2014 American Control Conference (ACC)
June 4-6, 2014. Portland, Oregon, USA

Accuracy Aspects in Motion Feedforward Tuning

Frank Boeren, Tom Oomen, and Maarten Steinbuch

Abstract— **Feedforward control can significantly improve the performance of a motion system through compensation of known disturbances. Recently, new feedforward algorithms have been proposed that exploit measured data from previous tasks and a suitable feedforward parametrization to attain high performance. The aim of this paper is to analyze the accuracy of these approaches. To achieve this, related results from closed-loop identification are exploited in feedforward control. Furthermore, a new algorithm is proposed that leads to optimal accuracy. The results are confirmed in a simulation study of a motion system.**

I. INTRODUCTION

Feedforward control is widely used to improve servo performance in motion systems, since feedforward can effectively compensate for the servo error induced by known disturbances. Typically, the key performance enhancement is obtained by using feedforward with respect to the reference. Indeed, many industrial applications have been reported where feedforward leads to a significant performance improvement. Relevant examples include model-based feedforward [1], [2] and Iterative Learning Control (ILC) [3].

Model-based feedforward leads in general to good performance for a class of reference signals. In model-based feedforward, a parametric model is determined that approximates the inverse of the system. The servo performance resulting from a model-based feedforward design is highly dependent on the model quality of the parametric model of the system and the accuracy of model-inversion [4]. Contrarily, ILC results in superior performance with respect to model-based feedforward for a single specific task, as it can compensate for all predictable disturbances. In ILC, the feedforward signal is updated by learning from previous tasks, under the assumption that the task is repetitive. In addition, ILC only requires an approximate model of the system.

In [5] an approach is presented that combines the advantages of model-based feedforward and ILC, resulting in a feedforward controller that attains high performance for a class of reference signals. To this end, basis functions are introduced in ILC. In [6], the need for an approximate model of the system, as is common in ILC, is eliminated by exploiting results from iterative feedback tuning (IFT) [7]. Extensions to multivariable systems and input shaping are provided in [8] and [9] respectively, while a comparative study of data-driven feedforward control is reported in [10]. In [11], a refinement is proposed such that a single task is sufficient to determine an unbiased update of the feedforward controller in the presence of noise. However, attention is restricted to obtaining unbiased estimates, without considerations for optimal accuracy.

The authors are with the Eindhoven University of Technology, Dep. of Mechanical Engineering, Control Systems Technology Group, Eindhoven, The Netherlands. Email: f.a.j.boeren@tue.nl. This research is supported by Philips Innovation Services and by the Innovational Research Incentives Scheme under the VENI grant Precision Motion: Beyond the Nanometer (no. 13073) awarded by NWO (The Netherlands Organisation for Scientific Research) and STW (Dutch Science Foundation).

Although iterative feedforward tuning is promising for motion control, the accuracy aspect of iterative feedforward control is not yet explored. This paper aims to analyze the accuracy obtained with the approaches proposed in [11], [6], and develop a solution that leads to optimal accuracy. Therefore, expressions for the variance are exploited which are developed in open-loop identification [12, Chapter 5, 6] and closed-loop identification [13], [14]. The presented work extends this field of research towards iterative feedforward control. Related results are proposed in [15], [16], and [17]. The provided analysis of accuracy aspects enables the design of an algorithm that has optimal accuracy, by exploiting bootstrap methods as proposed in [14, Section 5] and [12, Chapter 5, 6]. A simulation study illustrates the performance enhancement in terms of servo error that is obtained with the proposed algorithm compared to existing approaches.

Notation. The variable q denotes the forward shift operator $qu(t) = u(t+1)$. For a vector x, $||x||_W^2 = x^T W x$. A positive definite matrix A is denoted as $A > 0$. Also, $A - B > 0$ is denoted as $A > B$. Furthermore, $\mathbb{E}(x) = \int_{-\infty}^{\infty} x f(x) dx$, with probability density function $f(x)$, $\bar{\mathbb{E}}(x) = \lim_{N\to\infty} \frac{1}{N}\sum_{t=0}^{N-1} \mathbb{E}(x)$ where N is the number of data points, and $R_{xy} = \sum_{t=0}^{N-1} x(t)y(t)$. The asymptotic sample mean $\bar{\theta}$ for m realizations is defined as $\bar{\theta} = \frac{1}{m}\sum_{l=1}^{m} \hat{\theta}_{N,l}$ where $\hat{\theta}_{N,l}$ is the estimate from the l^{th} realization.

II. PROBLEM DEFINITION

A. Feedforward Control Goal

The goal in feedforward control is to improve performance by compensating for known exogeneous input signals that affect the system. Consider the two degree-of-freedom control configuration as depicted in Fig. 1. The true unknown system P is assumed to be discrete-time, single-input single-output, and linear time-invariant. The control configuration consists of a given stabilizing feedback controller C_{fb}, a feedforward controller C_{ff} and a prefilter C_y, as in [9]. In this paper, attention is restricted to the case with $C_y = 1$. Extending the results in this paper to the general case is conceptually straightforward. Furthermore, let r denote the known reference signal, v the disturbance, u_{ff} the feedforward signal, and e the servo error. The unknown disturbance v is assumed to be given by $v = H\epsilon$, where H is monic and

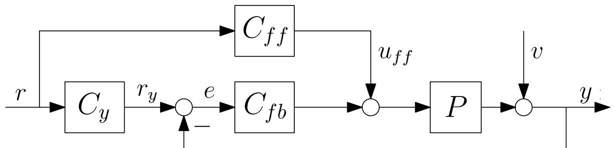

Fig. 1: Two degree-of-freedom control configuration.

