Multivariable Repetitive Control: Decentralized Designs with Application to Continuous Media Flow Printing

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Abstract—The production speed and medium size in traditional wide-format printers are limited by positioning errors caused by step-wise medium transportation. The aim of this paper is to develop a multivariable repetitive control (RC) framework that enables continuous media flow printing with enhanced positioning accuracy and increased productivity. The developed technical framework explicitly addresses the trade-off between performance and modeling requirements. In particular, systematic design approaches for RC are developed, where unmodeled interaction is explicitly addressed as uncertainty, i.e., through robustness. The result is a range of solutions, including i) independent single-input single-output (SISO) designs, and ii) sequential SISO design. Experimental results confirm that the developed RC framework in conjunction with continuous media flow printing outperforms existing approaches.

I. INTRODUCTION

Production speed, print quality, medium size, and medium versatility in wide-format roll-to-roll inkjet printing systems are limited by medium positioning errors. Positioning errors are induced during step-wise medium transportation in between print-passes, and are dominantly caused by internal medium deformations due to, e.g., friction and hysteresis [1], and medium-dependent dynamics, which may be nonlinear and uncertain. Positioning errors deteriorate the alignment of print-passes, and distort the printed product. To conceal these adverse effects, typically multiple overlapping print-passes are performed at the cost of production speed [2].

The reproducibility of positioning errors in step-wise transportation is exploited in [3] through the use of iterative learning control (ILC). Batches of medium position measurements are collected using a scanner, which are used to iteratively improve medium positioning in subsequent print passes. However, the achievable accuracy is limited by the uncertain and nonlinear nature of the medium dynamics.

Although substantial improvements have been made to compensate positioning errors due to step-wise transportation, these approaches are inherently limited by the ability to model and compensate the medium-dependent dynamics and deformations. The aim of the present paper is to investigate a fundamentally different approach, which is to transport the medium with constant longitudinal velocity throughout the printing process, such that the uncertain and nonlinear dynamics essentially can be represented as a constant disturbance.

The proposed process, called continuous media flow printing, requires that the carriage, which contains the printheads, accurately tracks a repetitive multidimensional trajectory over the print surface, see Figures 1 and 2. This requires simultaneous actuation of multiple axes, which is enabled by flatbed printing systems. The gantry beam offers the required motion freedom in medium transport direction, which is usually not present in traditional wide-format printers, see [3].

To achieve the tracking accuracy required for printing, a repetitive control (RC) [4], [5] framework for multi-input multi-output (MIMO) systems is required, which enables to learn from reproducible errors as in [3], yet in continuous instead of batchwise operation. RC enables to track or reject disturbances that repeat continuously in time, see, e.g., [6], [7] for applications. In contrast, the ILC in [3] aims to track repeating finite-length trajectories that are unrelated in time.

Robust stability of RC algorithms is crucial to deal with modeling errors. Most robust RC design approaches for MIMO systems are based on optimization techniques, such as $H_{\infty}/\mu$-synthesis, see, e.g., [8], [9], [10], [11]. These approaches require a MIMO parametric nominal model and its uncertainty in a certain prespecified form to guarantee robust stability. Despite being very systematic, this imposes a large burden on the modeling requirements [12], which may hamper industrial implementation. Especially for lightly damped motion systems such as flatbed printers, see Figure 1, these models can be difficult and expensive to obtain due to complex dynamics [13], [14], and numerical issues [15].

In view of multivariable continuous media flow printing, as
is developed in the present paper, a fundamentally different RC framework is essential to meet the performance and modeling cost requirements. In many applications, see, e.g., [5], [16], [17], repetitive controllers are designed using manual loop-shaping in the frequency domain, which is often preferred by control engineers. Robust stability can be verified by using inexpensive frequency response function (FRF) measurements to model the uncertainty [18], [19]. However, such manual design approaches are typically limited to single-input single-output (SISO) systems, and their application to MIMO systems is largely undeveloped. In particular, interaction is typically ignored, which can lead to robust stability issues.

The contributions of this paper are twofold.

C1) A systematic framework is developed for robust decentralized RC design for general MIMO systems, that explicitly addresses the trade-offs between modeling requirements and performance.

C2) The potential of the proposed RC framework is experimentally demonstrated for continuous media flow printing, enabling increased productivity and medium sizes.

The proposed design techniques, constituting the framework in C1, rely on manual SISO loop-shaping design tools and SISO parametric models, hence considerably simplify the design, yet guarantee robust stability of the MIMO system through non-parametric FRF measurements. In particular, interaction is addressed as structured uncertainty, i.e., through robust stability. The design approaches include i) independent robust SISO designs, and ii) sequential SISO design, building upon results in [20], [21], [22], including the use of the structured singular value (SSV) [23, Section 11.2]. The framework relates to [24] for MIMO ILC, yet in addition allows sequential design, whereas this is not directly relevant for ILC. The present paper generalizes preliminary results in [25] with new theoretical and application results, and detailed proofs.

