (6) small;
R3) Low modeling requirements.

Two sub-cases of R3 are considered: i) parametric models needed for the design of L, which are often expensive to obtain, especially for MIMO systems, and ii) nonparametric models, e.g., FRF measurements, for the design of Q, which are often accurate, inexpensive and fast to obtain [13], [14].

III. DECENTRALIZED ILC IGNORING INTERACTION

In this section, the design of decentralized ILC schemes is analyzed. Although tuning and design of decentralized ILC schemes is relatively simple and intuitive, it is shown that ignoring interaction can lead to non-convergent systems. As an almost straightforward extension of existing results, the ILC scheme is *a posteriori* extended by a robustness filter to account for ignored interaction, which is shown to lead to potential conservatism. It is assumed that $P \in \mathcal{R}^{p \times p}(z)$, possibly after a squaring-down process, see, e.g., [29].

A. Design of Decentralized ILC Ignoring Interaction

In this section, P is assumed given. Often, through appropriate input/output selection and decoupling, the interaction is low, e.g., up to the control bandwidth. Indeed, this is often done in practice for feedback design. Such systems allow for decentralized ILC design. In this section, decentralized ILC design is analyzed when ignoring interaction.

Advantages of decentralized ILC include i) simple design procedures, and ii) relatively small modeling requirements. This shows from considering the decentralized learning filter

$$L = \mathrm{diag}\{L_1, L_2, \ldots, L_p\},$$

with $L_i(z) \in \mathcal{R}(z)$. Typically, it is aimed to design each L_i such that $L_i(S_P)_{ii} = 1$, see, e.g., [1], [6], [12], for example using ZPETC [17] or stable inversion procedures, see, e.g., [30], [31]. Indeed, to this purpose only p SISO models of $(S_P)_{ii}$ are required. These models are often obtained based on i) independent identification of the elements $(S_P)_{ii}$, or ii) a set of parametric models based on first principles modeling.

If all loops are independent, i.e., no interaction is present, then Q needs only to be designed to account for modeling errors in each independent loop. That is, each element of

$$Q = \operatorname{diag}\{Q_1, Q_2, \ldots, Q_p\}, \tag{7}$$

with $Q_i(z) \in \mathcal{R}(z)$, is designed to satisfy

$$\left|Q_i(e^{j\omega})(1 - L_i(e^{j\omega})(S_P)_{ii}(e^{j\omega}))\right| < 1, \; \forall i, \omega \in [0, 2\pi]. \tag{8}$$

Note that (8) can be verified based on FRF measurements of each $(S_P)_{ii}$. For the diagonal case, condition (8) guarantees monotonic convergence of the ILC system, see Theorem 2.

B. Accounting for Interaction Through Robustness

In the previous section, traditional SISO design procedures are described for decentralized ILC. However, interaction in the system is ignored. In this section, a decentralized Q-filter is synthesized, providing robustness to ignored interaction *a posteriori*. This is a direct extension of existing results.

The decentralized Q-filter is designed as $Q = Q_d I$, with $Q_d(z) \in \mathcal{R}(z)$. Then, the control problem simplifies to the design of SISO filter Q_d. Rewriting (5) shows that the decentralized ILC system is monotonically convergent if

$$\begin{aligned}
&\bar{\sigma}\left(Q(e^{j\omega})(I - L(e^{j\omega})S_P(e^{j\omega}))\right) \\
&\leq \bar{\sigma}(Q(e^{j\omega}))\bar{\sigma}\left(I - L(e^{j\omega})S_P(e^{j\omega})\right) \\
&= \left|Q_d(e^{j\omega})\right| \bar{\sigma}\left(I - L(e^{j\omega})S_P(e^{j\omega})\right) < 1, \; \forall \omega \in [0, 2\pi].
\end{aligned} \tag{9}$$

