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Exploiting Rational Basis Functions in Iterative Learning Control

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Abstract— **Iterative learning control approaches often suffer from poor extrapolability with respect to exogenous signals, including setpoint variations. The aim of this paper is to introduce rational basis functions in ILC. Such rational basis function have the potential to both increase performance and enhance extrapolability. The key caveat that is associated with these rational basis function lies in a significantly more complex optimization problem when compared to using polynomial basis functions. In this paper, a novel iterative optimization procedure is proposed that enables the use of rational basis functions in ILC. A simulation example confirms i) the advantages of rational basis functions compared to pre-existing results, and 2) the efficacy of the proposed iterative algorithm.**

I. INTRODUCTION

Iterative learning control (ILC) [1] can significantly enhance the performance of systems that perform repeated tasks. After each repetition the command signal for the next repetition is updated by learning from past executions. A key assumption in ILC is that the task of the system is invariant under the repetitions. As a consequence, the learned command signal is optimal for the specific fixed task only. In general, extrapolation of the learned command signal to other tasks leads to a significant performance deterioration.

Several approaches have been proposed to enhance the extrapolability of ILC to a class of reference signals. In [2] a segmented approach to ILC is presented and applied to a wafer stage. This approach is further extended in [3], where the complete task is divided into subtasks that are learned individually. On the other hand, in [4], extrapolability of ILC is enhanced through the use of basis functions. To fit into the standard ILC solution, *polynomial* basis functions are employed. Essentially, such polynomial basis functions are essential to retain the analytic solution of the ILC controller [5]. In [6], the polynomial basis functions in [7] are implemented in the ILC framework of [4] and successfully applied to an industrial wafer stage system, whereas in [8] an application to a wideformat printer application is reported. Extensions towards input shaping are presented in [9].

Although the use of polynomial basis functions enhances extrapolability of ILC algorithms, the polynomial nature of the basis functions severely limits the achievable performance and extrapolability. Essentially, the basis functions typically constitute a model inverse of the true system [10], [11]. The use of polynomial basis functions implies that only a perfect inverse can be obtained if the model only contains poles and no zeros. Since many physical systems are modeled using rational models containing both poles and zeros, this implies that existing results necessarily lead to biased model inverses. Consequently, both achievable performance and extrapolability is limited.

This paper aims to introduce a new ILC framework that can achieve perfect performance and extrapolability for the general class of rational systems. Hereto, in this paper it is proposed to introduce *rational* basis functions in ILC. The key technical difficulty associated with these basis functions is that the analytic solution of standard optimal ILC [5] and the basis function approach in [4] is lost. In fact the resulting optimization problem in general is non-convex.

The main contribution of this paper is a novel introduction of rational basis functions in iterative learning control and a new solution that resorts to an iterative procedure of convex optimization problems. The proposed solution has strong connections to common algorithms in both time domain [12, Section 10] and frequency domain system identification [13, Section 9.8.3], [14]. Early contributions in this direction are presented in [15] and [16], respectively. Interestingly, the results in [4], [6], [9] are directly recovered as a special case of the proposed solution.

In the next section, the problem formulation is formally stated. Then, in Section III, the new parameterization is proposed. Section III-A contains a novel iterative solution to the optimization problem, which constitutes the main contribution of this paper. Section IV establishes connections to pre-existing results that employ basis functions in ILC. In Section V, a simulation example is presented that reveals the advantages of employing rational basis functions and efficacy of the proposed iterative solution.

Notation: For a vector x, $||x||_W = x^T W x$. All signals and systems are discrete time and often tacitly assumed of length n. The i^{th} element of a vector θ is denoted $\theta[i]$. Also, $H(\theta, z) = \sum_{i=1}^{m} \xi_i(z)\theta[i]$, with z a complex indeterminate. In addition q^{-1} is defined as the backward shift operator $qu(t) = u(t-1)$. For $u, y \in \mathbb{R}^{n\times 1}$, $y = H(\theta, q^{-1})u$ is equivalent to $y = H(\theta)u$ and $y = \Psi_{Hu}\theta$, where $H(\theta, z)$ is a transfer function with parameters $\theta \in \mathbb{R}^{m\times 1}$, basis functions $\xi_i(z)$ with $i = 1, 2, \ldots, m$ and $H(\theta) \in \mathbb{R}^{n\times n}$ is the corresponding convolution matrix and $\Psi_{Hu} = [\xi_1(q)u, \xi_2(q)u, \ldots, \xi_m(q)u] \in \mathbb{R}^{N\times m}$.