Notation. All systems are discrete-time, MIMO, and linear time-invariant. Let $\mathcal{R}(z)$ denote the set of rational discrete-time transfer matrices. The space of real rational functions bounded on the unit circle and analytic in $|z| > 1$ is denoted $\mathcal{RH}_\infty$. The imaginary unit is denoted $i$, i.e., $i^2 = -1$. The $ij$-th element of a matrix $A$ is denoted $a_{ij}$.

II. Problem Description

The printing operation for continuous media flow is introduced, and the repetitive control design problem is formulated.

A. Printing Operation with Continuous Media Flow

Figure 1 depicts an industrial flatbed printer with roll-based media supply, which enables to print on flexible media. The printheads, which contain many closely spaced nozzles that deposit ink onto the medium, are located underneath the carriage. In traditional operation, the carriage performs lateral passes over the stationary medium, which is translated step-wise in between passes. For continuous media flow (b), the carriage performs a multivariable motion while printing on the medium, which moves with constant longitudinal velocity. An animation of these processes is provided in the supplemental video.

During a print pass, the carriage should maintain its position relative to the medium in transport direction. Varying carriage step-sizes in between print passes can be used for multi-pass printing, see [2], resulting in asymmetrical trajectories. To achieve the required printing accuracy while tracking the repeating motion, a multivariable RC framework is developed.

B. Repetitive Control Setup

Consider the control scheme shown in Figure 3, consisting of plant $G \in \mathcal{R}^{n_y \times n_y}(z)$, stabilizing feedback controller $C \in \mathcal{R}^{n_u \times n_y}(z)$, and a repetitive controller $R \in \mathcal{R}^{n_y \times n_u}(z)$, connected as an add-on controller, see, e.g., [26]. The presented results can be appropriately modified for other RC implementations, e.g., serial configurations. The control objective is to reject a periodic exogenous disturbance with period $N \in \mathbb{N}$, e.g., $r(k) \in \mathbb{R}^{n_y}$ with $r(k + N) = r(k)$. That is, minimize the tracking error $e = r - y$ given by

$$
e = (I + GC(I + R))^{-1}(r - d)$$
$$= (I + GC + (I + GC)(I + GC)^{-1}GCR)^{-1}(r - d)$$
$$= ((I + GC)(I + TR))^{-1}(r - d) = S_RS(r - d),$$

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2.png}
\caption{In traditional operation (a), the carriage makes lateral passes over the stationary medium, which is translated step-wise in between passes. For continuous media flow (b), the carriage performs a multivariable motion while printing on the medium, which moves with constant longitudinal velocity. An animation of these processes is provided in the supplemental video.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig3.png}
\caption{Control configuration with add-on repetitive controller $R$.}
\end{figure}
with sensitivity $S = (I + GC)^{-1} \in RH_{\infty}^{ny \times ny}$ the transfer function matrix from $r$ to $e$ with $R = 0$, complementary sensitivity $T = (I + GC)^{-1}GC = I - S$, and modifying sensitivity $S_R = (I + TR)^{-1}$.

According to the internal model principle [27], perfect asymptotic rejection of an exogenous disturbance is achieved if a model of the disturbance generating system is included in a stable feedback loop. For general $N$-periodic disturbances, this corresponds to a memory loop with $N$ samples delay. Typically, the repetitive controller is designed as, see Figure 4,

$$R = Lz^{-N}Q(I - z^{-N}Q)^{-1},$$

(2)

with learning filter $L \in RH_{\infty}^{ny \times ny}$ and robustness filter $Q \in RH_{\infty}^{ny \times ny}$ such that $z^{-N}L(z)Q(z) \in RH_{\infty}^{ny \times ny}$. That is, $L$ and $Q$ can have finite preview, i.e., be non-causal, provided that $R(z)$ is causal, by embedding their preview in the delay $z^{-N}$. Although in this work a single memory loop is used, the controller structure can be directly extended using multiple memory loops for uncertain $N$, see, e.g., [16], [28], [29].

C. Background: RC Design for SISO Systems

For the SISO case, i.e., $n_y = n_u = 1$, design techniques for $L$ and $Q$ are well developed. Fundamental to these techniques is the following well-known result for stability, see, e.g., [5], which is recovered as a special case of the multivariable case in Section III. Assuming stable $S$, $T$, $L$, $Q$, the closed-loop system (1) is stable for all $N \in \mathbb{N}$ if

$$|(1 - T(e^{j\omega})L(e^{j\omega}))Q(e^{j\omega})| < 1 \quad \forall \omega \in [0, \pi]$$

(3)

Based on this result, typically the following two-step design procedure is considered [4], [26], [28].

Procedure 1. Frequency-domain SISO RC design

1) Given a parametric model of $T(z)$, construct $L(z)$ as an approximate stable inverse, i.e., $L(z) \approx \frac{n}{\pi \tau}$, with learning gain $\alpha \in (0, 1]$.

2) Using a non-parametric FRF model of $T(e^{j\omega})$, design $Q(z)$ such that (3) is satisfied.