ϵ is normally distributed white noise with zero mean and variance λ_ϵ^2. Hence, v and r are uncorrelated.

For servo systems, the goal of feedforward control is to attain high performance by compensating for the servo error induced by the reference signal. From Fig. 1 it follows that

$$e(t) = S(q)(1 - P(q)C_{ff}(q))r(t) - S(q)v(t),$$

where $S(q) = (1 + P(q)C_{fb}(q))^{-1}$, implying that the contribution of e induced by a nonzero r is eliminated if $C_{ff}(q) = P^{-1}(q)$.

B. Iterative Feedforward Control

In iterative feedforward control, measurement data is exploited to update C_{ff} after each task, i.e.,

$$C_{ff}^{j+1}(q) = C_{ff}^{j}(q) + C_{ff}^{\Delta}(q), \tag{1}$$

as in Fig. 2, where the update $C_{ff}^{\Delta}(q)$ is based on the measured signals $e_m^j(t)$ and $y_m^j(t)$ obtained during the j^{th} task of the closed-loop system with $C_{ff}^j(q)$ implemented. For $C_{ff}(q)$, a general polynomial parametrization is adopted that encompasses common parametrizations in feedforward control, including those in [18], [6], [8] and [9]. Herein, (1) becomes

$$\begin{aligned} C_{ff}^{j+1}(q, \theta^{j+1}) &= C_{ff}^{j}(q, \theta^{j}) + C_{ff}^{\Delta}(q, \theta^{\Delta}) \\ &= \sum_{i=1}^{n_\theta} \psi_i(q^{-1})(\theta_i^j + \theta_i^\Delta), \end{aligned}$$

where $\theta_i^{j+1} = \theta_i^j + \theta_i^\Delta$, and $\psi_i(q^{-1})$ are basis functions. For this parametrization, the update $C_{ff}^{\Delta}(q, \theta^\Delta)$ is given by

$$C_{ff}^{\Delta}(q, \theta^\Delta) = \sum_{i=1}^{n_\theta} \psi_i(q^{-1})\theta_i^\Delta, \tag{2}$$

with $\theta^\Delta = [\theta_1^\Delta, \theta_2^\Delta, \ldots, \theta_{n_\theta}^\Delta]^T \in \mathbb{R}^{n_\theta \times 1}$ and $\Psi(q) = [\psi_1(q^{-1}), \psi_2(q), \ldots, \psi_{n_\theta}(q^{-1})] \in \mathbb{R}[q^{-1}]^{1 \times n_\theta}$. The parameters θ^Δ result from the optimization problem

$$\hat{\theta}^\Delta = \arg\min_{\theta^\Delta} V(\theta^\Delta).$$

In [11], a general framework is proposed for iterative feedforward control based on instrumental variables. The rationale is that unbiased estimates of $\hat{\theta}^\Delta$ are obtained in the presence of v, in contrast to least-squares estimation. In this case, the criterion

$$V(\theta^\Delta) = \left|\left| \frac{1}{N} \sum_{t=0}^{N-1} z^T(t) L(q) \hat{e}(t, \theta^\Delta) \right|\right|_W^2,$$

is employed, where $z(t) \in \mathbb{R}^{1 \times n_z}$ are instrumental variables, W is a positive-definite weighting matrix, $n_z \geq n_\theta$, $L(q)$ is a prefilter and $\hat{e}(t, \theta^\Delta)$ is the predicted error in the $(j+1)^{\text{th}}$ task, given by

$$\hat{e}(t, \theta^\Delta) = e_m^j(t) - S(q)P(q)\Psi(q)\theta^\Delta r(t). \tag{3}$$

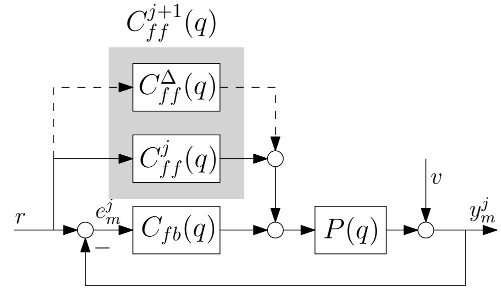

Fig. 2: The feedforward controller C_{ff}^{j+1} is determined based on the reference r, and measured signals y_m^j and e_m^j in the j^{th} task.

The key feature in such iterative feedforward control approaches is that $\hat{\theta}^\Delta$ is estimated without modeling SP, e.g., as in [7]. By exploiting the commutative property of SISO systems in (3) and the following result derived in [6]

$$\mathbb{E}\left\{(C_{fb}(q) + C_{ff}^j(q))^{-1} y_m^j(t)\right\} = S(q)P(q)r(t),$$

the predicted error $\hat{e}(t, \theta^\Delta)$ becomes $\hat{e}(t, \theta^\Delta) = e_m^j(t) - \varphi^j(t)\theta^\Delta$, with $\varphi^j(t) = \Psi(q)(C_{fb}(q) + C_{ff}^j(q))^{-1} y_m^j(t) \in \mathbb{R}^{1 \times n_\theta}$. Then, $\hat{\theta}^\Delta$ is the solution to

$$\hat{\theta}^\Delta = (R_N^T W R_N)^{-1} R_N^T W r_N, \tag{4}$$

where $R_N = \frac{1}{N}\sum_{t=0}^{N-1} z^T(t)L(q)\varphi^j(t)$ is nonsingular and $r_N = \frac{1}{N}\sum_{t=0}^{N-1} z^T(t)L(q)e_m^j(t)$.