II. PROBLEM FORMULATION

In this section, the problem that is addressed in this paper is defined. First, in Section II-A, the general ILC setup is introduced. Then, in Section II-B, optimization-based ILC is introduced. This is further tailored towards polynomial basis functions in Section II-C, followed by a definition of the problem that is addressed in the present paper.

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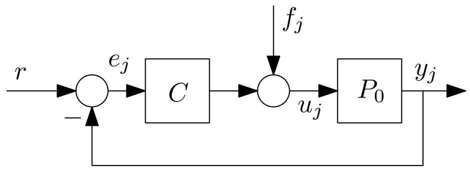

Fig. 1. ILC setup.

A. Problem setup

The considered ILC setup is shown in Fig. 1. The setup consists of a feedback controller C, and system P_0, where both are assumed linear time invariant (LTI) and causal. During an experiment with index j, the reference r and system output y_j are measured. The feedforward command signal is denoted f_j. Note from Fig. 1 that

$$e_j = S_0 r - P_0 S_0 f_j, \quad (1)$$

with $S_0 := \frac{1}{1+CP_0}$ the sensitivity. In ILC, the feedforward is generated by learning from measured data of previous experiments, also called trials. The objective is to minimize e_{j+1}, i.e., the predicted tracking error for the next experiment. From (1), it follows that

$$e_{j+1} = S_0 r - P_0 S_0 f_{j+1}. \quad (2)$$

Since r is constant, $S_0 r$ is eliminated from (1) and (2), yielding the error propagation from trial j to trial $j+1$

$$e_{j+1} = e_j + P_0 S_0 (f_j - f_{j+1}).$$

B. Norm optimal ILC

Norm optimal ILC is an important class of ILC algorithms, e.g., [5], [17]–[20], where f_{j+1} is determined from the solution of an optimization problem. The optimization criterion is typically defined as follows.

Definition 1 (Norm optimal ILC)**.** *The optimization criterion for norm optimal ILC algorithms is given by*

$$\mathcal{J}(f_{j+1}) := ||e_{j+1}||_{W_e} + ||f_{j+1}||_{W_f} + ||f_{j+1} - f_j||_{W_{\Delta f}}, \quad (3)$$

with $W_e \succ 0$, and $W_f, W_{\Delta f} \succeq 0$.

In (3), $W_e \succ 0$, and $W_f, W_{\Delta f} \succeq 0$ are user-defined weighting matrices to specify performance and robustness objectives, including i) robustness with respect to model uncertainty (W_f) ii) convergence speed and sensitivity to trial varying disturbances ($W_{\Delta f}$). The corresponding feed forward update is given by

$$f_{j+1} = \arg\min_{f_{j+1}} \mathcal{J}(f_{j+1}). \quad (4)$$

In view of (2), a norm optimal ILC controller that minimizes (3) is optimized for a specific reference trajectory r. As a result, changing r implies that the command signal f_j is not optimal in general. In the next section, norm optimal ILC with *polynomial* basis functions is investigated that aims to learn command signals for a class of references signals, thereby enhancing the extrapolability properties of ILC.

C. Norm optimal ILC with polynomial basis functions

In, e.g., [4], basis functions have been introduced in ILC that are of the form

$$f_j = \Psi \theta_j, \quad (5)$$

where f_j is a linear combination of user-selected vectors $\Psi = [\psi_1, \psi_2, \ldots, \psi_m]$. Notice that the basis functions in (5) are in lifted notation. A typical example of a basis Ψ constitutes of filtered reference signals [6]. To ensure that the basis is linear in the parameters, this is typically chosen as a polynomial function that is automatically linear in the parameter θ_j. Hence, the basis in [6] is referred to as *polynomial basis functions*.