Procedure 1 enables a systematic and inexpensive robust design, particularly since no accurate parametric model of the system is required. In step 1, the inverse approximation can be based on an, often deliberately, coarse parametric model. In case $T$ is non-minimum phase, algorithms for inversion include ZPETC [26], FIR models [30], and $H_{\infty}$-synthesis with [31] and without finite preview [9]. Then, in step 2, robustness to modeling errors and uncertainty, originating from step 1, can effectively be dealt with through the use of non-parametric FRF models, which are for mechatronic systems often inexpensive, accurate and fast to obtain [18]. In addition, in view of robust stability with respect to plant uncertainty, the use of FRF models straightforwardly allows to include confidence intervals, which are typically readily available for FRF estimates that result from experimental data [18].

D. Problem Formulation and Contributions

Despite the availability of Procedure 1 for SISO RC design, a systematic RC design for MIMO systems, which addresses model quality and model requirements, is not yet available. The aim of the present paper is to bridge this gap by extending and generalizing Procedure 1 to the multivariable case.

The first contribution (C1) is the development of a range of robust decentralized design techniques. The designs require only SISO parametric models, since robustness to modeling errors and uncertainty, including multivariable interaction, can be dealt with through the use of FRF models. The second contribution (C2) is an experimental demonstration of the proposed RC framework for continuous media flow printing.

In the next section, the multivariable design problem is analyzed, forming the basis for the decentralized designs in Sections IV and V. The experimental validation on a flatbed printer with continuous media flow is presented in Section VI.

III. Analysis of Multivariable RC Design

Next, the multivariable RC problem is analyzed. A stability theory is developed that is fundamental to forthcoming developments, and the implications of decentralized design on modeling requirements are investigated.

A. Stability Analysis

Stability is investigated of the system (1). Observe that

$$S_R = (I + TR)^{-1} = \left(1 + TLz^{-N}Q(I - z^{-N}Q)^{-1}\right)^{-1}$$

$$= \left((I - z^{-N}Q + TLz^{-N}Q)(I - z^{-N}Q)^{-1}\right)^{-1}$$

$$= (I - z^{-N}Q)(I - (I - TL)z^{-N}Q)^{-1}.$$  

The following results, see, e.g., [5], enable subsequent derivations, including decentralized RC designs.

Theorem 1 (A Nyquist stability theorem for MIMO RC).

Consider the control configuration of Figure 3 with $R$ as in (2). Suppose all poles of $S$, $T$, $L$ and $Q$ are in the open unit disk, and there are no unstable pole-zero cancellations between $S$, $(I - z^{-N}Q)$, and $(I - (I - TL)z^{-N}Q)^{-1}$. Then, the closed-loop multivariable system (1) is stable if and only if the image of $\det(I - (I - TL)z^{-N}Q)$

- does not encircle the origin,
- and does not pass through the origin, as $z$ traverses the Nyquist contour $\Gamma$, depicted in Figure 5.

Theorem 2 (Stability independent of $N$).

Let the assumptions of Theorem 1 hold. If $(I - TL)z^{-N}Q$ is strictly proper, then
the closed-loop system (1) with R as in (2) is stable for all \( N \in \mathbb{N} \) if

\[
\rho \left( (I - T(e^{i\omega})L(e^{i\omega})) Q(e^{i\omega}) \right) < 1 \quad \forall \omega \in [0, \pi].
\]

(5)

Proofs of Theorem 1 and Theorem 2 are provided in Section A. The results are crucial for forthcoming developments. It is emphasized at this point that Theorem 1 gives a necessary condition for stability for a fixed value of \( N \), and Theorem 2 guarantees stability for all \( N \in \mathbb{N} \).

**Remark 1.** Theorem 1 reveals the importance of factorization (4) for RC design. Note that in Theorem 1, no encirclements of the origin are required for stability. In sharp contrast, directly applying Nyquist’s theorem to \( S_R = (I + TR)^{-1} \) may severely complicate RC design, since the loop-gain \( TR \) may contain unstable poles. For example, selecting \( Q = I \) in (2) yields \( N \) open-loop unstable poles on the unit circle. In the multivariable case, the Nyquist contour in Figure 5 should be adapted to include indentations into the unit disk, around the poles on the unit circle [32]. Hence, for stability, the image of \( \det(I + TR) \) should encircle the origin \( N \) times in a counterclockwise direction.

**Remark 2.** From a practical perspective, Theorem 2 is crucial, since it enables design of stabilizing RC controllers independent of \( N \). To see this, suppose that \( \rho((I - TL)e^{-i\omega N}Q) < 1 \) is violated for some frequency \( \omega \neq 0 \) and fixed \( N \). Since the phase of \( e^{-i\omega N} \) decreases very fast, up to \( N \pi \) radians at the Nyquist frequency, the image of \( \det(I - (I - TL)e^{-i\omega N}Q) \) may encircle the origin when varying \( N \).

Motivated by Theorem 2 and the internal model principle [27], typical design aims for multivariable RC are \( TL \approx \alpha I \), \( \alpha \in (0, 1) \), and \( Q \) as close to \( I \) as possible, while satisfying (5). This requires a multivariable parametric model of \( T \). To avoid this requirement, a decentralized approach can be taken.

