Suppressing non-collocated disturbances in inferential motion control:
with application to a wafer stage

Nic Dirkx\textsuperscript{1,2}, Noud Mooren\textsuperscript{2} and Tom Oomen\textsuperscript{2}

Abstract—Structural deformations are a key obstruction to achieve the ever-increasing performance demands in high-precision positioning systems. This applies especially to systems in which the performance variables are not directly measured and the disturbance inputs do not coincide with the actuated inputs. The aim of this paper is to establish a robust observer-based inferential motion control framework, wherein non-collocation of both the performance variables with the measured variables, and the disturbance variables with the actuated variables is addressed explicitly. Experimental results from an industrial wafer stage setup confirm that the developed techniques lead to a significant performance improvement compared to classical approaches.

I. INTRODUCTION

Structural mechanical deformations are a hampering factor in achieving the ever-increasing performance demands in high-precision positioning systems. An example of such system is the wafer stage, used in the production of integrated circuits [1]. To meet the often nanometer-tight performance requirements, traditional control techniques that assume the stage dynamics to be represented as a rigid body [2] are no longer adequate. Control techniques are required that explicitly address the flexible dynamics in their design to counteract the effect of structural deformations [3]. Structural deformations lead to a significant increase in complexity of the control design. This applies in particular to systems where the performance variables cannot directly be measured [4]. So, these must be estimated (inferred) from the measurements using a model that represents the dynamics between the measured and performance variables [5]. Additional complexity arises when the disturbances are not collocated with the control inputs. In the presence of these so-called non-collocated disturbances, also a model of the dynamics between the disturbance and control inputs must be incorporated explicitly in the control design. Evidently, the general inferential control problem inherently requires a model-based design approach that is significantly more involved than the traditional control problem [4].

Addressing robustness is especially crucial in inferential control problems, since the achieved performance relies heavily on inevitably uncertain models. A \(H_{\infty}\)-optimal robust inferential control design framework is presented in [4]. This framework extends upon the optimal robust control methods for the traditional control problem as presented in [2], [6], [7], [8]. A disadvantage of the methods in [4] is that the resulting general two degree-of-freedom (DOF) controller is difficult to interpret since it lacks an intuitive internal structure, which complicates the design in practice. Furthermore, non-collocated disturbances are not addressed in [4], hence the methods are not compatible with the general inferential control problem.

In contrast to the general two-DOF inferential controller structure, the observer-based controller provides a structure wherein the estimation and control problem are transparently separated [9]. Furthermore, this structure explicitly enables the incorporation of model knowledge of unmeasured performance variables and (non-collocated) disturbances in the controller design, e.g., [10], [11], [12]. However, the observer-based controller structure leads to fixed-structure controller synthesis problems that cannot be dealt with by standard synthesis algorithms [13], [14]. A sequential design approach that avoids these issues, and is user-friendly and intuitive, is presented in [15]. However, robust control performance requirements cannot be enforced throughout the design sequence, leading to non-optimal performance. Although significant progress on inferential motion control has been made, at present design methods that i) can deal with non-collocated disturbances in addition to unmeasured performance variables, and ii) lead to a transparent controller structure, and iii) address robustness explicitly and non-conservatively, are not available. The aim of this paper is to establish a transparent observer-based controller design framework for optimal robust inferential control, that can deal with both unmeasured performance variables and non-collocated disturbances.

The main contributions in this paper are:

1. An inferential control-relevant \(H_{\infty}\)-optimal observer design approach that leads a transparent and optimal observer-based inferential control structure.
2. A weighting filter design approach for positioning systems with unmeasurable performance variables and non-collocated input disturbances.
3. An approach for constructing robust-inferential-control-relevant model sets for systems with non-collocated input disturbances.
4. Experimental validation on a wafer stage setup.

Proofs are omitted throughout this paper to conserve space.

Notations: The pair \(\{N,D\}\) is a right coprime factorization (RCF) of \(P\) if \(P = ND^{-1}\) with \(N,D \in RH_{\infty}\), and \(\exists X_{r}, Y_{r} \in RH_{\infty}\) such that \(X_{r}D+Y_{r}N = I\). Left coprime factorizations (LCF) are obtained by dual definitions.
II. PROBLEM FORMULATION

In this section, the inferential control problem is formulated and put in perspective of a wafer stage control problem.