C. Contribution of this paper

Existing approaches for iterative feedforward control as proposed in [11] and [6] focus on obtaining unbiased estimates in the presence of noise. In this paper, the accuracy properties of these approaches are explored in terms of variance. The contribution of this paper is threefold:

C1. Analyze the accuracy of existing approaches in iterative feedforward control.
C2. Propose a new algorithm that attains optimal accuracy.
C3. Compare the proposed approach in C2 with existing approaches in an simulation study of a motion system.

III. OPTIMAL FEEDFORWARD BASED ON INSTRUMENTAL VARIABLES

In this section, an expression for $z(t)$ is derived that results in optimal accuracy for iterative feedforward control. In the next section, this optimal instrument $z_{\text{opt}}(t)$ and the corresponding covariance matrix P_{IV} are exploited as a benchmark to evaluate existing approaches in terms of accuracy, see Table I.

A. Asymptotic Distribution

The optimal instrumental variable z_{opt} is determined based on the asymptotic distribution of $\hat{\theta}^\Delta$. In this section, it is assumed that $C_{fb}(q)$ is designed such that $S(q)H(q) = 1$. This is typically achieved in control design, e.g., by using

TABLE I: Comparison of existing approaches in [6] and [11], the proposed and optimal method.

Method	Instrumental variables	Covariance matrix P_{IV}	Optimal accuracy
Optimal	$z_{\text{opt}}(t) = \varphi_r^j(t)$	$P_{IV}^{\text{opt}} = \lambda_\epsilon^2 \left[\bar{\mathbb{E}}(\varphi_r^j(t))^T \varphi_r^j(t)\right]^{-1}$	yes
Method 1 [11]	$z_1(t) = \Psi(q)r(t)$	$P_{IV,1} = \lambda_\epsilon^2 \left[\bar{\mathbb{E}}(\Psi r(t))^T \varphi_r^j(t)\right]^{-1} \left[\bar{\mathbb{E}}(\Psi r(t))^T \Psi r(t)\right] \left[\bar{\mathbb{E}}(\Psi r(t))^T \varphi_r^j(t)\right]^{-T}$	no
Method 2 [6]	$z_2(t) = \varphi_2(t)$	$P_{IV,2} = \lambda_\epsilon^2 \left[\bar{\mathbb{E}}\varphi_2^T(t)\varphi_r^j(t)\right]^{-1} \left[\bar{\mathbb{E}}\varphi_2^T(t)\varphi_2(t)\right] \left[\bar{\mathbb{E}}\varphi_2^T(t)\varphi_r^j(t)\right]^{-T}$	no
Proposed	$z_p(t) = \hat{\varphi}_r^j(t)$	$P_{IV,p} = \lambda_\epsilon^2 \left[\bar{\mathbb{E}}(\hat{\varphi}_r^j(t))^T \varphi_r^j(t)\right]^{-1} \left[\bar{\mathbb{E}}(\hat{\varphi}_r^j(t))^T \hat{\varphi}_r^j(t)\right] \left[\bar{\mathbb{E}}(\hat{\varphi}_r^j(t))^T \varphi_r^j(t)\right]^{-T}$	yes

LQG optimal control. In addition, the basic IV method is used as advocated in [11] and [6], i.e., $n_z = n_\theta$, $L(q) = 1$ and $W = I$. Then, the asymptotic distribution of $\hat{\theta}^\Delta$ is given by

$$\sqrt{N}(\hat{\theta}^\Delta - \theta_0^\Delta) \xrightarrow{\text{dist}} \mathcal{N}(0, P_{IV}), \tag{5}$$

where θ_0^Δ is the asymptotic parameter estimate, and the covariance matrix $P_{IV} \in \mathbb{R}^{n_\theta \times n_\theta}$ is given by

$$P_{IV} = \lambda_\epsilon^2 R^{-1} J R^{-T}, \tag{6}$$

with

$$J = \bar{\mathbb{E}} z^T(t) z(t), \quad R = \bar{\mathbb{E}} z^T(t) \varphi_r^j(t),$$

and $\varphi_r^j(t) = \Psi(q)(C_{fb}(q) + C_{ff}^j(q))^{-1} y_r^j(t)$, where the reference-induced contribution to the measured output signal $y_m^j(t)$, denoted $y_r^j(t)$, is given by

$$y_r^j(t) = S(q)P(q)(C_{fb}(q) + C_{ff}^j(q))r(t).$$

Conditions for the existence of (5) are provided in App. I. The asymptotic distribution is a special case of the general framework for extended IV methods without assuming $S(q)H(q) = 1$, and is derived along similar lines as in [19, Appendix A8.1] for open-loop identification and [20, Section 3] for closed-loop identification. Details are omitted for brevity.

