

# Improved Local Rational Method by incorporating system knowledge: with application to mechanical and thermal dynamical systems.

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**Abstract:** A key step in experimental modeling of mechatronic systems is Frequency Response Function (FRF) identification. Applying these techniques to systems where measurement time is limited leads to a situation where the accuracy is deteriorated by transient dynamics. The aim of this paper is to develop a local parametric modeling technique that improves the identification accuracy of a range of systems by exploiting prior knowledge. The method is to impose a prior on the approximate locations of the system poles. This leads to better fit results and enables an accurate variance characterization. As a special case, traditional LPM is recovered.

*Keywords:* Frequency Response Function, Local Polynomial Method, Orthonormal Basis Functions, Transient Reduction

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## 1. INTRODUCTION

Frequency Response Function (FRF) identification is fast, inexpensive and accurate, and often used in applications. These functions are used either directly, e.g., for controller tuning (Karimi and Zhu, 2014) or stability analysis (Evers et al., 2017), or as a basis for parametric identification (Pintelon and Schoukens, 2012). Examples include mechatronics and flexible dynamics in Geerardyn et al. (2015); Voorhoeve et al. (2015), thermal systems in Guo (2014); Monteyne et al. (2013) and electrical systems in Relan et al. (2017) and combustion systems in van Keulen et al. (2017).

Identification of FRFs has been substantially advanced over recent years, particularly by explicitly addressing transients errors. Indeed, one of the underlying assumptions is that the system is in steady state, which is often not valid for experimental systems. The Local Polynomial Method (LPM) (Pintelon et al., 2010) exploits the assumed smoothness of the transient response and approximates locally the transfer function by a polynomial such that the transient can be estimated and removed. A generalization of this method (McKelvey and Guérin, 2012) is the Local Rational Method (LRM) which uses a rational function in the local approximation. As a consequence, the local estimation problem is no longer linear in the parameters which poses additional challenges compared to the LPM method. The LRM has proven to be successful in attaining improved estimation quality (Geerardyn and

Oomen, 2017) and extensions include LPV-LRM (van der Maas et al., 2015). A different related approach is The Transient Impulse response Modelling Method (TRIMM) (Hägg and Hjalmarsson, 2012; Gevers et al., 2016) that uses a global impulse response model of the transient component, but estimates its coefficients by assembling a large set of local windows to construct an over-determined set of equations.

Analysis of different papers has revealed that the LRM often leads to improved results. On the one hand LPM is recovered as a special case, and the additional freedom allows to capture dynamics, especially lightly damped, more accurately, see for a theoretical analysis Verbeke and Schoukens (2017); Schoukens et al. (2013) and Geerardyn et al. (2015) for experimental evidence. On the other hand, the rational parametrization leads to a non-convex optimization problem, which is approximated in typical LRM approaches as in Levy (1959). Indeed, at present solving the non-convex optimization through an iterative algorithm has not yet lead to acceptable results (Geerardyn et al., 2015). Furthermore, as a direct consequence, the variance results, which are valid for LPM, are only accurate for the LRM for sufficiently high Signal to Noise Ratio (SNR) due to the inclusion of the measured data in the least squares regressor matrix.

Important progress in FRF identification has been made by eliminating transients, at present rational parametrizations often lead to highly accurate FRFs yet suffer from inaccurate variance results and either computationally demanding optimizations or an approximation. The aim of the present paper is to investigate alternative parametrizations, which are also recovered as a special case of the LRM, yet are linear in the parameters while exploiting the advantages of rationally parametrized model structures. The main idea is to exploit prior knowledge, since often an

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initial guess can be made where the system poles lie, e.g., damping ratio in mechanical systems (Geerardyn et al., 2015), or poles on the real axis (Relan et al., 2017; Guo, 2014). Indeed, in this respect in the LPM the poles are tacitly selected at 0, which is often arbitrary. The current paper selects these poles at a different location, making use of the freedom in the parameterization. The new approach is again linear in the parameters and therefore maintains the associated benefits of an unbiased estimator and possible variance analysis. This is achieved by utilizing orthonormal rational basis functions, which are well studied (de Vries and Van den Hof, 1998; Ninness and Gustafsson, 1997; Heuberger et al., 2005), to form the basis for the local regression problem. By forming the local parametrization as a linear combination of rational basis functions it is shown that additional benefits can be achieved over the polynomial approach.