The basis functions in (5) are general and encompass standard norm optimal ILC as in Section II-B and specific choices that enhance extrapolability. Indeed, on the one hand note that the special case of norm optimal ILC is recovered by setting $\Psi = I$. On the other hand, the essence of enhancing extrapolation of the feed forward command signal to different tasks lies in choosing f_j to be a function of r. Hereto, let $f_j = F(\theta_j) r$. Subsequent substitution into (1) yields

$$\begin{aligned} e_j &= S_0 r - P_0 S_0 F(\theta_j) r \\ &= (I - P_0 F(\theta_j)) S_0 r. \end{aligned} \quad (6)$$

Equation (6) reveals that parametrizing the feed forward is in terms of the reference r, the error in (6) can be made invariant under the choice of r if $F(\theta_j)$ is selected as $F(\theta_j) = P_0^{-1}$. The results in [6] and [8] use a polynomial basis that is linear in the parameters for $F(\theta_j)$, and is a special case of the ILC with basis functions framework with Ψ as follows

$$f_j = \Psi_r \theta_j,$$

here $\Psi_r = [\xi(q) r, \xi^2(q) r, \ldots, \xi^m(q) r]$ with $\xi(z) = 1 - z^{-1}$, i.e., a differentiator.

By virtue of (6), the use of basis functions can significantly enhance the extrapolability properties of ILC. The following theorem reveals that an analytic solution exists to the norm optimal ILC problem with the basis functions defined in (5) and criterion defined in (3).

Theorem 2. *Given an model P of P_0 and $S = (1+CP)^{-1}$, Ψ with full column rank, W_e, W_f s.t. $P^T S^T W_e P S + W_f \succ 0$ and given measurements e_j, f_j, and $f_j = \Psi \theta_j$, then* (3) *is minimized with parameter update*

$$\begin{aligned} \theta_{j+1} &= L e_j + Q f_j \\ L &= \left[\Psi^T \left(P^T S^T W_e P S + W_f + W_{\Delta f}\right) \Psi\right]^{-1} \left[\Psi^T P^T S^T W_e\right] \\ Q &= \left[\Psi^T \left(P^T S^T W_e P S + W_f + W_{\Delta f}\right) \Psi\right]^{-1} \Psi^T \left(P^T S^T W_e P S + W_{\Delta f}\right) \end{aligned} \quad (7)$$

with L and Q the learning filters. In addition, (7) leads to monotonic convergence of $||f_j||$ for P sufficiently close to P_0.

Proof. The proof of Theorem 2 following along the same lines as norm-optimal ILC, e.g., [5] and is based on the necessary condition for optimality $\frac{\partial \mathcal{J}}{\partial \theta_{j+1}} = 0$, and solving this linear equation for θ_{j+1}, yielding the parameter update in (7). □

Although the polynomial basis (5) enhances extrapolability of ILC (viz. (6)) and retains the analytic solution (7), these results only hold exactly if $F(\theta_j) = P_0^{-1}$, hence P_0 is restricted to be a rational function with a unit numerator, i.e., no zeros. In case this condition is violated, the achievable performance and extrapolability of ILC is severely deteriorated. Since typical physical systems are modeled using rational models that contain both poles and zeros, a unit numerator is highly restrictive for practical applications.

D. Problem formulation: ILC with rational basis functions for enhancing performance and extrapolability

In view of the limitations imposed by the polynomial basis functions in Section II-C, this paper aims to investigate more general parameterizations that enhance both i) tracking performance, and ii) extrapolability of the learned feed forward command signal. In this paper, general rational basis functions are proposed and the following aspects are explicitly addressed.

Q1. The introduction of the rational feedforward parameterization with parameters in both the numerator and denominator

Q2. How to perform the actual optimization for more general rational parameterizations, i.e., how to compute the parameter update?

In the next section, the first aspect, i.e, Q1, is addressed.

III. A New Framework for Iterative Learning Control with Rational Basis functions

In this section, the main contribution of this paper is presented: the formulation, analysis, and synthesis of optimal ILC with rational basis functions. As is argued in Section II-A, the motivation for using such basis functions stems from (6), which reveals that parameterizing the feed forward command signal in terms of the reference signal enables extrapolation of the learned feed forward command signal to other reference trajectories. The motivation for using *rational* basis functions stems from the fact that optimal feed forward control designs typically involve a system inversion, see (6) and [10], i.e., optimal tracking performance is achieved when $f_j = P_0^{-1} r$, furthermore in this case $e_j = 0$ and is independent of r, yielding perfect extrapolation capabilities. For typical LTI systems, the optimal system inverse is rational. The following parameterization is considered.