It can be observed in (9) that Q_d provides robustness to *all* modeling errors and interaction in the system combined. This is illustrated through factorization of (5) according to

$$\begin{aligned}
&\bar{\sigma}\left(Q(e^{j\omega})(I - L(e^{j\omega})S_P(e^{j\omega}))\right) \\
&= \bar{\sigma}\left(Q(e^{j\omega})(I - L(e^{j\omega})(S_P)_{\mathrm{d}}(e^{j\omega}))(I + E(e^{j\omega}))\right) \\
&< 1, \; \forall \omega \in [0, 2\pi],
\end{aligned} \tag{10}$$

where E denotes the interaction in $I - LS_P$, normalized with respect to the diagonal elements, defined by

$$E = -(I - L(S_P)_{\mathrm{d}})^{-1}(LS_P - L(S_P)_{\mathrm{d}}). \tag{11}$$

Two key observations are made on the basis of (10). First, satisfaction of (8) does not imply (5). This illustrates that ignoring interaction in decentralized ILC design is potentially dangerous. Second, when designing Q_d according to (9), combined robustness is provided to both modeling errors, contained in $I - L(S_P)_{\mathrm{d}}$, and interaction, i.e., E. Consequently, e.g., large modeling errors in a single loop affects the Q-filter design for all other loops, leading to conservatism.

C. Concluding Remarks

In this section, the design of decentralized ILC is investigated, and is it shown that only SISO models of the system are required. An analysis shows that ignoring interaction is however potentially dangerous, as it can lead to non-convergent systems. To this purpose, a Q-filter is synthesized which *a posteriori* provides robustness. Clearly, Q may be unnecessarily conservative as it accounts for the *worst-case* interaction. This motivates the development systematic independent Q-filter designs, where each decentralized ILC loop i is *robustified* through the corresponding Q_i separately.

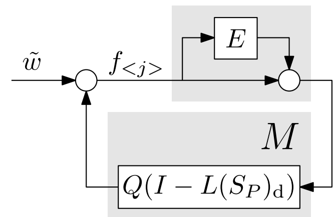

Fig. 2. Feedforward propagation structure operating in the trial domain for studying the effect of interaction on ILC. Note that $I + E$ is invariant to Q, and measures on $I + E$ can be developed to design for convergence.

IV. ACCOUNTING FOR INTERACTION THROUGH INDEPENDENT Q-FILTER DESIGNS

In this section, the first main contribution is presented. A technique based on the structured singular value (SSV) is proposed to account for interaction in each loop independently. For SISO systems, Q needs only to be designed for the modeling error, whereas for MIMO systems Q must be designed to account for modeling errors and interaction. Indeed, these two aspects are mixed up in the approach presented in Section III-B. In this section, the objective is to propose a unified approach, which individually addresses modeling errors and interaction. This potentially leads to less conservative design of Q, and hence improved performance.

Motivated by (10), the propagation (3) is expressed as

$$f_{j+1} = Q(I - LS_P)f_j + \tilde{w} = M(I + E)f_j + \tilde{w}, \tag{12}$$

where $\tilde{w} = QLSr$, $M = Q(I - L(S_P)_{\mathrm{d}})$, and E as (11). Figure 2 schematically illustrates the iterative scheme (12), operating in the domain of j. It is key to observe that the interaction term E is not exploited in the design of Q in (9). This is the main motivation for the presented approach.

The magnitude of E is often used as an interaction, or uncertainty, measure. In the framework of feedback interconnections, see, e.g., [16, Chapters 9, 11] and [21, Chapter 8], the magnitude of E can be used to analyze robust stability of the uncertain interconnected system. For ILC, E can be used to analyze convergence of the system to be designed.

Lemma 1. *Consider the control configuration in Figure 2, and suppose $f_j, e_j \in \ell_2$. Then, the ILC algorithm* (2) *converges to a fixed point f^* if*

$$\bar{\sigma}(M(e^{j\omega})) < \mu_M^{-1}(I + E(e^{j\omega})), \quad \forall \omega \in [0, 2\pi], \tag{13}$$

where $\mu_M(I + E)$ is the structured singular value, see, e.g., [16], of $I + E$ with respect to the (diagonal) structure of M.