The main novel contributions of this paper are:

- C1 An approach for LRM that uses a linearly parametrized basis, leading to an efficient and exact optimization in addition to bias and variance analyses.
- C2 Exploitation of a single complex pole Orthonormal Basis Function (OBF).
- C3 Application of the method on relevant systems, e.g., demonstrating resonant dynamics and a first order thermal example.

The paper is constructed as follows: First, an overview of the local parametric methods is given, where a comparison is made between the LPM and LRM and highlighting possible challenges using the rational parametrization. Second, the new approach is introduced together with an exposition of the rational orthonormal basis functions that are used. Third, the method is applied to two example systems, involving 1) lightly damped resonant dynamics and 2) first order thermal dynamics. And finally, a conclusion based on the simulation results is presented.

## 2. LOCAL PARAMETRIC APPROACH AND PROBLEM FORMULATION

This section presents a concise overview of FRF developments, followed by the problem formulation. The response  $y(n)$  of a discrete Linear Time Invariant (LTI) system to an input  $u(n)$  can be represented as

$$y(n) = \sum_{k=-\infty}^{\infty} u(k)g(n-k) + v(n) \quad (1)$$

where  $g(n)$  is the impulse response of the LTI system and  $v(n)$  the noise contribution. By then applying a Discrete Fourier Transform (DFT)

$$X(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n)e^{-\frac{i2\pi kn}{N}} \quad (2)$$

on a finite interval of (1) the convolution can be represented as a multiplication

$$Y(k) = G(e^{i\omega_k})U(k) + T(e^{i\omega_k}) + V(k) \quad (3)$$

in the Fourier domain, where  $G(e^{i\omega_k})$  is the frequency response function of the dynamic system,  $Y(k)$ ,  $U(k)$ ,  $V(k)$  are the DFT of  $y(n)$ ,  $u(n)$ ,  $v(n)$  and  $k$  denotes the  $k$ -th frequency bin. Where  $T(e^{i\omega_k})$  accounts for the transients of both the system response  $T_G(e^{i\omega_k})$  and the noise  $T_V(e^{i\omega_k})$ .

These transients are caused by the transition of an infinite to a finite interval. The additional term account for this “leakage” such that (3) is again an exact relation.

### 2.1 Local Polynomial Method

The fundamental concept underlying both the LPM and LRM is the smoothness of both  $G(e^{i\omega_k})$  and  $T(e^{i\omega_k})$  in (3) such that the transient can be eliminated if both terms are explicitly estimated. Since both functions are assumed to be smooth, see Pintelon and Schoukens (2012), they are approximated in a local window  $[k-n : k+n]$  by a Taylor series expansion of degree  $R$ , i.e.,

$$G(e^{i\omega_{k+r}}) \approx G(e^{i\omega_k}) + \sum_{s=1}^R g_s(k)r^s \quad (4)$$

$$T(e^{i\omega_{k+r}}) \approx T(e^{i\omega_k}) + \sum_{s=1}^R t_s(k)r^s. \quad (5)$$

For this the input  $U(k)$  is assumed to vary significantly wild over the full input spectrum such that  $G(e^{i\omega_k})U(k)$  can be distinguished from  $T(e^{i\omega_k})$ , this constraint can be fulfilled by either broadband noise excitation or random phase multisines (Pintelon and Schoukens, 2012). The parameters of the expansion consisting of the elements in  $g_s$  and  $t_s$  are contained in  $\theta$  and can be estimated in a linear Least Squares (LS) problem

$$\hat{\theta} := \min_{\theta} \sum_{r=-n}^n |Y(k+r) - G(e^{i\omega_{k+r}})U(k+r) - T(e^{i\omega_{k+r}})|^2 \quad (6)$$

over the local frequency window  $[k-n : k+n]$ . Here, care has to be taken such that the Degree Of Freedom (DOF) of the LS problem in (6)

$$q = 2n - 2R - 1 \quad (7)$$

remains greater than 0. Moreover, since the LS in (6) is linear in the parameters it is computationally efficient and allows for a variance and bias analysis, see, e.g., Geerardyn (2016).

