Kernel-Based Regression of Non-Causal Systems for Inverse Model Feedforward

Estimation of inverse models

Inverse models are often used in control

- Feedforward control
- Iterative learning control

\[ u \rightarrow y \]

\[ y \rightarrow u \]

Two approaches to estimate model of \( P^{-1} \):

1. Estimate forward model \( \hat{P} \), and invert: \( (\hat{P})^{-1} \)
   - Stability?
   - Model quality?

2. Directly estimate inverse model \( \hat{P}^{-1} \) [1,2]

Aim: estimation of \( P^{-1} \) using I/O data \( \{y(t), u(t)\}_{t=1}^{N} \) from normal operating conditions (short, limited excitation)

Control aspects:

- ‘Stability’ of \( P^{-1} \)
- Model quality ⇔ feedforward performance

Stability or causality?

If \( P \) has zeros in \( \mathbb{E} \), then \( P^{-1} \) has poles in \( \mathbb{E} \)

- Standard interpretation: causal & unbounded response ("unstable")

Approach: interpret \( P^{-1} \) as non-causal operator on \( L^{2} \)

- exploit preview
- bilateral Z-transform: \( P^{-1}(z) = \sum_{k=-\infty}^{\infty} \theta_{k} z^{-k} \)

Example: what is the impulse response of \( \frac{1}{z^{2} - 1} \)?

A: 4

B: 0.9

Key step 1: kernel-based regression in \( L^{2} \)

- Non-causal FIR model: \( \hat{P}^{-1}(q, \theta) = \sum_{k=-\infty}^{\infty} \theta_{k} q^{-k} \)
- Kernel-based regression [3] in inverse setting:
  \[ \hat{\theta} = \arg \min_{\theta} \| u_{N} - \Phi_{N}\theta \|_{2}^{2} + \sigma^{2} \theta^{T} D^{-1} \theta \]

- Kernel \( D \) improves model quality, by encoding
  - smoothness
  - exponential decay of \( \theta_{k} \): stability
  - resonant dynamics [4,5]

Note: existing kernels are causal (SS, DC, ROBFs, ...)

Key step 2: non-causal kernels in \( L^{2} \)

Approach: define kernels \( D(k, k') \) also for \( k, k' < 0 \)

- e.g., using non-causal ROBFs in \( L^{2} \) [6]
- also applies to SS, DC, ...

Results: inverse model feedforward

Compared regularization techniques:

- **Proposed non-causal kernel** (ROBFs in \( L^{2} \)): low mean square error of \( \hat{\theta} \rightarrow \) near-optimal \( \| e \|_{2} \)
- **Traditional causal kernel** (ROBFs in \( H_{2} \)): high variance of non-causal estimates \( \rightarrow \) moderate performance
- **No regularization**: high variance \( \rightarrow \) high \( \| e \|_{2} \)

Histogram of tracking performance \( \| e \|_{2} / \sqrt{N} \) with estimated inverse models implemented as feedforward filters. Results using optimal kernel (oracle) in gray. Monte Carlo simulations with 1000 realizations.