B. Optimal Accuracy for Iterative Feedforward Control

In this section, a lower bound on P_{IV} is provided as a function of $z(t)$. For iterative feedforward control, the lower bound P_{IV}^{opt} exists under the condition that $z(t)$ consist of filtered $r(t)$. Then, P_{IV}^{opt} is given by

$$P_{IV} \geq P_{IV}^{\text{opt}},$$

where

$$P_{IV}^{\text{opt}} = \lambda_\epsilon^2 \left[\bar{\mathbb{E}}(\varphi_r^j(t))^T \varphi_r^j(t)\right]^{-1}. \tag{7}$$

From P_{IV} in (6) and P_{IV}^{opt} in (7) it follows that

$$P_{IV} = P_{IV}^{\text{opt}},$$

if the instruments $z(t)$ are selected as

$$z_{\text{opt}}(t) = \varphi_r^j(t). \tag{8}$$

In the next section, P_{IV}^{opt} in (7) and $z_{\text{opt}}(t)$ in (8) are exploited to analyze the accuracy properties of the approaches in [6] and [11].

IV. ANALYSIS OF EXISTING APPROACHES

In this section, the accuracy properties of the iterative feedforward tuning approaches in [6] and [11] are compared with the optimal approach derived in Sect. III. An overview of this comparison is given in Table I.

A. Analysis of Method 1

Consider the approach proposed in [11], where $z(t)$ is selected as $z_1(t) = \Psi(q)r(t)$, with $\Psi(q)$ the basis functions of $C_{ff}(q)$. Substitute $z_1(t)$ in (6) to obtain the corresponding covariance matrix

$$P_{IV,1} = \lambda_\epsilon^2 \left[\bar{\mathbb{E}}(\Psi r(t))^T \varphi_r^j(t)\right]^{-1} \left[\bar{\mathbb{E}}(\Psi r(t))^T \Psi r(t)\right] \times \left[\bar{\mathbb{E}}(\Psi r(t))^T \varphi_r^j(t)\right]^{-T}.$$

By using [19, Lemma A.4],

$$P_{IV,1} > P_{IV}^{\text{opt}}.$$

This result implies that optimal accuracy is not attained for the approach proposed in [11].

B. Analysis of Method 2

Consider the approach proposed in [6], where $z(t)$ is selected as $z_2(t) = \varphi_2(t)$, where $\varphi_2(t) = \Psi(q)(C_{fb}(q) + C_{ff}^j(q))^{-1} y_m(t)$ is determined based on measured data from an additional task. Hence, this approach requires measured data from two tasks to determine a single update $\hat{\theta}^\Delta$. Furthermore, it is assumed that

$$\mathbb{E} R_{v_1 v_2} = 0, \tag{9}$$

i.e., $v_1(t)$ in the first task and $v_2(t)$ in the second task are uncorrelated. The covariance matrix $P_{IV,2}$ for this approach is given by

$$P_{IV,2} = \lambda_\epsilon^2 \left[\bar{\mathbb{E}}\varphi_2^T(t)\varphi_r^j(t)\right]^{-1} \left[\bar{\mathbb{E}}\varphi_2^T(t)\varphi_2(t)\right] \times \left[\bar{\mathbb{E}}\varphi_2^T(t)\varphi_r^j(t)\right]^{-T}.$$

By using condition (9) and [19, Lemma A.4],

$$P_{IV,2} > P_{IV}^{\text{opt}}. \tag{10}$$

As a result, it is concluded that the lower bound P_{IV}^{opt} is not reached for the approach in [6].

In addition to the provided analysis with respect to P_{IV}^{opt}, there is a second contribution that hampers obtaining optimal accuracy. Note that the asymptotic distribution of $\hat{\theta}^{\Delta}$ is given by

$$\sqrt{N}(\hat{\theta}^{\Delta} - \theta_0^{\Delta}) \xrightarrow{\text{dist}} \mathcal{N}(0, P_{IV}),$$

while measured data is used from two tasks of each N samples. Clearly, for instrumental variable methods based on measured data from a single task, the asymptotic distribution for the same number of tasks is given by

$$\sqrt{2N}(\hat{\theta}^{\Delta} - \theta_0^{\Delta}) \xrightarrow{\text{dist}} \mathcal{N}(0, P_{IV}),$$

revealing a $1/\sqrt{2}$ reduced accuracy in the first approach. That is, since an additional task is required to determine $z(t)$, N samples are not used to reduce the variance of $\hat{\theta}^{\Delta}$. Hence, in combination with (10), it is concluded that optimal accuracy is not attained for the approach proposed in [6].

C. Concluding Remarks

In this section, the existing approaches in [6] and [11] are compared with the optimal approach derived in Sect. III. It is shown that optimal accuracy is not obtained with these approaches, as summarized in Table I.

V. PROPOSED APPROACH ACHIEVING OPTIMAL ACCURACY

A. Caveat of the Optimal Instruments

The approach provided in Sect. III leads to optimal accuracy for iterative feedforward control. It seems straightforward to use the optimal instrumental variable $z_{\text{opt}}(t) = \varphi_r^j(t)$, as derived in Sect. III-B. However, the caveat associated with constructing $z_{\text{opt}}(t)$ is that $\varphi_r^j(t)$ can not be determined without modeling $S(q)P(q)$. To illustrate this statement, recall that $\varphi_r^j(t)$ is given by

$$\varphi_r^j(t) = \Psi(q)(C_{fb}(q) + C_{ff}^j(q))^{-1} y_r^j(t),$$

where the reference-induced contribution to the measured output signal $y_m^j(t)$, denoted $y_r^j(t)$, is equal to

$$y_r^j(t) = S(q)P(q)(C_{fb}(q) + C_{ff}^j(q))r(t).$$

Clearly, a model for $S(q)P(q)$ is required to construct $y_r^j(t)$, which is conflicting with the pursued data-driven approach. In the remainder of this section, the known reference signal $r(t)$ is used to determine an approximate implementation of $z_{\text{opt}}(t)$ without modeling $S(q)$ and $P(q)$.