### 2.2 Local Rational Method

The LRM is developed in McKelvey and Guérin (2012) and is a generalization of the LPM. It uses rational functions to approximate the terms  $G(e^{i\omega_k})$  and  $T(e^{i\omega_k})$  in (3). Such that in the local window

$$Y(k+r) = \frac{N_{k+r}}{D_{k+r}}U(k+r) + \frac{M_{k+r}}{D_{k+r}} + V(k+r) \quad (8)$$

where

$$N_{k+r} = \sum_{s=0}^{N_n} n_s(k)r^s \quad (9)$$

$$M_{k+r} = \sum_{s=0}^{N_m} m_s(k)r^s \quad (10)$$

$$D_{k+r} = 1 + \sum_{s=1}^{N_d} d_s(k)r^s \quad (11)$$

and  $N_n$ ,  $N_m$ ,  $N_d$  denote the order of the plant and transient numerator and common denominator respectively. Collect-

ing again the expansion parameters in a vector  $\theta$  one aims to solve the following LS problem

$$\hat{\theta} := \min_{\theta} \sum_{r=-n}^n |Y(k+r) - \frac{N_{k+r}}{D_{k+r}}U(k+r) - \frac{M_{k+r}}{D_{k+r}}|^2. \quad (12)$$

Here the LPM is recovered as a special case by selecting  $N_d = 0$ , i.e.,  $D = 1$ . However, clearly (12) is Non-Linear (NL) in the parameters, which complicates the computation of the optimizer. At least two approaches have been persued to achieve this: (Geerardyn, 2016; Verbeke and Schoukens, 2017) 1) iteratively solving the LS problem or 2) multiplying the problem with  $D_{k+r}$ , similar to Levy (1959), to change the cost criterion of the problem. It has been shown that iteratively solving the NL problem, besides being computationally demanding, is not guaranteed to lead to satisfactory results (Geerardyn, 2016). The second approach, re-parameterizing (12) by multiplying with  $D_{k+r}$  leads to a biased estimator and variance analyses are only valid for sufficiently high SNR.

### 2.3 Problem formulation

Although the LRM offers a richer parameterization and the method has been successfully applied in a large number of applications, at present challenges remain. The aim of this paper is to investigate a different parameterization that enables the added estimation benefits of the rational functions and the bias and variance analyses as offered by the LPM. This is done by exploiting the freedom in linearly parameterized parameterizations (Voorhoeve et al., 2015) by selecting a different basis than the monomial basis used in the LPM but such that the LPM can be recovered as a special case.

## 3. LOCAL RATIONAL METHOD: PRE-SPECIFIED POLES

It is shown that the richer rational parameterization of the LRM is used to achieve superior estimation quality (Geerardyn and Oomen, 2017). The following section aims to combine the merits of the LPM and the LRM by taking a different approach than the linearization (Levy, 1959) of the non-convex problem (12) by pre-specifying the system poles.

Consider again a local window around a DFT bin  $k$  such that locally the terms in (3) can be approximated by

$$\begin{aligned} G(e^{i\omega_{k+r}}) &= \sum_{b=1}^{N_b} \theta_{G_b} B_b(e^{i\omega_{k+r}}) \\ T(e^{i\omega_{k+r}}) &= \sum_{b=1}^{N_b} \theta_{T_b} B_b(e^{i\omega_{k+r}}) \end{aligned} \quad (13)$$

where  $G(e^{i\omega_{k+r}})$  and  $T(e^{i\omega_{k+r}})$  are now linearly parameterized by a set of basis functions  $B_b(e^{i\omega_{k+r}})$  and their parameters  $\theta_{G_b}, \theta_{T_b}$ . Recognize that the LPM is recovered as a special case when  $B_b(e^{i\omega_{k+r}})$  in (13) is chosen to be a monomial basis and pre-specifying all poles to be at 0. The parameterization in (13) potentially allows for the improved estimation performance of the LRM. Clearly, if the basis functions  $B_b$  contain the true system dynamics of  $G(w_{k+r})$  and  $T(e^{i\omega_{k+r}})$  then the basis in (13) can approximate the system in the local window arbitrarily well.

Different approaches are possible, e.g., adapting poles to the local window, uniform distribution of poles over the complex plane or using global basis functions with prescribed poles. For clarity and brevity of the exposition the latter is used throughout, the main idea is that by exploiting the linear parameterization in (13) additional freedom is gained in selecting the bases.

Generally, the exact pole locations of the true systems are unknown, however, a certain degree of prior-knowledge is expected. Consider, e.g., the damping ratio and frequency range of flexible dynamics, or the location on the real-axis of first order poles in thermal systems (Monteyne et al., 2013). The main goal of the current work is to incorporate this prior knowledge into a parametrization used in (13).