Definition 3 (Rational basis for optimal ILC)**.** *The rational basis for use in optimal ILC is defined as*

$$f_j = F(\theta_j, q^{-1}) r,$$

with $F \in \mathcal{F}$,

$$\mathcal{F} = \left\{ (\theta_j, z) \middle| F(\theta_j, z) = \frac{A(\theta_j, z)}{B(\theta_j, z)}, \theta_j \in \mathbb{R}^{m_a + m_b} \right\} \quad (8)$$

and

$$A(\theta_j, z) = \sum_{i=1}^{m_a} \xi_i^A(z) \theta_j[i],$$
$$B(\theta_j, z) = 1 + \sum_{i=1}^{m_b} \xi_i^B(z) \theta_j[i + m_a].$$

Here, $\xi_i^A(z) \in \mathbb{R}[z]$ *and* $\xi_i^B(z) \in \mathbb{R}[z]$ *are user-chosen polynomial basis functions, and* $\theta_j = [\theta_j^A, \theta_j^B]^T$.

A. Synthesis of optimal ILC controllers with rational basis functions

In this section, a new ILC algorithm is presented that enables optimal controller synthesis using rational basis functions. The main idea is to solve a sequence of least-squares problems (similar to Theorem 2) and to consider the nonlinear terms as a priori unknown weighting functions. Hereto, note that (3) can be recast as

$$\begin{aligned}\mathcal{J}(\theta_{j+1}) = &\left\| B(\theta_{j+1}, q^{-1})^{-1} [B(\theta_{j+1}, q^{-1}) e_{j+1}] \right\|_{W_e} + \\ &\left\| B(\theta_{j+1}, q^{-1})^{-1} [B(\theta_{j+1}, q^{-1}) f_{j+1}] \right\|_{W_f} + \\ &\left\| B(\theta_{j+1}, q^{-1})^{-1} [B(\theta_{j+1}, q^{-1}) f_{j+1}] - f_j \right\|_{W_{\Delta f}} .\end{aligned} \quad (9)$$

In (9), $\mathcal{J}(\theta_{j+1})$ is nonlinear in θ_{j+1} in the term $B(\theta_{j+1}, z)^{-1}$. In contrast, $\mathcal{J}(\theta_{j+1})$ is linear in θ_{j+1} in the terms $B(\theta_{j+1}, z) e_{j+1}$ and $B(\theta_{j+1}, z) f_{j+1}$. In view of this distinction, an auxiliary index k is introduced, i.e., $\theta_{j+1}^{<k>}$ and $\theta_{j+1}^{<k-1>}$. These are substituted into (9)

$$\begin{aligned}&\mathcal{J}_k(\theta_{j+1}^{<k>}) = \\ &\quad \left\| B(\theta_{j+1}^{<k-1>}, q^{-1})^{-1} \left[B(\theta_{j+1}^{<k>}, q^{-1}) e_{j+1}^{<k>} \right] \right\|_{W_e} + \\ &\quad \left\| B(\theta_{j+1}^{<k-1>}, q^{-1})^{-1} \left[B(\theta_{j+1}^{<k>}, q^{-1}) f_{j+1} \right] \right\|_{W_f} + \\ &\quad \left\| B(\theta_{j+1}^{<k-1>}, q^{-1})^{-1} \left[B(\theta_{j+1}^{<k>}, q^{-1}) f_{j+1} \right] - f_j \right\|_{W_{\Delta f}} .\end{aligned} \quad (10)$$

where

$$e_{j+1}^{<k>} = e_j + P_0 S_0 f_j - \frac{A(\theta_{j+1}^{<k>}, q^{-1})}{B(\theta_{j+1}^{<k>}, q^{-1})} P_0 S_0 r.$$

Notice that (9) is recovered by setting $\theta_{j+1} = \theta_{j+1}^{<k>} = \theta_{j+1}^{<k-1>}$. In addition, notice that if $\theta_{j+1}^{<k-1>}$ is known, then $\mathcal{J}_k(\theta_{j+1}^{<k>})$ is a quadratic function of $\theta_{j+1}^{<k>}$. Hence, $\theta_{j+1}^{<k>}$ can be calculated along similar lines as in Theorem 2. The key idea is indeed to fix the nonlinear $B(\theta_{j+1}^{<k-1>}, z)^{-1}$ at iteration k and interpret it as an iterative weighting function. By iterating over θ_{j+1}, is is hoped that the a priori unknown weighting by the nonlinear term is effectively compensated after convergence of the iterative procedure. Clearly, this necessitates a solution to (10) for $\theta_{j+1}^{<k>}$, given $\theta_{j+1}^{<k-1>}$. The following theorem provides the required L and Q that minimize $\mathcal{J}_k(\theta_{j+1}^{<k>})$.