**B. Decentralized RC Design: Stability Considerations**

The aim is to design the decentralized controllers

\[
L(z) = \text{diag}\{l_1(z), l_2(z), \ldots, l_n(z)\},
\]

(6)

\[
Q(z) = \text{diag}\{q_1(z), q_2(z), \ldots, q_n(z)\}
\]

(7)

such that the multivariable closed-loop system is stable. A crucial aspect in decentralized designs is dealing with unmodeled interaction, i.e., the off-diagonal terms of \( T(z) \), which can potentially destabilize the system. Several approaches can be taken to derive explicit bounds on \( Q \) from (5) that guarantee robust stability. The restrictiveness of these bounds depends on the assumptions made on the structure of \( Q \). For instance, \( Q(z) = q_d(z)I \) with SISO filter \( q_d(z) \) leads to the next result.

**Corollary 1.** Let the assumptions of Theorem 2 hold. Then, the closed-loop multivariable system (1) with \( R \) as in (2), \( L \) in (6) and \( Q(z) = q_d(z)I \) is stable for all \( N \in \mathbb{N} \) if

\[
|q_d(e^{i\omega})|\rho((I - T(e^{i\omega})L(e^{i\omega})) < 1 \quad \forall \omega \in [0, \pi].
\]

(8)

The bound on \( q_d(z) \) in (8) is nonconservative for the case \( Q(z) = q_d(z)I \). However, \( q_d \) accounts for the worst-case modeling errors and interaction over all loops. Consequently, e.g., large modeling errors in a single loop affect the \( Q \)-filter applied to all other loops, which may deteriorate performance. This motivates the development of systematic procedures for independent \( Q \)-filter design, where each loop \( i \) is robustified through the corresponding \( q_i \), separately.

**IV. DECENTRALIZED RC: INDEPENDENT DESIGNS WITH ROBUSTNESS TO INTERACTION**

In this section, the MIMO design problem of Theorem 2 is reformulated as sets of independent SISO design problems, that account for interaction through robustness in \( Q \). The developed techniques are closely related to decentralized feedback interconnections, see, e.g., [20, 33], yet differ fundamentally regarding multivariable interaction. This is analyzed in Subsection IV-A. Based on this, sets of independent robust SISO design conditions are developed in Subsections IV-B and IV-C. These result in a design procedure for independent decentralized RC design in Subsection IV-D.

**A. Factorization of Interaction**

A factorization is performed to analyze the role of interaction and enable robust designs. Let \( M = I - TL \). Then,

\[
MQ = (I + E)M_dQ,
\]

(9)

where \( E = (M - M_d)M_d^{-1} \) represents normalized interaction in \( M \), and \( M_d = \text{diag}\{M_{11}, M_{22}, \ldots\} \). Hence, reformulating Theorem 2, stability is achieved for all \( N \in \mathbb{N} \) if

\[
\rho((I + E)M_dQ) < 1 \quad \forall \omega \in [0, \pi].
\]

(10)

The diagonal term \( M_dQ \) is to be designed, and the interaction term \( I + E \) can be used to analyze robust stability.

**Remark 3.** The pursued factorization-based approach for decentralized RC is closely related to decentralized feedback control, see, e.g., [20] and [33, Section 10.6], yet fundamentally differs regarding the employed factorization.

In decentralized feedback design, i.e., \( K = \text{diag}\{k_i\} \) with loop gain \( GK \), typically the return difference is factored as

\[
I + GK = (I + ET_d)(I + G_dK),
\]

(11)

with \( E = (G - G_d)G_d^{-1} \) and \( \tilde{T}_d = \text{diag}\{\frac{q_{d,k_i}}{1 + g_{d,k_i}}\} \). Assuming stable \((I + G_dK)^{-1}\) and \( \tilde{T}_d \), the closed-loop system is stable if \( \rho(ET_d) < 1 \), \( \forall \omega \in [0, \pi] \), see [20, Thm. 2], [33, Sec. 10.6].
This is in sharp contrast with decentralized RC. The difference can be observed from (4). Essentially, the term \((I - (I - TL)z^{-N}Q)^{-1}\) constitutes a positive feedback interconnection with loop gain \((I - TL)z^{-N}Q\). In (9), this loop gain is factored. As a result, \(E\) appears affinely in (10).

Alternative to this approach, also in RC the return difference can be factored (i.e., dual to decentralized feedback design):

\[
I - (I - TL)z^{-N}Q = I - (I + E)M_d z^{-N}Q \tag{11}
\]

Similar to feedback design on the basis of (11), conditions on \(\rho(E\hat{T}_{RC})\) can be developed for stability. It is emphasized that since \(\hat{T}_{RC}\) is a rational function of \(z^{-N}\), such conditions in general guarantee stability only for a specific value of \(N\).

Factorizations (9) and (12) are both suitable alternatives. In the present paper, the former is pursued, i.e., factoring the loop gain, since it enables design independent of \(N\). The presented results can be appropriately modified for decentralized design based on (12), i.e., by factoring the return difference.

Next, sufficient conditions for robust stability are developed based on the use of 1) Gershgorin’s theorem, and 2) the SSV.

B. Independent design based on Gershgorin bounds

Application of Gershgorin’s theorem, see, e.g., [34, Theorem 8.2], to the factorization (9) leads to the following result.