A. Inferential control problem

The linear time invariant system

\[
\begin{bmatrix}
  z_p \\
  y_p 
\end{bmatrix} = P \begin{bmatrix}
  d_p \\
  u_p 
\end{bmatrix}, \quad P = \begin{bmatrix}
  P_{zd} & P_{zu} \\
  P_{yn} & P_{yu} 
\end{bmatrix}
\]  

(1)

is considered, where \( z_p \) denotes the unmeasured performance variables, \( u_p \) the measured variables, \( u_d \) the controlled inputs, and \( d_p \) the unactuated disturbance inputs. The entries of \( P \) have common dynamics and are detectable and stabilizable. The inferential control problem is the minimization of the inferential control error \( e_z = r_z - z_p \), with \( r_z \) a reference signal, by appropriate feedback control design.

B. Wafer stage setup

The inferential control problem in Section II-A is encountered in the control of motion platforms such as the prototype wafer stage shown in Fig. 1, which is considered as the experimental setup in this paper. The stage is designed to exhibit dominant flexible dynamic behavior, envisaging the lightweight designs of future wafer stages. The stage is equipped with four actuators \( u_p \) and three sensors \( y_p \) in the out-of-plane directions, as indicated in Fig. 1. The control goal is to position the point \( z_p \) at the lower right corner, which represents the performance variable. For identification and validation purposes, the stage is equipped with a sensor at this location, which is not used for feedback control. In addition, the stage is equipped with an actuator that is utilized to mimic a non-collocated disturbance \( d_p \). During normal operation, the controller does not have access to this actuator.

C. Robust control problem formulation

The inferential control goal is the minimization of the inferential control error \( e_z = r_z - z_p \) by the design of a feedback controller \( u_p = K(r_z, y_p) \). The notation \( \hat{\cdot} \) will be clarified later.

The optimal inferential controller is defined as follows.

**Definition 1 (Optimal inferential controller):** Given the true system \( P_o \), the optimal inferential controller is

\[
\hat{K}^{opt} = \arg \min_K J(P_o, K),
\]

(2)

where \( J \) is the inferential control objective function.

The true system \( P_o \) is unknown and therefore \( \hat{K}^{opt} \) in (2) cannot be computed in practice. Instead, the computation of a controller relies on a (nominal) model \( \hat{P} \) that reflects the knowledge of \( P_o \). The achievable control performance therefore hinges on the quality of the model. This applies particularly to the inferential control problem, since the performance variables cannot be measured and hence must be inferred from the model. This emphasizes the need to address model uncertainty explicitly, such that the performance of the model-based controller when implemented on the true system \( P_o \) is guaranteed. Hereeto, the nominal model \( \hat{P} \) is extended towards a model set \( \mathcal{P} \) that encompasses \( P_o \),

\[
P_o \in \mathcal{P}(\hat{P}, \Delta),
\]

(3)

where \( \Delta \) represents model uncertainty [17]. This enables the formulation of a worst-case control criterion

\[
J_{WC}(\mathcal{P}, \hat{K}) = \sup_{P \in \mathcal{P}} (P, \hat{K}).
\]

(4)

The optimal robust inferential controller is then expressed in terms of the worst-case model in the set.

**Definition 2 (Optimal robust inferential controller):**

Given the model set \( \mathcal{P}(\hat{P}, \Delta) \), the optimal robust inferential controller is

\[
\hat{K}^{RP} = \arg \min_K J_{WC}(\mathcal{P}, K).
\]

(5)

The following performance is guaranteed when the robust controller \( \hat{K}^{RP} \) is implemented on the true system

\[
J(P_o, \hat{K}^{opt}) \leq J(P_o, \hat{K}^{RP}) \leq J_{WC}(\mathcal{P}, \hat{K}^{RP}).
\]

(6)

Achieving (5) demands a systematic control design framework. The following design requirements are imposed.