Theorem 4. *Given $\theta_{j+1}^{<k-1>}$, f_j and e_j criterion* (10) *is minimized by*

$$\theta_{j+1}^{<k>} = L^{<k>} e_j + Q^{<k>} f_j,$$

with

$$L^{<k>} = \left[\Psi_1^T W_e \Psi_1 + \Psi_2^T (W_f + W_{\Delta f})\Psi_2\right]^{-1} \Psi_1^T W_e B(\theta_{j+1}^{<k-1>}, z)^{-1}$$

$$Q^{<k>} = \left[\Psi_1^T W_e \Psi_1 + \Psi_2^T (W_f + W_{\Delta f})\Psi_2\right]^{-1} \left(\Psi_2^T W_{\Delta f} + \Psi_1^T W_e B(\theta_{j+1}^{<k-1>}, z)^{-1} PS\right)$$

where

$$\Psi_1 = B(\theta_{j+1}^{<k-1>}, q^{-1})^{-1} \left[\Psi_{PSr}^A, \quad -\Psi_{e_j}^B - \Psi_{PSf_j}^B\right],$$
$$\Psi_2 = B(\theta_{j+1}^{<k-1>}, q^{-1})^{-1} \left[\Psi_r^A, \quad 0\right].$$

Proof. Criterion (10) is quadratic in $\theta_{j+1}^{<k>}$, hence the proof follows along identical lines of Theorem. 2. ☐

Note that Theorem 4 once in general does not lead to the optimal solution of (4) since $B(\theta_{j+1}^{<k-1>}, z)^{-1}$ is unknown. The proposed solution is to iteratively solve for $\theta_{j+1}^{<k>}$ in (10), given an $\theta_{j+1}^{<k=0>}$, for increasing k. In this approach, $B(\theta_{j+1}^{<k-1>}, z)^{-1}$ can be interpreted as an a priori unknown weighting in the cost function that is compensated for during each iteration of the solution procedure by updating L and Q. These steps are formulated in the following iterative parameter update procedure.

Procedure 5. *Given f_j and e_j, set $k = 0$ and initialize $\theta_{j+1}^{<k-1>} = \theta_j$ then:*

1) *Determine $L^{<k>}, Q^{<k>}$*
2) *Determine $\theta_{j+1}^{<k+1>} = Q^{<k>} f_j + L^{<k>} e_j$*
3) *Set $k \to k+1$ and go back to (1) until an appropriate convergence condition is met: $\theta_{j+1}^{<k\to\infty>} = \theta_{j+1}$*

Theorem 4 and Procedure 5 provide a new solution and algorithm to minimize $\mathcal{J}(\theta_{j+1})$ in (3), constituting the main result of this paper. The key novelty of these results lies in their use in optimal ILC algorithms. Indeed, similar algorithms [15], [16], [12, Section 10], [13, Section 9.8.3], [14] have been used abundantly in system identification. Despite the fact that the objective function is non-convex in general, practical use has revealed good convergence properties and in fact global convergence has been established for specific cases, e.g., in [21].

Remark 1. *The stable inversion approach in [10] can be adopted in the case where the filtering operation in Theorem 4 involves a non-minimum phase $B(\theta_{j+1}^{<k-1>}, z)$ and hence unstable $B(\theta_{j+1}^{<k-1>}, z)^{-1}$.*

IV. Connections to pre-existing approaches

As already mentioned throughout the preceding sections, specific polynomial basis function choices are recovered as special cases, including the results in [6] and [9]. As these special cases will be compared in the next section, these specific parameterizations are introduced next. In the next section, these specific parameterizations are compared with respect to the unified framework and solution as proposed in the present paper.