**Theorem 3** (Independent RC design based on Gershgorin’s theorem). Let the assumptions of Theorem 2 hold. Then, the closed-loop system (1) with \(R\) in (2), \(L\) in (6), and \(Q\) in (7) is stable for all \(N \in \mathbb{N}\) if either

\[
\left|1 - t_{ii}(e^{i\omega})l_i(e^{i\omega})q_i(e^{i\omega})\right| < \frac{1}{\sum_{j \neq i} |(I + E(e^{i\omega}))_{ij}|} \quad \forall i, \omega \in [0, \pi] \tag{13}
\]

\[
\left|1 - t_{ii}(e^{i\omega})l_i(e^{i\omega})q_i(e^{i\omega})\right| < \frac{1}{\sum_{j \neq i} |(I + E(e^{i\omega}))_{ij}|} \quad \forall i, \omega \in [0, \pi] \tag{14}
\]

A proof is provided in Section A. Theorem 3 provides individual bounds for each loop. Yet, these bounds impose no restriction on the structure of \(Q\), which is potentially conservative for decentralized design. Next, alternative conditions are investigated that exploit the diagonal structure of \(Q\).

C. Independent design based on the structured singular value

Robust stability conditions are developed using the structured singular value (SSV), see, e.g., [20], [23]. The key idea is to exploit the structured form (9) in Theorem 2.

**Definition 1.** For \(A \in \mathbb{C}^{n \times n}\), the SSV \(\mu_{\Delta}(A)\) is defined

\[
\mu_{\Delta}(A) = \frac{1}{\min_{\sigma} \{ \sigma(\Delta) : \Delta \in \Delta, \det(I - A\Delta) = 0 \}},
\]

where \(\Delta = \{ \text{diag}\{\Delta_1, \ldots, \Delta_m\} : \Delta_j \in \mathbb{C}^{m_j \times m_j}, \sum_{j=1}^{m} m_j = n \} \) a prescribed set of block diagonal matrices, unless no \(\Delta \in \Delta\) makes \(I - A\Delta\) singular, in which case \(\mu_{\Delta}(A) = 0\).

**Theorem 4** (Independent RC design based on SSV). Let the assumptions of Theorem 2 hold. Then, the closed-loop system (1) with \(R\) in (2), \(L\) in (6), and \(Q\) in (7) is stable for all \(N \in \mathbb{N}\) if

\[
\left|(1 - t_{ii}(e^{i\omega})l_i(e^{i\omega}))q_i(e^{i\omega})\right| < \frac{1}{\mu_1(I + E)} \quad \forall i, \omega \in [0, \pi] \tag{15}
\]

where \(\mu_1(\cdot)\) is taken with respect to a diagonal structure.

A proof is provided in Section A. In Theorem 4, the SSV is employed in a fundamentally different way than in traditional stability analyses of feedback interconnections. In robust control approaches, see, e.g., [23, Chapters 9, 11], [33, Chapter 8], typically \(\mu_\Delta(M)\) is taken with respect to a structured uncertainty \(\Delta\), and \(M\) denotes a nominal model. In sharp contrast, here \(I + E\) has the role of nominal model, and \(M_d Q\) is the structured uncertainty, i.e., is yet to be designed.

D. Design Considerations and Procedure

Theorems 3 and 4 enable systematic robust decentralized design using only SISO parametric models. Interaction does not have to be included in parametric models, since the right-hand-sides of (13), (14), (15) can be computed based on FRF models. This gives rise to the following design procedure.

**Procedure 2.** Independent decentralized RC design

1) Given SISO parametric models of \(t_{ii}(z), i = 1, \ldots, n_y\), construct \(l_i(z)\) as approximate stable inverses, i.e., \(l_i(z) \approx \frac{1}{t_{ii}(z)}\).
2) Given a MIMO non-parametric FRF model of \(T(e^{i\omega})\), design \(q_i(z)\) according to joint evaluation of Theorems 3 and 4, such that for each separate frequency \(\omega \in [0, \pi]\), at least one of (13), (14), (15) is satisfied.

**Remark 4.** In the SISO case, Theorems 3 and 4 recover the SISO condition for stability (3), since in this case \(E = 0\).

**Remark 5.** The sufficient conditions for stability (13), (14), (15) complement each other in the sense that the ordering of their tightness may vary as a function of frequency, see [24]. Hence, they should be considered jointly during design, see Procedure 2. Note that they can not be combined over loops \(i\): stability is guaranteed only if, for each \(\omega \in [0, \pi]\), at least one condition is satisfied for all loops \(i\) simultaneously. Further results on their tightness can be found in literature, e.g., [20], [23], [35] and [34, Chapter 8].

In Procedure 2, interaction is dealt with through independent robust design of each SISO filter \(q_i\). Alternatively, an approach for sequential SISO design is presented next, at the expense of a more involved design procedure. Only interaction from previously designed loops need to be taken into account.