Note that in Lemma 1, the SSV is employed in a fundamentally different way than in traditional stability analyses of feedback interconnections. In robust control approaches, see, e.g., [16, Chapters 9, 11] and [21, Chapter 8], normally $\mu_\Delta(M)$ is taken, where M is the nominal model, and the SSV is taken with respect to the structure of uncertainty Δ. Here, one may intuitively think that E is the uncertainty, but this is not the case. Indeed, E has the role of nominal model, and the diagonal ILC, i.e., M, is still unknown and to be designed. Hence, M constitutes the uncertainty with diagonal structure. The following result is essential for designing M.

Theorem 3. *Consider the control configuration in Figure 2 with decentralized Q-filter* (7)*, and suppose* $f_j, e_j \in \ell_2$. *Then, the ILC algorithm* (2) *converges to a fixed point* f^* *if*

$$|Q_i(e^{j\omega})(1 - L_i(e^{j\omega})(S_P)_{ii}(e^{j\omega}))| < \mu_M^{-1}(I + E(e^{j\omega})), \quad \forall i, \omega \in [0, 2\pi]. \tag{14}$$

Theorem 3 is crucial for the design of decentralized Q-filter (7), since the MIMO design problem of Q is formulated as a set of independent SISO designs. The bound on each individual design can be computed a priori based on non-parametric FRF measurements, and guarantees convergence of the MIMO system when satisfied for each independent loop. This design approach is summarized next.

Procedure 1. *Design procedure for decentralized ILC*

1) *Obtain a parametric model of the elements* $(S_P)_{ii}$.
2) *Compute* L_i *through inversion of* $(S_P)_{ii}$*, e.g., using ZPETC [17] or stable inversion, see, e.g., [30], [31].*
3) *Obtain a (non)parametric model of* S_P*, e.g., using FRF measurements.*
4) *Compute* $\mu_M(I + E)$*, as in* (13).
5) *Design each* Q_i *based on the SISO condition* (14).

In this section, a technique is proposed for independent SISO Q-filter designs, which provide robustness for ignored interactions in the L-filter design. In the next section, approaches are presented which explicitly account for interaction through centralized L-filter designs, thus potentially imposing smaller requirements on robustness.

V. ACCOUNTING FOR INTERACTION THROUGH CENTRALIZED L-FILTER DESIGNS

In this section, the second main contribution of this paper is presented. Several approaches are presented to design centralized L-filters, which explicitly account for interaction. The main goal is to compute an $L \in \mathcal{R}^{q\times p}(z)$ for the possibly non-square system $S_P \in \mathcal{R}^{p\times q}(z)$, such that $LS_P = I$. This potentially improves performance with respect to decentralized ILC design, as discussed in previous sections. The approaches all require a MIMO parametric model of S_P.

In Section V-A, an extension is proposed of the widely-used ZPETC [17] towards multivariable systems. In Section V-B, a stable inversion procedure is described which allows for inversion of non-minimum phase models of S_P.

A. Centralized L-Filter Design Based on ZPETC

In this section, a design is proposed for a centralized L-filter. Often, for decentralized L-filter designs the well-known ZPETC algorithm is used [17]. ZPETC yields an approximate, but stable, inverse of systems with non-minimum phase dynamics, i.e., unstable zeros. Here, an extension to multivariable systems is proposed, which recovers the traditional ZPETC for SISO systems as a special case. The procedure exploits the Smith-McMillan form of rational matrices. First, the required theory is presented. Then, a procedure is provided for the design of the L-filter.

1) Smith-McMillan Form: First, S_P is reduced to its Smith-McMillan form $\mathcal{M}$, see, e.g., [16, Lemma 3.26].

Lemma 2. *(Smith-McMillan form). Let* $S_P(z) \in \mathcal{R}^{p\times q}(z)$ *be of normal rank* r*. Then, there exist unimodular matrices* $U(z) \in \mathbb{R}^{p\times p}[z]$, $V(z) \in \mathbb{R}^{q\times q}[z]$ *such that*

$$U(z)S_P(z)V(z) = \mathcal{M}(z) = \mathrm{diag}\{\tfrac{\varepsilon_1(z)}{\psi_1(z)}, \ldots, \tfrac{\varepsilon_r(z)}{\psi_r(z)}, 0\}, \tag{15}$$

where each pair $\varepsilon_i(z)$ *and* $\psi_i(z)$ *is coprime,* $\varepsilon_i(z)$ *divides* $\varepsilon_{i+1}(z)$*, and* $\psi_i(z)$ *divides* $\psi_{i+1}(z)$.