A. FIR structure

In [6] a polynomial basis F is used that is linear in the parameters. Given its close resemblance to the well-known FIR basis, the parameterization in [6] is referred to as FIR parameterization. This FIR parameterization connects to the rational basis functions in Definition 3 by setting $m_b = 0$ and $m_a = m$, hence $B(\theta_j, z) = 1$. Furthermore, $W_e = I$ and $W_f = W_{\Delta f} = 0$. Finally, let $\xi = 1 - z^{-1}$, then the basis functions used are $\xi_1^A(z) = \xi(z), \xi_2^A(z) = \xi(z)^2, ..., \xi_m^A = \xi(z)^m$.

B. Extended FIR

In [9], a more general polynomial basis is presented that extends the FIR parameterization in [6]. This parameterization is referred to as extended FIR. In fact, this parameterization can be interpreted as a combination between parallel and serial ILC, see [1] for appropriate definitions. Next, by exploiting the commutativity property for SISO LTI systems, the framework in [9] can be recast in the form of (8). As a result, the results in [9] fit in the unified framework of the present paper by selecting: $W_e = I, W_f = W_{\Delta f} = 0$, let $\xi = 1 - z^{-1}$, then the basis functions used are $\xi_1^A(z) = \xi(z), \xi_2^A(z) = \xi(z)^2, ..., \xi_{m_a}^A = \xi(z)^{m_a}$ and $\xi_1^B(z) = \xi(z), \xi_2^B(z) = \xi(z)^2, ..., \xi_{m_b}^B = \xi(z)^{m_b}$. $B(\theta_{j+1}, z)^{-1} := 1$, ignoring the nonlinear part of (9), and hereby introducing an a priori unknown weighting $B(\theta_{j+1}, z)$ in the performance criterion.

V. Motivating example

The novel controller structure is compared with pre-existing results in a simulation study. The considered true system P_0 is a fourth order motion system that is controlled by a stabilizing feedback controller C. State space realizations of P_0 and C are given by

$$x_0(t+1) = A_0 x_0(t) + B_0 u_j(t)$$
$$y_j(t+1) = C_0 x_0(t)$$

$$x_c(t+1) = A_c x_c(t) + B_c e_j(t)$$
$$u_j^c(t+1) = C_c x_c(t) + D_c e_j(t)$$

with

$$A_0 = \begin{bmatrix} 1 & 1.76\cdot10^{-1} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 9.97\cdot10^{-1} & -6.27\cdot10^{-2} \\ 0 & 0 & 6.27\cdot10^{-2} & 9.97\cdot10^{-1} \end{bmatrix}, B_0 = \begin{bmatrix} -2.24\cdot10^{-6} \\ -4.64\cdot10^{-4} \\ -2.86\cdot10^{-3} \\ 2.78\cdot10^{-3} \end{bmatrix},$$

$$C_0 = \begin{bmatrix} -1.22\cdot10^{-3} & -1.02\cdot10^{-4} & -2.86\cdot10^{-3} & -2.76\cdot10^{-3} \end{bmatrix},$$

$$A_c = \begin{bmatrix} 8.97\cdot10^{-1} & 6.92\cdot10^{-2} & 1.01\cdot10^{-2} \\ -6.92\cdot10^{-2} & 9.5\cdot10^{-1} & -1.88\cdot10^{-2} \\ 1.02\cdot10^{-2} & 1.88\cdot10^{-2} & 9.97\cdot10^{-1} \end{bmatrix}, B_c = \begin{bmatrix} -12.3 \\ -1.07 \\ 8.70\cdot10^{-1} \end{bmatrix},$$

$$C_c = \begin{bmatrix} 12.27 & -1.07 & -8.70\cdot10^{-1} \end{bmatrix}, D_c = 1341.$$

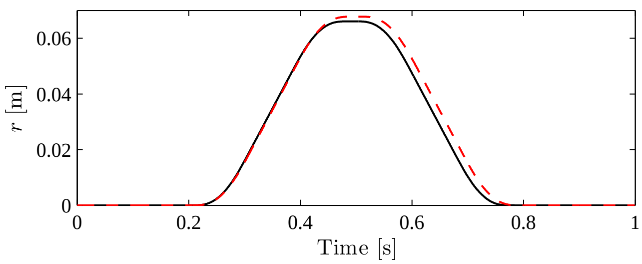

Fig. 2. Refrences r_1 (black solid) and r_2 (red dashed).