**Definition 3 (Design framework requirements):**

R1 Non-collocated disturbances must be optimally attenuated in view of the unmeasured performance variables.

R2 The structure of the controller \( \hat{K} \) must be transparent and intuitive to facilitate design and interpretation.

R3 The model set \( \mathcal{P} \) must non-conservatively encompass systems with unmeasured performance variables and non-collocated disturbances.

In the next section, a design framework is established that addresses R1 and R2.

III. OPTIMAL OBSERVER-BASED INFERENTIAL CONTROL DESIGN FRAMEWORK

This section presents the \( \mathcal{H}_\infty \)-optimal design framework for inferential controllers. The key mechanism to addressing both requirements R1 and R2, is the optimization of an observer-based controller directly in terms of the inferential control criterion, enabling optimal control performance with a transparently structured controller.
A. Controller structure selection

The inferential control problem imposes different requirements on the controller structure than the classical control problem that aims at minimization of the measured error $e_y = r_y - y_p$. In particular, achieving joint disturbance rejection and reference tracking in the inferential context requires a two-DOF controller structure, instead of the classical single-DOF controller structure [16]. The two-DOF controller is defined as $\bar{K} = [\bar{K}_r \ \bar{K}_y]$ such that $u_p = \bar{K}[r_z \ y_p]^T$. Various implementations of two-DOF controllers exist [17, p.147]. In this paper, the observer-based controller form is considered, see the dashed box in Fig. 2. This this form intuitively reflects the intrinsic mechanism of inference and control that underlies the inferential control problem. The two-DOF observer-based controller is governed by

$$
\bar{K}(\bar{O}, K_{fb}) : \quad u = S\bar{O}K_{fb} \begin{bmatrix} I & -\bar{O}_y \end{bmatrix} \begin{bmatrix} r_z \\ y_p \end{bmatrix},
$$

(7)

where $S\bar{O} = (I + K_{fb}O_u)^{-1}$ and where

$$
\bar{O} : \quad \hat{z}_p = \begin{bmatrix} \bar{O}_u & \bar{O}_y \end{bmatrix} \begin{bmatrix} u_p \\ y_p \end{bmatrix},
$$

(8)

is an observer that generates an estimate $\hat{z}_p$ of the performance variable $z_p$. Thus, this controller structure satisfies requirement R2, while the two-DOF structure facilitates R1.

B. Inferential control goal

The design of $\mathcal{H}_\infty$-optimal controllers requires the selection of the generalized inputs and outputs such that the control criterion (2) reflects the control goal. The presented selection is shown in Fig. 2 and is motivated as follows: inputs $r_z, d_p$ are the inputs that must be tracked or suppressed, respectively. Input $d_y$ is included to enable high-frequency controller roll-off, and $d_u$ is included to bound the input sensitivity function $S_i$, i.e., to include robustness margins. Output $e_z$ is the inferential control error and $u_p$ is the control command that should remain bounded. This selection results in the closed-loop TFM $\bar{T}(P, \bar{O}, K_{fb}) : \bar{w} \mapsto \bar{z}$,

$$
\bar{T} = \begin{bmatrix} -P_{zu} \\ I \end{bmatrix} S_i[K_r \ \bar{K}_y -\bar{K}_yP_{yu} I] + \begin{bmatrix} I & 0 & -P_{zd} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},
$$

(9)

where $S_i = (I + \bar{K}_yP_{yu})^{-1}$ and

$$
\bar{w}^T = [r_z^T \ \bar{d}_y^T \ \bar{d}_p^T \ \bar{d}_u^T]^T, \quad \bar{z}^T = [e_z^T \ u_p^T]^T.
$$

The TFM $\bar{T}$ in (9) is an eight-block problem and extends upon the classical four-block problem, e.g., [20].