The roots of polynomials $\prod_{i=1}^r \psi_i(z)$ and $\prod_{i=1}^r \varepsilon_i(z)$ in (15) are the poles and transmission zeros of S_P, respectively. Thus, $\mathcal{M} = \mathrm{diag}\{\mathcal{M}_i, 0\}$ contains the poles and zeros of S_P on its diagonal. Next, ZPETC for SISO systems is discussed, which is consequently extended to MIMO systems.

2) Zero-Phase Error Tracking Control for SISO Systems: Considering each element $\mathcal{M}_i$ of $\mathcal{M}$ as an independent SISO subsystem, the ZPETC algorithm [17] can be applied straightforwardly. For $\mathcal{M}_i$, the ZPETC $L_{\mathcal{M},i}$ is given by

$$L_{\mathcal{M},i}(z) = z^{d_i}\frac{\psi_i(z)\varepsilon_{i,u}(z^{-1})}{\beta_i\varepsilon_{i,s}(z)}, \qquad i = 1, \ldots, r, \tag{16}$$

where d_i represents a delay or time advance of $\mathcal{M}_i$, $\varepsilon_{i,s}$ contains the remaining zeros of ε_i inside the unit disk, and $\varepsilon_{i,u}$ contains the zeros of ε_i outside the unit disk. This yields

$$L_{\mathcal{M},i}(z)\mathcal{M}_i(z) = \frac{\varepsilon_{i,u}(z)\varepsilon_{i,u}(z^{-1})}{\beta_i}, \qquad i = 1, \ldots, r,$$

which has zero phase. Parameter β_i can be chosen such that $|L_{\mathcal{M},i}(z)\mathcal{M}_i(z)| = 1$ at some frequency, e.g., $\omega = 0$.

3) Centralized Learning Filter: In (16), a stable filter $L_\mathcal{M}$ is designed which approximates $\mathcal{M}^{-1}$. Next, a centralized L-filter is constructed for S_P. To achieve this, select

$$L = VL_\mathcal{M}U, \tag{17}$$

such that $LS_P = L\left(U^{-1}\mathcal{M}V^{-1}\right) = VL_\mathcal{M}\mathcal{M}V^{-1}$. Three observations are made. First, L is stable since $L_\mathcal{M}$ is stable and U, V are unimodular. Second, in case S_P has no unstable zeros, $LS_P = I$ for all frequencies. Third, if S_P has unstable zeros, $L_\mathcal{M}\mathcal{M} \neq I$ for some frequencies. Although $L_\mathcal{M}\mathcal{M}$ has zero phase, there is no guarantee that LS_P has zero phase due to operations with transfer matrix V.

Using the results stated above, the procedure for centralized L-filter design is summarized next.

Procedure 2. *Centralized L-filter design based on ZPETC*

1) *Obtain a parametric model of* S_P.
2) *Determine the Smith-McMillan form* $\mathcal{M}$ *of* S_P*, as in* (15).
3) *Compute a SISO ZPETC for each* $\mathcal{M}_i$ *as in* (16).
4) *Construct the MIMO* L *in* (17)*, with* U, V *from step 2.*

In this section, a design for a stable centralized L-filter is provided. In case the model of S_P has non-minimum phase dynamics, L provides phase cancellation for these dynamics, and as such $LS_P \approx I$. In the next section, an approach is presented which enables exact inversion of non-minimum phase models S_P through a stable inversion procedure.

TABLE I

OVERVIEW OF APPROACHES COMPARED IN SECTION VI.