### 3.1 Orthonormal basis functions

Encoding prior knowledge into the selected rational parameterization can be directly done by using rational Orthonormal Basis Functions (OBFs). These basis functions have been extensively studied and are often employed in system identification applications (de Vries and Van den Hof, 1998; Heuberger et al., 2005; van Herpen et al., 2016) and control applications (Blanken et al., 2017). Moreover, they often lead to a computational (Ninness et al., 2000) advantageous parameterization for the local estimation problem due to their orthogonality.

Discrete time OBFs can be generated by a series connection of all-pass elements. A general form (Ninness and Gustafsson, 1997) is given by

$$B_b(z) = \left( \frac{z\sqrt{1-|\zeta_n|^2}}{z-\zeta_n} \right) \prod_{k=0}^{n-1} \left( \frac{1-\bar{\zeta}_k z}{z-\zeta_k} \right) \quad (14)$$

where  $\zeta = \{\zeta_0, \zeta_1, \dots, \zeta_p\}$  are the pre-specified poles  $p$  for the all pass functions. The parameterization in (14) is known as the Takenaka-Malmquist (Takenaka, 1925; Ninness and Gustafsson, 1997) functions and are incorporated in (13) to form a linear combination of rational functions. Moreover, prior knowledge can be straightforwardly incorporated by selection of the poles  $\zeta$ . The generalization simplifies to the well-known Laguerre functions by taking  $\zeta_k = \zeta \in \mathbb{R}$  such that the rational functions are of first order with real-valued poles and impulse responses. Modeling second order dynamics while maintaining a real-valued impulse response is done by selecting  $\zeta_{k+1} = \bar{\zeta}_k \in \mathbb{C}$  such that all complex poles appear in complex conjugate pairs, resulting in the Kautz basis functions. Finally, taking  $\zeta = 0$  results in the FIR-basis (Ljung, 1999). The basis formed by (13) using (14) is complete (Ninness and Gustafsson, 1997), i.e., able to model the (local) linear dynamics arbitrarily well with respect to a certain norm  $\mathcal{H}_p$  with  $1 < p < \infty$ , if and only if

$$\prod_{k=0}^{\infty} (1 - |\zeta_k|) = \infty. \quad (15)$$

### 3.2 Local Rational Parameterization

In literature (Ninness et al., 2000; Heuberger et al., 2005) the poles of the OBFs are often chosen as either real

(Laguerre) or complex conjugate pairs (Kautz), such that the impulse response of the basis functions are real valued. However, for the local modeling in (13) no such property is required. Since a direct local approximation in the frequency domain is constructed, single complex poles can be allowed. Indeed such model structures with complex parameters are standard in the LPM/LRM. This allows for the parameterization of resonant behavior with fewer parameters than using a Kautz basis function. The basis functions are implemented as a state-space realization. The complex basis functions are generated by a sequence of inner functions  $G_{b,k}(z)$

$$G_{b,k}(z) = \frac{1 - \bar{\zeta}_k z}{z - \zeta_k} \quad (16)$$

who's realization in state space form is given by

$$(A_{b,k}, B_{b,k}, C_{b,k}, D_{b,k}) = (\zeta, \sqrt{1 - \zeta^2}, \sqrt{1 - \zeta^2}, -\zeta) \quad (17)$$

where  $\zeta \in \mathbb{C}$  is the complex pole prescribed for each of the inner functions. Such that an orthonormal basis is formed by multiplying the vector functions

$$V_k(z) = \psi_k(z) \prod_{j=1}^{k-1} G_{b,j}(z) \quad (18)$$

where  $\psi_k(z) = (zI - A_{b,k})^{-1} B_{b,k}$  with an Euclidean basis vector  $e_i \in \mathbb{R}^{n_{b,k}}$  such that

$$\phi_{k,i}(z) = e_i^T V_k(z), \quad 1 \leq i \leq n_{b,k} \quad (19)$$

form an orthonormal basis. For a detailed exposition and proof of the state-space realization of orthonormal basis functions see, e.g., de Hoog (2001).

#### 4. PRIOR KNOWLEDGE BY PRE-SCRIBING POLES

The proposed method, denoted as Local Rational Method with Pre-scribed poles (LRMP), of encoding prior-knowledge is applied to two example systems to illustrate the potential estimation benefit of the investigated rational parameterization. The first system is described by

$$G_0(s) = \frac{\omega_1^2}{s^2 + 2\xi\omega_1 s + \omega_1^2} + \frac{\omega_2^2}{s^2 + 2\xi\omega_2 s + \omega_2^2} \quad (20)$$

in continuous time, where  $\omega_1 = 5$ ,  $\omega_2 = 3\omega_1$  and  $\xi = 0.1$ , see Monteyne et al. (2013). The system is discretized using a sample time of  $T_s = 0.1$  [s]. The discrete system then has two sets of complex conjugated poles at  $z_1 = 0.8359 \pm 0.4540i$ ,  $z_2 = 0.0673 \pm \pm 0.8581i$ .