To specify the performance requirements, the weighting matrices

$$
W = \begin{bmatrix} W_z & 0 \\ 0 & W_u \end{bmatrix}, \quad V = \begin{bmatrix} V_r & 0 & 0 & 0 \\ 0 & V_{dy} & 0 & 0 \\ 0 & 0 & V_{dp} & 0 \\ 0 & 0 & 0 & V_{du} \end{bmatrix},
$$

(10)

are introduced, where $W, V, W^{-1}, V^{-1} \in \mathcal{R}\mathcal{H}_\infty$, such that $w = V\bar{w}$ and $z = W\bar{z}$. The control criterion (2) is then defined as

$$
T(P, \bar{O}, K_{fb}) = W\bar{T}(P, \bar{O}, K_{fb}) V
$$

(11)

and

$$
\mathcal{J}(P, \bar{O}, K_{fb}) = \parallel T(P, \bar{O}, K_{fb}) \parallel_\infty.
$$

(12)

C. Inferential control-relevant observer design

To achieve optimal control performance, it is essential that both the controller and observer are designed such to minimize the criterion (12). Joint synthesis of the two components generally involves solving a non-convex optimization problem, which may prohibit achieving optimal results [14]. Optimality is neither achieved for sequential design approaches [15], e.g., wherein first the observer is designed using the Kalman-Bucy filtering approach [18], followed by a controller synthesis step.

This section presents a method that achieves optimality by synthesizing the observer directly in view of the control criterion (12). This leads to the definition of the inferential-control-relevant (ICR) observer.

Definition 4: For a given controller $K_{fb}$, the ICR observer is computed by

$$
\bar{O}^{ICR}(P, K_{fb}) = \arg \min_{\bar{O}} \mathcal{J}(P, \bar{O}, K_{fb}).
$$

(13)

The achieved performance in terms of (12) is invariant to the choice of $K_{fb}$ under the following conditions.

Theorem 1: The controller $\hat{K}(\bar{O}^{ICR}, K_{fb})$ of the form (7) and with $\bar{O}^{ICR}$ as in Definition 4, minimizes the control criterion (12), i.e.,

$$
\mathcal{J}(P, \bar{O}^{ICR}, K_{fb}) = \min_{\bar{O}, K_{fb}} \mathcal{J}(P, \bar{O}, K_{fb}),
$$

(14)

and additionally $\hat{z}_p$ is bounded, for any biproper and minimum-phase $K_{fb}$.

By virtue of Theorem 1, Definition 4 enables optimal observer-based controller design, while avoiding structured synthesis problems like encountered in [13], [14]. Choosing $K_{fb}$ as a high-gain controller in conjunction with appropriate weighting filters $W, V$, facilitates the interpretation of an interconnection between a high-gain controller and a high-gain observer.

D. Weighting filter design

The design of the weighting matrices (10) deserves special attention, as the eight-block problem (9) imposes more design restrictions than the classical four-block problem. This section presents a simple but effective frequency-domain weighting filter design approach, that enables parametrization of the filters in terms of the target bandwidth.
Definition 5: The bandwidth $f_{bw}$ is the frequency where $\bar{\sigma}(\bar{K}_y P_y u)$ first crosses 1 from above.

The filter design consists of a) a static scaling step and b) a loop shaping step. The filters are composed accordingly, i.e., $W = W^b W^c$, $V = V^c V^b$. For the ease of exposure, the entries of the plant $P$ are assumed SISO in the following.

a) Scaling filter design $W^c, V^c$. The goal of scaling is to normalize the magnitudes of $T$ in (9) at the target bandwidth [2]. This is achieved by choosing

$$W^c = \begin{bmatrix} |P_{yu} f_{bw}^2| \frac{1}{2} \\ P_{yu} f_{bw}^2 \end{bmatrix}, \quad V^c = \begin{bmatrix} |P_{yu} f_{bw}^2| \frac{1}{2} \\ P_{yu} f_{bw}^2 \end{bmatrix} = \begin{bmatrix} |P_{yu} f_{bw}^2| \frac{1}{2} \\ P_{yu} f_{bw}^2 \end{bmatrix}.$$

The resulting system $\bar{T}^c = W^c T V^c$ forms a convenient basis for loop shaping in the next step.

b) Loop shaping filter design $W^b, V^b$. The following objectives are pursued in the loop shaping design:

(i) For reference tracking, a small closed-loop gain must be attained in $T_{11}^b$ below $f_r$, i.e., $T_{11}^b \ll 1 \forall f \in [0, f_r]$.  
(ii) Disturbance suppression requires a small closed-loop gain in $T_{13}^b$ below $f_d$, i.e., $T_{13}^b \ll 1 \forall f \in [0, f_d]$. 
(iii) For controller roll-off, a small closed-loop gain must be attained in $T_{22}^b$ beyond $f_c$: $T_{22}^b \ll 1 \forall f \in [f_c, \infty]$.