B. Approximate Implementation of the Optimal Instruments

In this section, an approximate implementation of the optimal instrument $z_{\text{opt}}(t)$ is proposed that leads to optimal accuracy. To this end, a bootstrap method is proposed, as in [14, Section 5] for closed-loop identification and [12, Chapter 5, 6] for open-loop identification.

The proposed instrument $z_{\text{p}}(t)$ exploits the reference $r(t)$ to approximate $z_{\text{opt}}(t)$, and is given by

$$z_{\text{p}}(t) = \hat{\varphi}_r^j(t) = \Psi(q)(C_{fb}(q) + C_{ff}^j(q))^{-1} r(t),$$

with $\Psi(q)$ the basis functions of $C_{ff}^j(q)$. The corresponding asymptotic distribution of $\hat{\theta}^{\Delta}$ is given by

$$\sqrt{N}(\hat{\theta}^{\Delta} - \theta_0^{\Delta}) \xrightarrow{\text{dist}} \mathcal{N}(0, P_{IV,p}),$$

with

$$\begin{aligned} P_{IV,p} = &\lambda_\epsilon^2 \left[\bar{\mathbb{E}}(\hat{\varphi}_r^j(t))^T \varphi_r^j(t)\right]^{-1} \left[\bar{\mathbb{E}}(\hat{\varphi}_r^j(t))^T \hat{\varphi}_r^j(t)\right] \\ &\times \left[\bar{\mathbb{E}}(\hat{\varphi}_r^j(t))^T \varphi_r^j(t)\right]^{-T}. \end{aligned}$$

For this design, it holds that $P_{IV,p}$ closely approximates the lower bound P_{IV}^{opt}, i.e., optimal accuracy, if

$$z_{\text{p}}(t) \approx z_{\text{opt}}(t). \tag{11}$$

To achieve optimal accuracy, the following iterative scheme is proposed for off-line computation of $\hat{\theta}^{\Delta}$.

Step 1. Construct the instrumental variable

$$z_{\text{p}}(t) = \hat{\varphi}_r^j(t) = \Psi(q)(C_{fb}(q) + C_{ff}^j(q))^{-1} r(t),$$

based on $C_{ff}^j(q, \theta^j)$ in the j^{th} task.

Step 2. Determine $\hat{\theta}^{\Delta}$ as in (4) based on the measured signals $e_m^j(t)$ and $y_m^j(t)$ in the j^{th} task, and $z_{\text{p}}(t)$ from Step 1. Then, construct $C_{ff}^{\Delta}(q, \hat{\theta}^{\Delta})$ according to (2).

Step 3. Update $z_{\text{p}}(t)$ based on $\hat{\theta}^{\Delta}$, i.e.,

$$z_{\text{p}}(t) = \hat{\varphi}_r^j(t) = \Psi(q)(C_{fb}(q) + C_{ff}^j(q) + C_{ff}^{\Delta}(q))^{-1} r(t).$$

Step 4. Determine $\hat{\theta}^{\Delta}$ as in (4) based on the measured signals $e_m^j(t)$ and $y_m^j(t)$ in the j^{th} task and $z_{\text{p}}(t)$ from Step 3.

Consider *Step 2* and *Step 3* in the proposed iterative scheme. In *Step 2*, $\hat{\theta}^{\Delta}$ is determined based on measured data from the j^{th} task and $z_{\text{p}}(t)$ in *Step 1*. Since $z_{\text{p}}(t)$ is uncorrelated with v, the analysis in App. I shows that $\hat{\theta}^{\Delta}$ is a consistent estimator. As a result, for N tends to infinity, it holds that $\hat{e}_r^j(t) = e_r^j(t) - \varphi^j(t)\hat{\theta}^{\Delta} = 0$. Since $z_{\text{opt}}(t) - z_{\text{p}}(t)$ in *Step 3* is given by

$$\begin{aligned} &z_{\text{opt}}(t) - z_{\text{p}}(t) = \varphi_r^j(t) - \hat{\varphi}_r^j(t) \\ &= \Psi(q)(C_{fb}(q) + C_{ff}^j(q) + C_{ff}^{\Delta}(q))^{-1} \left[e_r^j(t) - \varphi^j(t)\hat{\theta}^{\Delta}\right], \end{aligned}$$

it follows that $z_{\text{opt}}(t) \approx z_{\text{p}}(t)$ in *Step 3* if $e_r^j(t) - \varphi^j(t)\hat{\theta}^{\Delta} \approx 0$. This implies that $P_{IV,p}$ closely approximates the lower bound P_{IV}^{opt} for $\hat{\theta}^{\Delta}$ in *Step 4*. That is, optimal accuracy is approximated with the proposed iterative scheme. Depending on the measured signals $e_m^j(t)$ and $y_m^j(t)$ and feedforward controller $C_{ff}^j(q)$ in the j^{th} task, multiple iterations may be required to obtain (11).

Summarizing, an approximate implementation of the optimal IV method is proposed for iterative feedforward control. In contrast with existing methods as provided in Sect. IV, the proposed approach closely approximates the lower bound on the covariance matrix. In the next section, the theoretical results derived in this paper are illustrated in a simulation study of a motion system.