##### 4.1 Resonant system

The method described in Sec. 3 is applied to the example system, and a comparison is made for different sets of pre-scribed poles. In the implementation use is made of the robust LPM algorithm that uses periodic excitation and solely estimates  $T(e^{i\omega_k r})$  in (13), see, e.g., Monteyne et al. (2013); Pintelon and Schoukens (2012). The results are then compared to results obtained by using the LPM with a monomial basis and the classical Empirical Transfer Function Estimate (ETF), see, e.g., Ljung (1999). The excitation signal is 2 periods ( $P = 2$ ) of a full random phase multisine at 10 [Hz] with a period length of 102.4 [s] resulting in  $N = 1024$  samples per period. The simulations are performed without additional noise contributions, to facilitate a comparison between the methods. The size

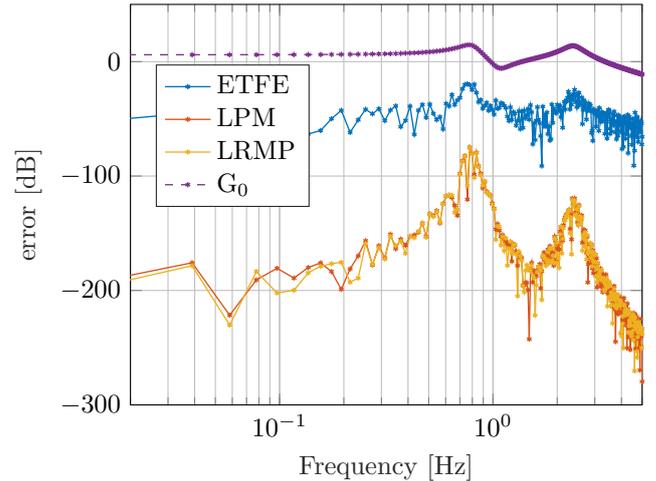


Fig. 1. Estimation error of the true plant  $G_0$ . Comparing the proposed method (LRMP) using  $\zeta = [0, 0, 0]$ , the Local Polynomial Method (LPM) and the Empirical Transfer Function Estimate (ETF). It is shown that the LRMP and LPM provide similar results, both significantly superior to the ETFE.

of the local window is  $n = 2$  and the order of the local polynomial approximation is  $R = 2$  such that the degree of freedom  $q$  in the least-squares equal to  $q = 1$ , since only  $T(e^{i\omega_k})$  is estimated. To facilitate a direct comparison the degree of freedom in the LS of the LRMP method as described in Sec. 3 is also kept at  $q = 1$ , i.e.  $N_b = 2n - q = 3$ . All comparisons are constructed by averaging the estimation error results per frequency of 100 Monte-Carlo simulations.

*No prior knowledge* Initially, no prior knowledge is assumed to be available, i.e., the poles of the OBF are taken as  $\zeta = [0, 0, 0]$  as in the LPM case. Fig. 1 shows the result of the simulated estimation using the ETFE, LPM and LRMP methods. In the noiseless situation, the ETFE will converge to  $G_0$  if  $N \rightarrow \infty$  (Pintelon and Schoukens, 2012). However, since  $N = 2048$  the remaining transient component is clearly dominating the estimation error. It can be seen that both the LPM and LRMP methods offer significantly superior results when compared to the ETFE by explicitly estimating and removing the transient component. Moreover, in this case the result of the LPM method is recovered by the LRMP method as no additional benefit is gained from the rational parameterization due to the chosen pole locations.

*Kautz basis* In the second case, some prior knowledge is assumed by constructing the basis of the OBFs using 2nd order Kautz functions. Since  $N_b$  is limited to 3 only one set of complex conjugated poles can be taken into account, e.g.,  $\zeta = [0.0673 + 0.8581i, 0.0673 - 0.8581i, 0]$  such that the second resonance mode is captured and a pole at 0 is added to complete the basis. Results in Fig. 2 show indeed an improved estimation around the second resonance mode, while around the first resonance mode no improvement is achieved.