V. Decentralized RC: Sequential Design for Interaction

The MIMO design problem of Theorem 1 is reformulated as a set of sequential SISO design problems, each of which explicitly accounts for interaction in previously designed loops. Compared to the independent designs in Subsection IV-A, this approach potentially reduces conservatism and improves performance. The presented approach for RC is related to sequential design of feedback controllers, see, e.g., [22], [36], yet has different implications on modeling requirements.
A. Sequential Design for Interaction

Considering Theorem 1 and for decentralized $L, Q$, it holds
\[
det(I-(I-TL)z^{-N})Q) = \prod_{i=1}^{n_y}(1-(1-\hat{t}_{ii} l_i)z^{-N} q_i) \quad (16)
\]
where $\hat{t}_{ii}$ denotes the SISO transfer function from $w_i$ to $e_i$, see Figure 4, with all preceding loops 1, ..., $i$ closed:
\[
\hat{t}_{ii} = F_u(X_i, z^{-N} Q_{i-1}),
\]
\[
X_i = \begin{bmatrix} I_{i-1} & 0 \\ 0 & I \end{bmatrix} + T_i \begin{bmatrix} -L_{i-1} & 0 \\ 0 & 1 \end{bmatrix} \quad (17)
\]
where $A_{i}$ denotes the submatrix formed from the first $i$ rows and columns of $A$, and the upper linear fractional transformation of $A$ with respect to $B$ is denoted $F_u(A, B) = A_{22} + A_{21} B(I - A_{11} B)^{-1} A_{12}$, given that the inverse exists, $B$ such that $A_{11} B$ is square, and $A$ partitioned as $A = \begin{bmatrix} A_{11} & A_{12} \end{bmatrix}$. Using (16) in Theorem 1 yields the following result.

**Theorem 5** (Sequential decentralized RC design). Let the assumptions of Theorem 1 hold. If $\hat{t}_{ii}$ is stable and strictly proper, $\forall i$, then the closed-loop system (1) with $R$ in (2), $L$ in (6), and $Q$ in (7) is stable if
\[
\left| (1 - \hat{t}_{ii}(e^{i\omega})) q_i(e^{i\omega}) \right| < 1 \quad \forall i, \omega \in [0, \pi] \quad (18)
\]
A proof is provided in Section A. The MIMO RC design problem is reformulated as a set of sequential SISO designs. Crucially, interaction is explicitly accounted for, since $\hat{t}_{ii}$ contains all interactions from all preceding loops 1, ..., $i$.

**Remark 6.** Note that since $\hat{t}_{ii}$ contains all repetitive controllers from preceding loops, it is a function of $z^{-N}$. Hence, Theorem 5 guarantees stability for a specific value of $N$ only.

B. Design Considerations and Procedure

It is emphasized that Theorem 5 offers clear advantages for designing $Q$, while sequential design of $L$ is substantially more involved. It seems attractive to design $l_i$ based on $\hat{t}_{ii}$, e.g., through approximate inversion. However, constructing parametric models of $\hat{t}_{ii}$ may be a cumbersome task, since i) the delays $z^{-N}$ from preceding loops may lead to very a high order of $\hat{t}_{ii}$, and ii) iterative redesign of $\hat{t}_{ii}$ and remodeling of $\hat{t}_{i}$ may be required, as is motivated next.

The loop-closing order in Theorem 5 must be selected with care, since the resulting closed-loop system may depend on the loop-closing order. Important considerations include, see, e.g., [22], i) performance deterioration of previously designed loops due to successive designs, and ii) influences of previously designed loops of subsequent designs. This implies that iterative redesign may be needed. Without loss of generality, the loop-closing order can be altered using a permutation matrix $P$, and replacing $T$ with $\hat{T} = PTP$. This leads to the following design procedure.

**Procedure 3.** Sequential decentralized RC design

Given a nonparametric MIMO model of $T(e^{i\omega})$ and decentralized filter $L(z)$, perform the following sequence of steps.
1) Choose the order in which the loops are designed.
2) Set the index $i = 1$, and perform the following steps.

\[a)\text{ Construct } \hat{t}_{ii}(e^{i\omega}) \text{ according to (17)}
\[b)\text{ Design } q_i \text{ according to (18) in Theorem 5, based on the nonparametric model of } T.
\[c)\text{ Until } i = n_y, \text{ set } i \rightarrow i + 1 \text{ and return to step 2a.}
\[d)\text{ If the resulting closed-loop system is unsatisfactory, reset } i = 1, \text{ return to 2a, and redesign } q_i.
\]
3) If the resulting closed-loop system after iterations is unsatisfactory, return to step 1 and change the loop closing order.

In the previous sections, decentralized design procedures are presented for robust multivariable RC. The benefits and differences of these approaches are demonstrated next.

VI. EXPERIMENTAL VALIDATION: CONTINUOUS MEDIA FLOW ON A FLATBED PRINTER

In this section, the decentralized RC design techniques are experimentally validated and compared on a flatbed printer. It is demonstrated that the proposed RC framework enables continuous media flow operation with the accuracy required for printing, constituting contribution C2 of the present paper. The experimental system is introduced next. In Subsection VI-B, the developed RC design techniques are applied to this system, and the corresponding results are provided in Subsection VI-C. Motivated by these results, guidelines for decentralized RC design are provided in Subsection VI-D.