Let $H_{LP}(f)$ and $H_{HP}(f)$ be bi-proper first-order normalized low-pass and high-pass filters, respectively, with corner frequency $f_i$. Then, the following weighting functions reflect the objectives (i)-(iii):

$$W^b = \begin{bmatrix} 1 \\ H_{LP}(f_r) \\ H_{HP}(f_c) \\ 1 \\ 1 \\ H_{LP}(f_d) \\ H_{HP}(f_c) \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}, \quad V^b = \begin{bmatrix} V_{sc} \\ V_{ls} \\ V_{sc} \\ V_{ls} \\ V_{sc} \\ V_{ls} \\ V_{sc} \\ V_{ls} \\ V_{sc} \\ V_{ls} \end{bmatrix}.$$

The characteristic frequencies $f_r, f_d, f_c$ are typically expressed as a (fixed) factor or fraction of the bandwidth $f_{bw}$ [2]. As a result, the weighting filter design can be performed by tuning of a single intuitive parameter $f_{bw}$.

IV. ROBUST INFERENTIAL CONTROL-RELEVANT MODEL SETS

The achievable robust control performance in terms of (5) is directly affected by the size of the model set $\mathcal{P}$. This motivates the construction of a robust inferential-control-relevant (RICR) model set, i.e., a set with minimal size in view of the control goal (5). This section presents a method to construct such RICR model sets for systems with unmeasurable performance variables and non-collocated disturbances that satisfies requirement R3.

A. Robust inferential control-relevant model set

The RICR model set is formulated as the dual of the robust controller design problem in Definition 2.

Definition 6: For a given two-DOF initial controller $\bar{K}^{init}$, the RICR model set is defined by

$$\mathcal{P}^{RICR} = \arg \min_P \mathcal{J}_{WC}(P, \bar{K}^{init})$$

subject to $P_o \in \mathcal{P}(\bar{P}, \Delta)$.

(15)

Definition 6 extends the control relevant sets presented in [4], [21], [22] to the general inferential control problem. Note that the RICR model set in (15) is expressed in terms of the initial controller $\bar{K}^{init}$, since the robust controller $\bar{K}^{RP}$ is yet to be designed. It is therefore desired that $\bar{K}^{init}$ resembles the robust optimal controller $\bar{K}^{RP}$. This paper considers the case in which $\bar{K}^{init}$ is of the form (7), but based on an initial observer $\bar{O}^{init}$, which may but needs not be equal to the observer $\bar{O}^{ICR}$. As discussed in, e.g., [21], the model set can be improved by iteratively refining $\bar{K}^{init}$, through alternatingly solving (5) and (15).

Constructing a RICR model set consists in the construction of a nominal model $\bar{P}$ and an uncertainty representation $\Delta$. Throughout, it is assumed that $\bar{P}$ is available, e.g., from first principles modeling or system identification [22]. Constructing the model uncertainty $\Delta$ that satisfies (15) is achieved by developing a particular dual-Youla-Kucera (dYK) parametrization. This is shown in the next section.

B. Towards a RICR model uncertainty structure

Obtaining a model set $\mathcal{P}$ that is RICR in the sense of Definition 6 imposes specific requirements on the model uncertainty structure:

(i) The true plant $P_o$ must be contained in the model set, i.e., constraint (3) must be satisfied.
(ii) The initial controller $\bar{K}^{init}$ must stabilize all candidates $P$ in the model set.
(iii) The initial controller $\bar{K}^{init}$ must achieve high performance for all models in the set.

None of these requirements are automatically satisfied for general uncertainty structures. The following results present a specific model uncertainty structure that achieves (ii).