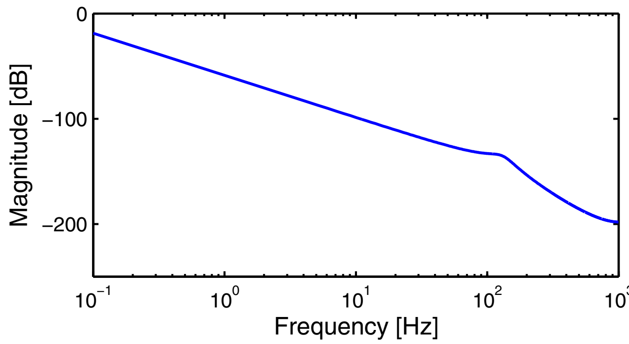

Fig. 3: Bode diagram of the system P.

VI. APPLICATION TO MOTION CONTROL

In this section, a simulation study is provided to illustrate that $z_{\mathrm{p}}(t)$ as proposed in Sect. V results in enhanced servo performance for a motion system compared to the approach in [11]. Consider the system $P(q)$ given by

$$P(q) = \frac{1.761 \times 10^{-9}}{1 - 3.69q^{-1} + 5.225q^{-2} - 3.38q^{-3} + 0.8451q^{-4}},$$

which corresponds to a two-mass spring damper system with noncollocated dynamics, see Fig. 3 for a Bode magnitude plot. The feedback controller $C_{fb}(z)$ is given by

$$C_{fb} = \frac{7.444 \times 10^4 q^{-1} - 1.47 \times 10^5 q^{-2} + 7.259 \times 10^4 q^{-3}}{1 - 2.736q^{-1} + 2.49q^{-2} - 0.7537q^{-3}}.$$

Similar to Sect. II-A, the unknown disturbance $v(t)$ is assumed to be given by $v(t) = H(q)\epsilon(t)$, where $H(q)$ is designed such that $S(q)H(q) = 1$, and $\epsilon(t)$ is normally distributed white noise with zero mean and standard deviation $\lambda_\epsilon = 2.5 \times 10^{-8}$. The system is excited by a 3^{rd}-order reference signal. The following parametrization is proposed

$$C_{ff}(q, \theta) = \psi_a(q^{-1})\theta_a + \psi_s(q^{-1})\theta_s,$$

with polynomial basis functions

$$\psi_a(q^{-1}) = \frac{1 - 2q^{-1} + q^{-2}}{T_s^2},$$
$$\psi_s(q^{-1}) = \frac{1 - 4q^{-1} + 6q^{-2} - 4q^{-3} + q^{-4}}{T_s^4},$$

with sampling time $T_s = 5 \times 10^{-4}$ s. Hence, $C_{ff}(q, \theta)$ consist of acceleration feedforward $\psi_a(q^{-1})\theta_a$ and snap feedforward $\psi_s(q^{-1})\theta_s$. Straightforward computations reveal that the true parameter vector θ_0 of $C_{ff}(q, \theta)$, as defined in Appendix I, is given by $\theta_0 = [22,\ 3 \times 10^{-5}]^T$. That is, the contribution of e induced by r is eliminated if $C_{ff}(q, \theta_0)$ is used as feedforward controller.

For the considered system, a comparison is provided between the approach in [11] with $z_1(t) = \Psi(q)r(t)$ and the proposed approach in Sect. V with $z_{\mathrm{p}}(t) = \hat{\varphi}_r^j(t)$. Monte Carlo simulations are performed for numerical illustration, where the number of samples and realizations are given by $N = 6000$ and $m = 200$, respectively. In addition, the initial parameter vector is given by $\theta^{\text{init}} = \left[16,\ 1 \times 10^{-5}\right]^T$.

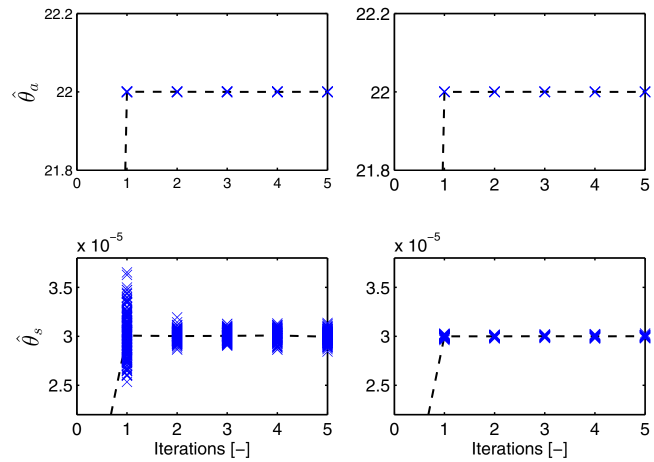

Fig. 4: Parameters $\hat{\theta}$ as a function of iteration for $m = 100$ realizations for $z_1(t)$ (left) and $z_{\mathrm{p}}(t)$ (right) show that the variance of $\hat{\theta}_a$ is comparable for both approaches, while the variance of $\hat{\theta}_s$ is significantly smaller for $z_{\mathrm{p}}(t)$ compared to $z_1(t)$.

From Fig. 4 it is clear that unbiased estimates of θ_0, i.e., $\bar{\theta} = \theta_0$, are obtained for $z_1(t)$ and $z_{\mathrm{p}}(t)$. Next, consider the variance of the estimates. On the one hand, the estimated parameters $\hat{\theta}^j$ as depicted in Fig. 4 visually confirm that θ_a is comparable for $z_1(t)$ and $z_{\mathrm{p}}(t)$. On the other hand, the variance of the snap feedforward parameter $\hat{\theta}_s$, is significantly smaller for $z_{\mathrm{p}}(t)$ when compared to $z_1(t)$. This confirms that the proposed approach significantly improves the accuracy of the estimated parameters.