A. Experimental Setup

An Océ Arizona 550GT flatbed printer is considered, see Figures 1 and 6. The system is controlled in the horizontal plane: the carriage translates along the gantry in $y$-direction, and the gantry translates in $x$-direction and rotates in $\varphi$. The inputs are the currents to the brushless electrical motors, located on the left and right side of the gantry, denoted $U_{l} [A]$ and $U_{r} [A]$, and along the carriage, $U_{y} [A]$. Static input transformations into the gantry translation and rotation yield a system $G(z)$ with inputs $U_{y}, U_{x}, U_{\varphi} [A]$ and outputs $y [m]$, $x [m]$, $\varphi$ [rad]. The encoder resolution is $10^{-6}$ m.

The system is controller in discrete time with sampling time 1 ms. A stabilizing feedback controller is implemented, yielding closed-loop bandwidths of 6, 3, 4 Hz in $y, x, \varphi$-directions, respectively (lowest frequencies where $|g_{ii}(e^{i\omega})c_{ii}(e^{i\omega})| = 1$). An FRF measurement of $T$ is depicted in Figure 7, together with parametric models $\hat{t}_{ii}$ for RC design.
B. Decentralized RC Designs

Four repetitive controllers are designed, according to Corollary 1, and Procedures 1, 2 and 3. Each \( t_i(z) \) is obtained through inversion of the minimum phase SISO models \( \hat{t}_{i} \), see Figure 7. The robustness filters \( q_i(z) \) are designed as 50th order zero-phase low-pass FIR filters, with associated cut-off frequencies \( f_{c,i} \) specified in Table I. The loop-closing order of the sequential approach is 2, 1, 3, i.e., in decreasing order of required robustness.

C. Experimental Results

The results are presented in Figures 9, 10, and 11. In Figure 9, the matrix norm \( \|e_j\|_F = \sqrt{\sum_{i,k} |e_j(i,k)|^2} \) is depicted as a function of periods, where \( e_j = [e_{j,x}, e_{j,y}, e_{j,\phi}]^T \in \mathbb{R}^{3 \times N} \) and \( e_j(k) = e(k+jN) \). The following observations are made:

- Application of multi-loop SISO RC designs, i.e., ignoring interaction, leads to an unstable system. This emphasizes the importance of accounting for interaction in multivariable RC design, including decentralized designs.
- Using robustly stable repetitive control, the tracking error is reduced by 99\% in terms of \( \|e\|_F \), compared to the case without repetitive control, i.e., \( \|e\|_F \approx 0.01 \).
- The sequential and independent design approaches outperform the robust SISO design by 38\% and 25\%, respectively, in terms of \( \|e\|_F \). Note that each approach uses exactly the same (non)parametric models; performance is increased only through more sophisticated design of \( Q \).
- This performance improvement is achieved by systematically designing for interaction. Through independent and sequential decentralized designs, see Table I, robustness is addressed separately per loop. Particularly in loops 1 and 3, less robustness is required, whereas the conservative robust SISO design applies the same filter \( q \) to each loop.
- Using sequential design, the peak error during printing is improved by a factor 9 in \( y \)-direction (from 45 \( \mu \)m to 5 \( \mu \)m), 37 in \( x \)-direction (from 150 \( \mu \)m to 4 \( \mu \)m), and 70 in \( \phi \)-direction (from 180 \( \mu \)rad to 2.5 \( \mu \)rad), compared to the case without RC. The corresponding time-domain errors are presented in Figure 10.
- The power spectra of the converged errors corresponding to the sequential design and the case without RC are depicted in Figure 11. It is observed that performance improvements are achieved up to the cut-off frequencies of the low-pass filters \( q_i(z) \), see Table I.

D. Design Guidelines

The insights obtained from the experimental results lead to guidelines on designing decentralized repetitive controllers:
If the period length \( N \) is a priori fixed, employ sequential designs (Procedure 3), as this in general yields the best performance.

- If RC is to be implemented with various \( N \), use independent designs (Procedure 2) to guarantee stability \( \forall N \in \mathbb{N} \).
- Only if the user effort in terms of algorithmic complexity is severely limited, use robust SISO design (Corollary 1).

VII. CONCLUSIONS

A design framework is developed for multivariable repetitive control that enables continuous media flow printing with enhanced positioning accuracy. The developed framework explicitly addresses the trade-offs between model knowledge, design complexity, and control performance. The presented decentralized designs require only SISO parametric models, and provide robustness to interaction through i) independent designs, including the use of the structured singular value, and ii) sequential design. The developed framework is experimentally validated on an industrial flatbed printing system, operating with continuous media flow. The results illustrate a large potential for high accuracy, high production speeds, and medium versatility in industrial printing.

