Optimal Experiment Design for a Wafer Stage: 
A Sequential Relaxation Approach

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1 Background
Ever-increasing performance requirements for high-precision positioning systems such as wafer scanners requires the availability of accurate models [1]. Frequency Response Function (FRF) identification from experimental data is considered a fast, inexpensive, and accurate method to obtain such models [2].

2 Problem
The choice of input signal determines the quality of the identified FRF, which motivates the employment of Optimal Experiment Design (OED) strategies. OED involves the optimization of the inputs to maximize the model accuracy within the available resources [3]. For Multiple Inputs Multiple Outputs (MIMO) systems, OED does not only require appropriate design of the magnitudes of the input signals, but also their directions. Furthermore, the design must address the specific constraints that are relevant for the considered system. For wafer stages, these constraints are typically related to specific elements of the systems, e.g., a power limitation for a specific actuator or a limitation of a local displacement.

The aim of this research is to develop OED methods that explicitly address directionality in excitation design within element-wise power constraints.

3 Approach
The OED problem is formulated as the constrained minimization of a cost function \(f(\Phi_w)\) related to the FRF accuracy over the input spectrum \(\Phi_w\):

\[
\begin{align*}
\text{minimize} & \quad f(\Phi_w) \\
\text{subject to} & \quad g(\Phi_w) \leq 0 \\
& \quad \text{rank}(\Phi_w) = 1.
\end{align*}
\]

Here, the combination of \(g()\) and the rank constraint function forms the set of element-wise power constraints. Due to the rank constraint, problem (1) is non-convex and hard to solve in most cases.

A Sequential Relaxation (SR) algorithm is proposed that replaces the rank constraint in (1) by an approximate convex function, and then solves the resulting sequence of convex optimization problems. The solution of the SR algorithm is guaranteed to converge to a local minimum of the original non-convex problem (1).

4 Results
The developed method is experimentally tested for the identification of the 4×4 dynamics of a wafer stage, see Fig. 1. The system is subject to element-wise output power constraints. Three different excitation designs are compared:

R1) Preliminary design, using a uniform excitation power distribution.

R2) Traditional optimized single input design approach with optimized excitation magnitude, but non-optimized directions.

R3) Optimal multivariable design using the SR algorithm, with optimized magnitudes and directions.

Fig. 1 shows entry (1,1) of the resulting FRFs, including 95% confidence regions (shades) and standard deviations (dashed). The multivariable design (R3) achieves a factor 2 lower standard deviation compared to the traditional approach (R2) by exploiting directionality in the design.

5 Ongoing work
Ongoing research includes development of methods for:

- dealing with peak amplitude constraints
- standard plant identification, including identification of unmeasurable performance variables.

References