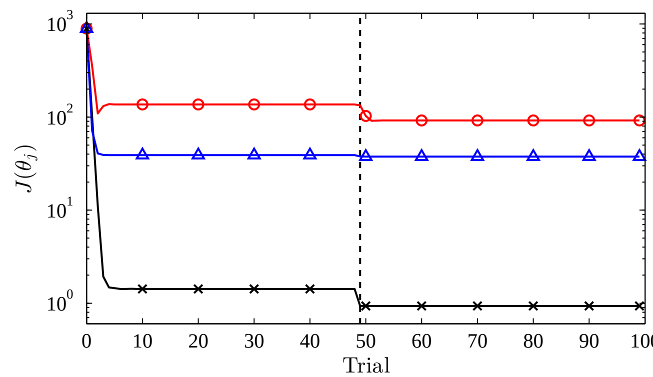

Fig. 3. Cost function $J(\theta_j)$, proposed (×), extended FIR (○), FIR (△), reference change from r_1 to r_2 (black dashed).

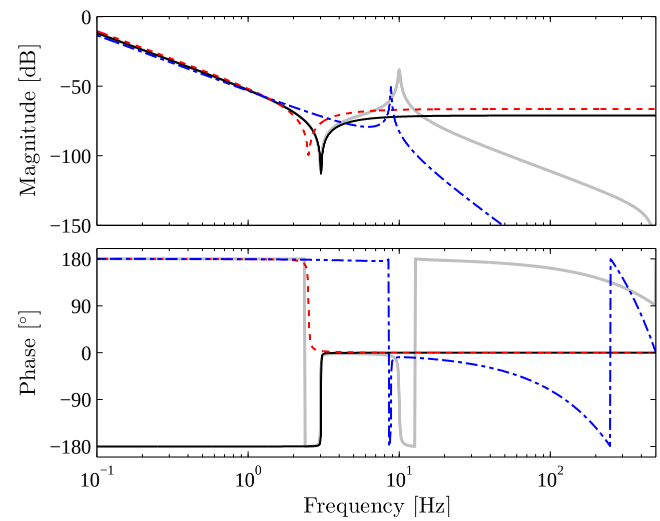

Fig. 4. P_0 and $F(\theta_\infty)^{-1}$, P_0 (grey solid), proposed (black solid), FIR (blue dash-dotted), extended FIR (red dashed).

The bandwidth (lowest frequency where $|CP_0(i\omega)| = 1$) of the feedback control system is approximately 1 Hz and the robustness margin of $\max|S_0| < 6$ dB is satisfied [22]. Given are the following design choices:

- $t_s = 1 \cdot 10^{-3}s$ (sampling time)
- $n = 1000$ samples (trial length)
- $W_e = I \cdot 10^3, W_f = W_{\Delta f} = 0$ (inverse model ILC)
- $P \neq P_0$ (10% modeling errors in mass, stiffness, and damping)
- Let $\xi(z) = 1 - z^{-1}$ be a differentiator, then $\xi_1^A(z) = \xi(z), \xi_2^A(z) = \xi(z)^2, ..., \xi_{m_a}^A = \xi(z)^{m_a}$ and $\xi_1^B(z) = \xi(z), \xi_2^B(z) = \xi(z)^2, ..., \xi_{m_b}^B = \xi(z)^{m_b}$
- $\theta_0 = 0$ (zero initial parameters for the first trial)

Clearly, the optimal solution is given by $F(\theta, z) = P_0^{-1}$ since this choice leads to $J(\theta) = 0$. For the proposed system, this choice corresponds to $m_a = 4$ and $m_b = 3$. However, to enable a fair comparison, also for the proposed rational basis a restricted complexity parameterization is pursued such that bias errors are present. In particular,

- Proposed $m_a = m_b = 2$
- FIR $m_a = 4, m_b = 0$
- Extended FIR $m_a = m_b = 2$