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APPENDIX A
PROOFS OF THEOREMS

Proof of Theorem 1. With $S$ and $(I - z^{-N}Q)$ are stable, the closed-loop $S_RS$ is stable iff $(I - H)^{-1}$ is stable with $H = (I - TL)z^{-N}Q$, see (4), assuming no unstable pole-zero cancellations. $(I - H)^{-1}$ essentially combines a positive feedback loop with loop gain $H$. The determinant of return difference $I - H$ can be expressed as [33, Section 4.9]:
\[
det(I - H(z)) = e^{\phi_{ol}(z)},
\]
where $\phi_{ol}(z)$ is the characteristic polynomial of $H(z)$, $\phi_{ol}(z)$ is the characteristic polynomial of $(I - H)^{-1}$, and $e$ is a non-zero constant if $(I - H)^{-1}$ is well-posed. The Cauchy argument principle states, assuming that
1) $\det(I - H(z))$ is analytic along $\Gamma$, i.e., $\phi_{ol}$ has no roots on $\Gamma$,
2) $\det(I - H(z))$ has $P$ poles inside $\Gamma$, i.e., $\phi_{ol}$ has $P$ roots inside $\Gamma$,
3) and $\det(I - H(z))$ has $Z$ zeros inside $\Gamma$, i.e., $\phi_{ol}$ has $Z$ roots inside $\Gamma$,
then the image of $\det(I - H(z))$ as $z$ traverses $\Gamma$ encircles the origin $Z - P$ times in a clockwise direction, see, e.g., [33, Lemma 4.10]. Hence, $(I - H)^{-1}$ is stable, i.e., has $Z = 0$ poles inside $\Gamma$, iff the image of $\det(I - H(z))$ encircles the origin $P$ times in a counterclockwise direction as $z$ traverses $\Gamma$. Since all poles of $H$ are inside the unit disk, i.e., $P = 0$, zero encirclements of the origin are needed. Note that as $H$ has no poles on $\Gamma$, assumption 1 is automatically satisfied.

Proof of Theorem 2. Let $H = (I - TL)z^{-N}Q$, and consider Theorem 1 with Nyquist contour $\Gamma$ in Figure 5. Note that if $H$ is strictly proper, which is the case for sufficiently large $N$, only the part of $\Gamma$ around the unit circle needs to be evaluated. This follows since $\lim_{|z| \to \infty} \det(I - H(z)) = 1$, i.e., the part of $\Gamma$ at infinity maps to $+1$, and the parallel branches along the real axis of $\Gamma$ do not influence the stability test [37, p. 86]. Next, consider $\det(I - (I - TL)e^{-i\omega N}Q) = \prod_i (1 - \lambda_i((I - TL)e^{-i\omega N}Q))$. Hence, if all $\lambda_i((I - TL)e^{-i\omega N}Q)$ are smaller than 1 for all frequencies, then $\det(I - (I - TL)e^{-i\omega N}Q)$ does not encircle the origin. Finally, since $|z^{-N}| = 1$ for $z$ on the unit circle, (5) implies stability for all $N \in \mathbb{N}$.

Proof of Theorem 3. First, since the products of two square matrices have identical spectra, note that $\rho((I + E)M_dQ) = \rho(M_dQ(I + E))$. Second, Gershgorin’s theorem states that the eigenvalues of a $n \times n$ matrix $A$ lie in the union of the set of disks defined by $|z - a_{ii}| \leq \sum_{j \neq i} |A_{ij}|$, and also in the union of the set of disks defined by $|z - a_{ii}| \leq \sum_{j \neq i} |a_{ji}|$. Hence, if $\sum_j |a_{ij}| < 1$, $\forall i$, or $\sum_j |a_{ji}| < 1$, $\forall i$, then $\rho(Ae^{i\omega}) < 1$. Combining these results with Theorem 2, closed-loop stability hence follows if either
\[
\sum_j |(I + E)M_dQ)_{jii}| = \sum_j |(I + E)_{jii}| |M_dQ_{ii}| < 1 \quad \forall i, \omega \in [0, \pi]
\]
\[
\sum_j |(M_dQ(I + E))_{jii}| = \sum_j |M_dQ_{ii}| |(I + E)_{jii}| < 1 \quad \forall i, \omega \in [0, \pi]
\]

Proof of Theorem 4. For a matrix $A$ and a matrix $\Delta \in \Delta$, see [35, Theorem 2], it holds
\[
\rho(A\Delta) \leq \bar{\sigma}(\Delta)\mu_\Delta(A),
\]
where $\mu_\Delta(A)$ is taken with respect to the structure of $\Delta$. Taking $\Delta = QM_d$, $A = I + E$ and $\Delta = \{\delta I : \delta \in \mathbb{C}\}$, closed-loop stability then follows from Theorem 2 and (9), if
\[
\rho((I + E)M_dQ) \leq \bar{\sigma}(M_dQ)\mu_d(I + E) < 1 \quad \forall \omega \in [0, \pi]
\]
where $\mu_d(I + E)$ is the structured singular value with respect to the diagonal structure of $M_dQ$. Finally, note that $\bar{\sigma}(M_dQ) = \max_i |M_dQ_{ii}|$ due to its diagonal structure.

Proof of Theorem 5. By substitution of factorization (16) into Theorem 1, it is observed that stability is achieved if and only if the images of $(1 - t_i\hat{\ell}_i)z^{-N}q_i$ make no net encirclements of the origin, as $z$ traverses the Nyquist contour $\Gamma$ in Figure 5. Clearly, this is guaranteed if $(1 - t_i\hat{\ell}_i)z^{-N}q_i$ does not exceed unit magnitude, $\forall i$. This is satisfied if $|(1 - t_i\hat{\ell}_i)q_i| < 1$ for all $|z| = 1$, see also the proof of Theorem 2.