Theorem 2: Let $\bar{K}^{init}$ internally stabilize $\bar{P}$ and let $\left\{ \begin{bmatrix} N_z \\ N_y \\ D_d \\ D_u \end{bmatrix}, \begin{bmatrix} \bar{D}_d \\ \bar{D}_u \end{bmatrix} \right\}$ and $\{N_z, D_k\}$ be an RCF of $\bar{P}$ and $\bar{K}^{init}$, respectively. Then, all systems $P$ that are internally stabilized by $\bar{K}^{init}$ are given by

$$P = \begin{bmatrix} \bar{N}_z + \Delta_1 \\ \bar{N}_y + D_k \Delta_2 \end{bmatrix} \begin{bmatrix} \bar{D}_d \\ \bar{D}_u - N_k \Delta_2 \end{bmatrix}^{-1},$$

(16)

where $\Delta = \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{bmatrix}$, with dimensions compatible to that of $\bar{P}$, and where $\Delta \in \mathbb{R}^{H\times\infty}$.

Theorem 2 extends the existing dYK parametrization, e.g., [23], to systems with non-collocated disturbances, unmeasured performance variables, and two-DOF controller structures. An important property of the parametrization in (16) is that it does not contain systems that are not stabilized by $\bar{K}^{init}$. This leads to the condition for requirement (i) to hold.

Theorem 3: Constraint (3) is satisfied if and only if $\bar{K}^{init}$ internally stabilizes $P_o$.

The RCFs in Theorem 2 are non-unique. This property is exploited to obtain a particular factorization that achieves requirement (iii), in the next section.
C. RICR coprime factorizations

The tightness of the model set depends on the particular choice of the non-unique coprime factors in Theorem 2. The following result presents a coprime factorization for $\hat{P}$ that leads to a RICR model set in terms of Definition 6.

**Theorem 4:** Let $T(\hat{P}, \hat{K}^{\text{init}}) \in \mathcal{RH}$ and let $\{\tilde{N}_e, \tilde{D}_e\}$ be a LCF of $\begin{bmatrix} \hat{K}^{\text{init}} V_r & \hat{K}^{\text{init}} V_y & \hat{V}_d \end{bmatrix}$, where $\tilde{N}_e = \begin{bmatrix} \tilde{N}_{e,r} & \tilde{N}_{e,y} & \tilde{N}_{e,d} \end{bmatrix}$ and consider

$$
\begin{bmatrix}
\tilde{N}_e \\
\tilde{N}_y \\
\tilde{D}_d \\
\tilde{D}_u
\end{bmatrix} = \begin{bmatrix}
\hat{P} \\
I \\
0 \\
\tilde{D}_e
\end{bmatrix} \begin{bmatrix}
0 \\
0 \\
\tilde{N}_{e,y} V_y^{-1}
\end{bmatrix} \begin{bmatrix}
\hat{P}_{yd} \\
\hat{P}_{yu}
\end{bmatrix}^{-1}.
$$

Then, $\begin{bmatrix} \tilde{N}_e \\
\tilde{N}_y \\
\tilde{D}_d \\
\tilde{D}_u \end{bmatrix}$ is an RCF of $\hat{P}$.

Suppose that a model set $\mathcal{P}^{\text{RICR}}$ is constructed by (16) with a norm-bounded uncertainty $\|\Delta\|_{\infty} \leq \gamma$, selected such that (3) holds, e.g., by a model validation procedure based on measurements of $P_{\text{r}}$ [24]. Then, the factorization in Theorem 4, in combination with a $\{W_u, W_z\}$-normalized RCF [22] of $K^{\text{init}}$ leads to the result

$$
\mathcal{J}_\text{WC}(\mathcal{P}^{\text{RICR}}, \hat{K}^{\text{init}}) \leq \mathcal{J}(\hat{P}, \hat{K}^{\text{init}}) + \gamma. 
$$

The crucial result is twofold: 1) the inequality (17) directly relates the size of the model uncertainty to the worst-case robust control performance, and 2) the upper bound is tight, in contrast to any other choice of coprime factorizations. The result (17) extends the results in [22], [4] to the general inferential control case and meets requirement R3.