The extrapolation properties of the controllers are tested using references r_1 and r_2, see Fig. 2. Initially reference r_1 is used, and at trial 49 the refrence is changed from r_1 to r_2 without reinitializing θ_j. The corresponding ILC algorithms are invoked, the results are presented in Fig. 3, Fig. 4, and Fig. 5. Since modeling errors are introduced, $P \neq P_0$, it requires a few trials until convergence of the cost function, see Fig. 3. Furthermore, Fig. 3 reveals that the converged results for all parameterizations lead to improved performance compared to feedback only (trial 0), it also shows the extrapolation capabilities since the cost function decreases at the reference change. The FIR structure leads to enhanced performance compared to the extended FIR structure. This is explained by the fact that the a priori unknown weighting function for the extended FIR filter essentially implies that a modified and hence wrong cost function is optimized in the extended FIR case. Finally, the results in Fig. 3 reveal that the proposed rational model parameterization leads to the best performance.

To interpret the converged feed forward, the parameterized feedforward is visualized using a Bode diagram. The results are depicted in Fig. 4, where $F(\theta_\infty)^{-1}$ for the different structures and P_0 are compared. Fig. 4 reveals that for frequencies up to 1 Hz the dynamics of P_0 are captured well by all approaches. The anti-resonance (i.e., complex conjugates zeros) around 3 Hz is only accurately captured by the proposed approach. The FIR structure does not have poles, hence its inverse does not have zeros. The inverse of the extended FIR structure has complex conjugated zeros. However, due to ignored weighting in the cost function the relative error in frequency of this errors is significant. The resonance is partly captured in the FIR structure case, which is enabled by the fact that its inverse has four poles (since $m_a = 4$), of which two are integrators (i.e., poles at 0 Hz). Summarizing, from visual inspection it is concluded that $F(\theta_\infty)^{-1}$ for the proposed approach in this paper has the best correspondence with the system.

The time-domain tracking for trials 48 (before refrence change) and 49 (directly after the reference change) are shown in Fig. 5. These results confirm earlier conclusions, since i) all approaches show extrapolation capabilities, ii) the proposed framework in this paper leads to the best results, and iii) the FIR structure leads to better performance when compared to the extended FIR structure.

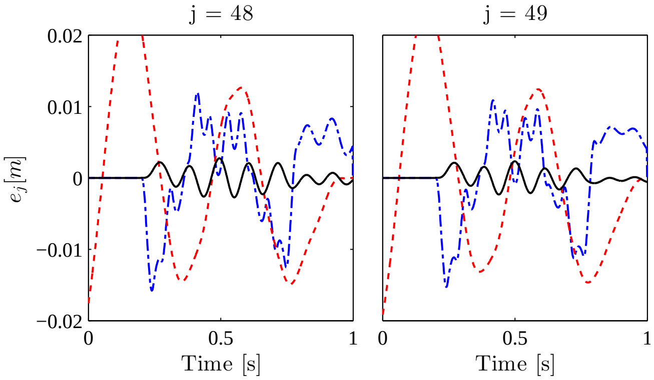

Fig. 5. Tracking errors e_48 (before reference change) and e_49 (after reference change): proposed (black solid), FIR (blue dash-dotted), extended FIR (red dashed).

VI. Conclusion

In this paper, ILC is extended towards rational basis functions. Herein, basis functions are adopted to enhance extrapolability of learned ILC feed forward command signals to other tasks, i.e., reference signals. The motivation for *rational* basis functions is based on the observation that the optimal basis functions should be chosen such that the model inverse is contained inside the parameterized model set. Since the inverse model generally is a general rational model, the proposed framework has the potential to significantly enhance performance and extrapolability compared to pre-existing results. Indeed, in pre-existing approaches, polynomial basis functions are employed that are optimal only for systems that have a unit numerator, i.e., no zeros. Hence, these pre-existing approaches are highly restrictive.

The caveat associated with rational basis functions lies in the synthesis of optimal ILC controllers, since the analytic solution to optimal ILC algorithms is lost in general. In this paper, we have proposed a novel ILC algorithm that has close connections to well-known and powerful iterative solution methods in system identification.

The advantages of using rational basis functions in ILC is confirmed in a relevant simulation study. In addition, the simulation results confirms the efficacy of the proposed iterative algorithm.

Ongoing results focuses on the actual implementation of the norm optimal ILC with rational basis functions in an experimental system. Furthermore, since the proposed approach can potentially cancel sampling zeros, the presented results necessitate a careful analysis of intersample behavior as in [23].

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