L-filter design, Section	Decentralized Q-filter design	Q_1 f_c [Hz]	Q_1 Order	Q_2 f_c [Hz]	Q_2 Order	
Decentr., III-A	Conservative (9)	2.4	1	2.4	1	□
Decentr., III-A	Independent (14)	1.8	1	4.1	1	○
Centr., V-A	Independent (14)	2.2	2	2.2	2	×
Centr., V-B	Independent (14)	2.2	2	2.2	2	◇

B. Centralized L-Filter Design Through Stable Inversion

In this section, a stable inversion approach is proposed for centralized L-filter design. A key aspect of this method is that non-minimum phase models S_P can be inverted exactly. Note that if $L = S_P^{-1}$ is unstable, the output can become unbounded. In stable inversion, the unstable filter is seen as a non-causal operator instead, see, e.g., [32, Section 1.5].

Next, the required steps for computing $f_{L,j} = Le_j$ are presented, see (2) with $f_{j+1} = Q(f_j + f_{L,j})$. It is assumed that L is a proper transfer matrix, which is a nonrestrictive assumption as the computation of $f_{L,j}$ is performed off-line and preview-based techniques can be used, see, e.g., [33].

1) Let $L(z)$ have a state-space realization (A, B, C, D).
2) Transform this system under similarity to

$$\begin{bmatrix} x_s(k+1) \\ x_u(k+1) \end{bmatrix} = \begin{bmatrix} A_s & 0 \\ 0 & A_u \end{bmatrix} \begin{bmatrix} x_s(k) \\ x_u(k) \end{bmatrix} + \begin{bmatrix} B_s \\ B_u \end{bmatrix} e_j(k),$$
$$f_{L,j}(k) = \begin{bmatrix} C_s & C_u \end{bmatrix} \begin{bmatrix} x_s(k) \\ x_u(k) \end{bmatrix} + De_j(k), \quad (18)$$

where all stable poles of L are contained in A_s, and the unstable poles in A_u.
3) Solve x_s forward in time, x_u backward in time, and construct $f_{L,j}$ as in (18).

C. Concluding Remarks

In this section, two algorithms are proposed for centralized L-filter design. Both approaches approximate the optimal choice for L, being $LS_P = I$. The approach in Section V-A uses a finite preview, and the approach in Section V-B involves boundary truncation effects. Future research includes investigation of these aspects with respect to performance.

VI. SIMULATION STUDY

In Sections III to V, a framework for (de)centralized design for ILC is presented. Next, the proposed approaches are analyzed and compared through simulations.

Consider the system $S_P \in \mathcal{R}^{2\times 2}(z)$ in Figure 3, corresponding to a 12$^{\text{th}}$-order motion system with flexible dynamics. The sampling frequency is 200 Hz. An 8$^{\text{th}}$ order parametric model of S_P is used for ILC design, which has two NMP zeros due to fast sampling. Since S_P is stable, all simulations are performed in open loop. The system is repeatedly excited by reference signals of length $n = 601$ samples, shown in Figure 4. In the first trial $j = 0$, $f_0 = 0$.

Table I lists the considered approaches. For decentralized ILC, each L_i is designed using ZPETC. All Q_i are low-pass Butterworth filters, with cut-off frequency and order specified in Table I. Each Q is applied using the Matlab$^{\text{TM}}$ function `filtfilt` to obtain zero-phase behaviour. Figure 5 depicts

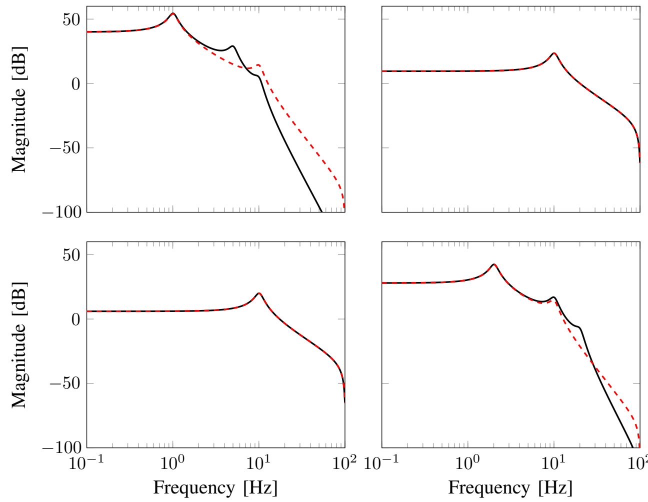

Fig. 3. Bode diagram of true system S_P (——) and model (– –).