It remains to be shown that an improved accuracy of the estimated parameters leads to an enhanced servo performance of the considered system. Therefore, consider the worst-case estimate of θ_0 in the j^{th} iteration given by

$$\hat{\theta}_{\text{wc}}^j = \max_{i=1,\dots,m} |\hat{\theta}_i^j - \theta_0|.$$

A comparison of the servo error $e(t)$ corresponding to $\hat{\theta}_{\text{wc}}^j$ and the nominal value $\bar{\theta}^j$ provides insight in the effect of a reduction in variance on the servo error $e(t)$. From Fig. 5 it becomes clear that the servo error e_{wc}^1 for $\hat{\theta}_{\text{wc}}^1$ is significantly reduced by improved accuracy of the estimated parameters. In fact, $e_{\text{wc}}^1(t)$ corresponding to $z_1(t)$ contains a significant contribution induced by the reference $r(t)$. In contrast, the reference induced contribution to $e_{\text{wc}}^1(t)$ for $z_{\mathrm{p}}(t)$ is negligible. These observations are confirmed by the cumulative power spectrum of the error signal as shown in Fig. 6. Hence, it is shown that $z_{\mathrm{p}}(t)$ as in Sect. V results in enhanced servo performance when compared to [11].

VII. CONCLUSIONS

In this paper, accuracy aspects are analyzed for iterative feedforward control. First, the accuracy properties of existing approaches are analyzed. Therefore, the covariance matrix of these approaches are evaluated with respect to the optimal covariance matrix. It is shown that optimal accuracy is not obtained with the existing approaches. Second, a new approach is proposed that attains optimal accuracy. In this

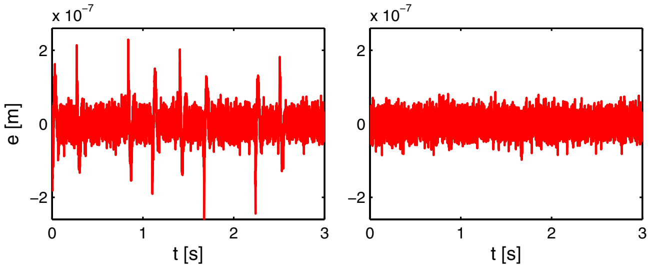

Fig. 5: The error signal $e^1_{\text{wc}}(t)$ in the 1st task corresponding to $\hat{\theta}^{\text{wc}}$ shows that $e^1_{\text{wc}}(t)$ contains a significant reference-induced component for $z_1(t)$ (left), while $e^1_{\text{wc}}(t)$ is dominantly stochastic for $z_{\text{p}}(t)$ (right).

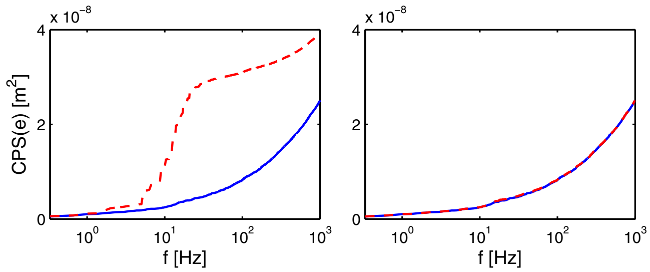

Fig. 6: Cumulative power spectrum of the error signal in the 1th iteration. Left: $\bar{\theta}$ (blue) and $\hat{\theta}^{\text{wc}}$ (red) for $z_1(t)$. Right: $\bar{\theta}$ (blue) and $\hat{\theta}^{\text{wc}}$ (red) for $z_{\text{p}}(t)$.

approach, an approximate implementation of the optimal instrumental variable is proposed based on a bootstrap method. A simulation study of a motion system illustrates that the proposed method i) leads to improved accuracy and ii) results in enhanced servo performance.

Future research focuses on an experimental confrontation of the proposed approach with an industrial motion system, multivariable generalizations, input shaping [9], inferential control, and positioning-varying effects [21].

Appendix I
Optimal Feedforward IV

The asymptotic distribution (5) in Sect. III exists under the following assumptions: i) there is an C^{j+1}_{ff} such that $C^{j+1}_{ff} = P^{-1}$ and ii) $\hat{\theta}^{\Delta}$ is a consistent estimator. First, consider the first assumption. For the polynomial parametrization of C^{j+1}_{ff} as defined in Sect. II-B, a necessary condition for $C^{j+1}_{ff} = P^{-1}$ to hold is that P is restricted to a rational function with unit numerator,

$$P(q) = \frac{1}{A_0(q^{-1})},$$

where $A_0(q^{-1}) \in \mathbb{R}[q^{-1}]$. Then, there is an unique θ^{Δ}_0 such that the reference-induced error $e^{j+1}_r(t)$, defined as

$$e^{j+1}_r(t) = S(q)(1 - P(q)C^{j+1}_{ff}(q))r(t),$$

is eliminated, i.e., $e^{j+1}_r(t) = e^j_r(t) - \varphi^j(t)\theta^{\Delta}_0 = 0$. As a result, $C^{j+1}_{ff}(q, \theta_0) = P^{-1}(q)$, where $\theta_0 = \theta^j + \theta^{\Delta}_0$.