V. EXPERIMENTAL VALIDATION

The developed framework is validated experimentally on the wafer stage setup described in Section II-B. The goals of the experimental study are 1) to illustrate the relevance of RICR model sets, and 2) to compare the performance of the inferential controller to a classical controller. The results in this section focus on a SISO controller element for the control of vertical displacements.

A. RICR model set identification

A parametric nominal model $\hat{P}$ is identified from non-parametric Frequency Response Function (FRF) measurements and is depicted in Fig. 3. The model has McMillan degree 22 and describes the rigid-body dynamics and the dominant flexibilities up to 1300Hz. The RICR model set $\mathcal{P}^{\text{RICR}}$ is constructed based on $K^{\text{init}} = \check{K}^{\text{NP}}$, which is presented in the next section. A dynamic uncertainty bound $\gamma$ is selected using the FRF measurements such that (3) is satisfied, leading to a norm-bounded uncertainty block $\Delta$. The $\gamma$-bound attains a maximum value of $\gamma_{\text{max}} = 2.9$. The resulting norm-bounded RICR model set $\mathcal{P}^{\text{RICR}}$ is visualized in Fig. 3 using the techniques in [25]. The model set tightly encloses the first three resonances, which is an indication of their relevance for inferential control.

B. Observer and controller design

The observer $O^{\text{ICR}}(P, K_{fb})$ in (13) is obtained through $\mathcal{H}_\infty$-optimization [17], based on the nominal model $\hat{P}$ and a PID controller $K_{fb}$ that achieves a 40Hz bandwidth. This leads to the nominally optimal two-DOF controller $\check{K}^{\text{NP}} = \check{K}(O^{\text{ICR}}(P, K_{fb}))$ of the form (7). Next, $O^{\text{ICR}}(\mathcal{P}^{\text{RICR}}, K_{fb})$ is computed by minimizing (13) using skewed-$\mu$ synthesis [17], given the model set $\mathcal{P}^{\text{RICR}}$ and $K_{fb}$. This leads to the robust two-DOF controller $\check{K}^{\text{RP}} = \check{K}(O^{\text{ICR}}(\mathcal{P}^{\text{RICR}}, K_{fb}))$. For both controllers, the target bandwidth $f_{\text{bw}}$ used in the weighting filters is 40Hz. For comparison, also a classical single-DOF robust controller $\check{K}^{\text{clas}}$ is synthesized with a 40Hz target bandwidth. The controllers are depicted in Fig. 4. Clearly, the classical controller achieves high gain at low frequencies due to integral action, while the gains of the inferential controllers roll off at a system-specific magnitude. This property is required to eliminate steady-state errors induced by non-collocated disturbances, which can be explained by means of the zero-spillover controller [19], [16].

C. Inferential control performance

a) Cost value: The initial and also nominal controller achieves a nominal performance of $\mathcal{J}(\hat{P}, \hat{K}^{\text{init}}) = 3.0$ and a robust performance of $\mathcal{J}_\text{WC}(\mathcal{P}^{\text{RICR}}, \hat{K}^{\text{init}}) = 5.9$. These values verify that the performance bound (17) is tightly satisfied for the RICR uncertainty description norm-bounded by $\gamma_{\text{max}} = 2.9$. The robust controller achieves $\mathcal{J}_\text{WC}(\mathcal{P}^{\text{RICR}}, \check{K}^{\text{RP}}) = 3.9$, which is close to the nominal performance, verifying that the model set $\mathcal{P}^{\text{RICR}}$ is RICR.
The inferential control design framework presented in this paper enables improving the performance of high-precision motion systems beyond classical control approaches. This is achieved by explicitly distinguishing between performance variables and measured variables, and between actuated inputs and disturbances in the controller design. In addition, an approach for model set construction is presented that enables dealing with model uncertainty in a non-conservative way to guarantee robust control performance. This is confirmed by an experimental validation on a wafer stage setup, which show a significant performance enhancement using the presented techniques, compared to classical approaches.

REFERENCES