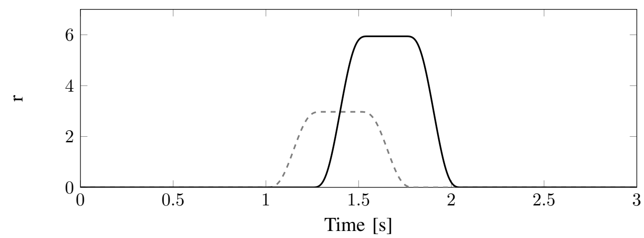

Fig. 4. Fourth order reference signals for input 1 (– –) and input 2 (——).

the matrix norm $\|e_j\|_2 = \bar{\sigma}(e_j)$, where $e_j \in \mathbb{R}^{n\times 2}$, as a function of iterations. The following observations are made:

- Considering decentralized ILC schemes, the cut-off frequencies of the independent Q-filter designs (○) vary significantly from the conservative design $Q = Q_d I$ (□). This is explained by modeling errors, see Figure 3. In loop 1, the modeling error plays a dominant role above 2 Hz. In loop 2 the modeling error is significantly smaller. Through independent designs of the Q-filter, modeling errors are addressed separately per loop. This key difference leads to an improved asymptotic error, see Figure 5.
- Both centralized L-filter designs outperform the decentralized L-filters in terms of asymptotic error. This is explained by the fact that interaction is explicitly accounted for through centralized L-filter design, whereas decentralized approaches require robustness to ignored interaction, which leads to performance degradation.

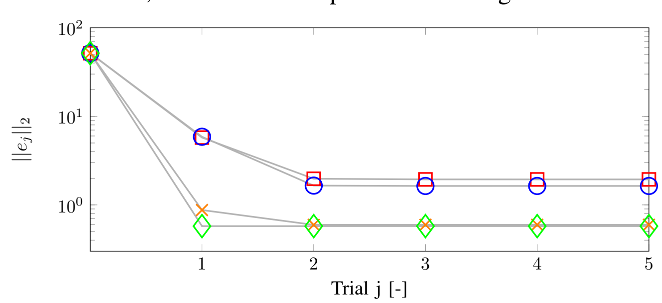

Fig. 5. Benefits of designing multivariable ILC for interaction: the centralized L-filter designs (×, ◇) outperform the decentralized designs (□, ○), which require robustness to both modeling errors and ignored interaction. Also, through independent Q-filter designs (○), ignored interaction is addressed appropriately, which potentially improves performance compared to conservative Q-filter design (□) for *worst-case* interaction.

- Both centralized L-filters, i.e., based on ZPETC (×) and stable inversion (◇), obtain similar performance after convergence. The convergence rate differs, since the ZPETC-based approach approximates S_P^{-1}, whereas the stable inversion approach provides an exact inverse of S_P.
- The performance of centralized ILC is limited by modeling errors in the diagonal elements. Indeed, modeling errors of interaction terms would further limit performance.

VII. CONCLUSIONS

In this paper, a framework is developed for multivariable ILC, ranging from decentralized to centralized designs. The proposed solutions differ in design, i.e., performance and robustness, and modeling requirements. In Section III, the consequences of interaction in ILC are analyzed, and it is shown that ignoring interaction may lead to non-convergent systems. The main contributions of this paper are presented in Sections IV and V: i) a decentralized ILC design, based on the structured singular value, which accounts for interaction in each loop independently, thereby only requiring SISO parametric models, and ii) several centralized ILC controllers, including extensions towards multivariable zero-phase error tracking control. These methods potentially lead to higher performance by explicitly compensating interaction, at the requirement of a full parametric MIMO model of the system, which is often expensive to obtain in practice. Hence, the proposed solutions are all suitable alternatives, and depending on the specific application at hand a suitable choice can be made. Benefits and differences of the proposed approaches are demonstrated through a simulation study.

VIII. ACKNOWLEDGMENTS

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