Next, consider the latter assumption. After establishing that there exist an unique θ^{Δ}_0, it remains to be shown that $\hat{\theta}^{\Delta}$ is a consistent estimator ($\text{plim}_{N\to\infty} \hat{\theta}^{\Delta} = \theta^{\Delta}_0$). Following a similar argument as in [14], $\hat{\theta}^{\Delta}$ is a consistent estimator if i) R is nonsingular and ii) $z(t)$ is uncorrelated with $v^j(t)$.

Acknowledgements

The authors thank the late Okko Bosgra for valuable discussions. Furthermore, Leon van Breugel and Dennis Bruijnen are acknowledged for their contribution.

References

[1] H. Zhong, L. Y. Pao, and R. A. de Callafon, “Feedforward control for disturbance rejection: Model matching and other methods,” in *Proceedings of the Chinese Conference on Decision and Control, Taiyuan, China*, 2012, pp. 3525–3533.

[2] J. Butterworth, L. Pao, and D. Abramovitch, “Analysis and comparison of three discrete-time feedforward model-inverse control techniques for nonminimum-phase systems,” *Mechatronics*, vol. 22, pp. 577–587, 2012.

[3] D. Bristow, M. Tharayil, and A. Alleyne, “A survey of iterative learning control,” *IEEE Control Systems Magazine*, vol. 26, no. 3, pp. 96–114, 2006.

[4] S. Devasia, “Should model-based inverse inputs be used as feedforward under plant uncertainty?” *IEEE Transactions on Automatic Control*, vol. 47, no. 11, pp. 1865–1871, 2002.

[5] J. van de Wijdeven and O. Bosgra, “Using basis functions in iterative learning control: analysis and design theory,” *International Journal of Control*, vol. 83, no. 4, pp. 661–675, 2010.

[6] S. van der Meulen, R. Tousain, and O. Bosgra, “Fixed structure feedforward controller design exploiting iterative trials: Application to a wafer stage and a desktop printer,” *Journal of Dynamic Systems, Measurement, and Control*, vol. 130, no. 5, pp. 0 510 061–05 100 616, 2008.

[7] H. Hjalmarsson, M. Gevers, S. Gunnarsson, and O. Lequin, “Iterative feedback tuning: theory and applications,” *IEEE Control Systems*, vol. 18, no. 4, pp. 26–41, 1998.

[8] M. Heertjes, D. Hennekens, and M. Steinbuch, “MIMO feed-forward design in wafer scanners using a gradient approximation-based algorithm,” *Control Engineering Practice*, vol. 18, no. 5, pp. 495–506, 2010.

[9] D. Bruijnen and N. van Dijk, “Combined input shaping and feedforward control for flexible motion systems,” in *Proceedings of the 2012 American Control Conference, Montréal, Canada*, 2012.

[10] H. Stearns, S. Yu, B. Fine, S. Mishra, and M. Tomizuka, “A comparative study of feedforward tuning methods for wafer scanning systems,” in *ASME Dynamic Systems and Control Conference, MI, USA*, 2008.

[11] F. Boeren and T. Oomen, “Iterative feedforward control: a closed-loop identification problem and a solution,” in *Proc. of the 52st Conference on Decision and Control, Firenze, Italy*, 2013, pp. 6694–6699.

[12] T. Söderström and P. Stoica, *Instrumental Variable Methods for System Identification*, A. Balakrishnan and M. Thoma, Eds. Springer-Verlag, Berlin, Germany, volume 57 of LNCIS, 1983.

[13] U. Forssell, “Closed-loop identification: methods, theory and applications,” Ph.D. dissertation, Linkoping university, 1999.

[14] M. Gilson and P. Van den Hof, “Instrumental variable methods for closed-loop system identification,” *Automatica*, vol. 41, pp. 241–249, 2005.

[15] S. Formentin, K. van Heusden, and A. Karimi, “A comparison of model-based and data-driven controller tuning,” *International Journal of Adaptive Control and Signal Processing, Early Access paper*, 2013.

[16] A. Karimi, M. Butcher, and R. Longchamp, “Model-free precompensator tuning based on the correlation approach,” *IEEE Transactions on Control Systems Technology*, vol. 16, no. 5, pp. 1013–1020, 2008.

[17] K.-S. Kim and Q. Zou, “A modeling-free inversion-based iterative feedforward control for precision output tracking of linear time-invariant systems,” *IEEE/ASME Transactions on Mechatronics*, vol. 18, no. 6, pp. 1767–1777, 2013.

[18] P. Lambrechts, M. Boerlage, and M. Steinbuch, “Trajectory planning and feedforward design for electromechanical motion systems,” *Control Engineering Practice*, vol. 13, pp. 145–157, 2005.

[19] T. Söderström and P. Stoica, *System Identification*. Prentice Hall, Hemel Hempstead, UK, 1989.

[20] T. Söderström, P. Stoica, and E. Trulsson, “Instrumental variable methods for closed loop systems,” in *In* 10*th* *IFAC World Congress on Automatic Control*, 1987, pp. 363–368.

[21] T. Oomen, R. van Herpen, S. Quist, M. van de Wal, O. Bosgra, and M. Steinbuch, “Connecting system identification and robust control for next-generation motion control of a wafer stage,” *IEEE Transactions on Control Systems Technology*, vol. 22, no. 1, pp. 102–118, 2014.