*Single complex poles* The previous simulation showed that due to the limited size of the basis the first resonance

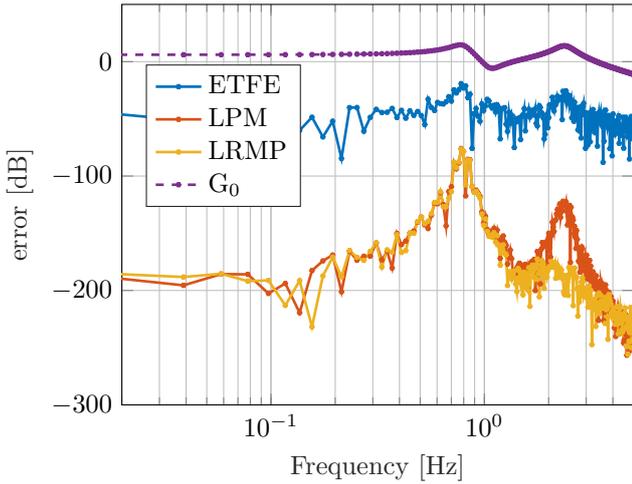


Fig. 2. Estimation error of the true plant  $G_0$ . Comparing the proposed method (LRMP) using a Kautz basis with  $\zeta = [0.0673 + 0.8581i, 0.0673 - 0.8581i, 0]$ , the Local Polynomial Method (LPM) and the Empirical Transfer Function Estimate (ETFE). It is shown that the LRMP provides superior estimation for the first resonance as it was included as prior knowledge.

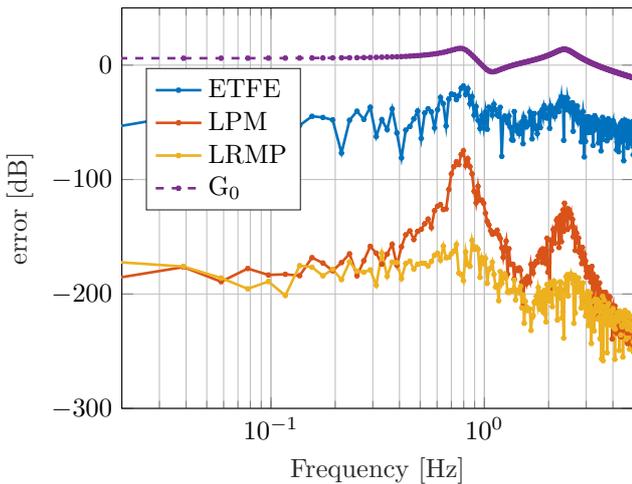


Fig. 3. Estimation error of the true plant  $G_0$ . Comparing the proposed method (LRMP) using a single complex pole basis with  $\zeta = [0.8359 + 0.4540i, 0.0673 + 0.8581i, 0]$ , the Local Polynomial Method (LPM) and the Empirical Transfer Function Estimate (ETFE). It is shown that the LRMP provides superior estimation for both resonance, even though the parameters are limited to  $N_b = 3$ .

mode was not captured. As described in Sec. 3 single complex poles are admissible for the proposed method, as apposed to the approach in most system identification literature where they are often avoided. Therefore, a basis is composed of single complex poles, e.g.,  $\zeta = [0.8359 + 0.4540i, 0.0673 + 0.8581i, 0]$  to capture both resonance modes with limited parameters. The results in Fig. 3 now show an improved estimation accuracy for both resonance modes, as opposed to the Kautz basis that only showed improvements for the second mode.

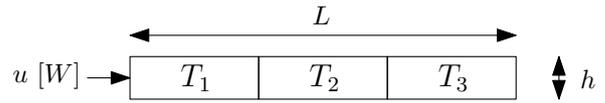


Fig. 4. Spatially discretized 1D heat equation, the relevant transfer is  $G_0 = \frac{T_3}{u}$ .

#### 4.2 Thermal system

The second example system is a 1D PDE heat equation representation of an aluminum rod of length  $L = 0.1[m]$  and height  $h = 0.01[m]$ , spatially discretized using a central difference method into 3 sections, see Fig. 4. The relevant transfer function is that between the input  $u [W]$  located at the first node and the temperature of the third node  $T_3$ . Resulting in a system with 3 first-order low-frequency poles described in the  $z$  domain by

$$G_0(z) = \frac{4.5 \cdot 10^{-9} z^2 + 1.773 \cdot 10^{-8} + 4.42 \cdot 10^{-9}}{z^3 - 2.985 z^2 + 2.97 z - 0.9851} \quad (21)$$

such that the poles are  $\zeta(G_0) = [0.988, 0.996, 0.999]$  which are all close to the unit circle.

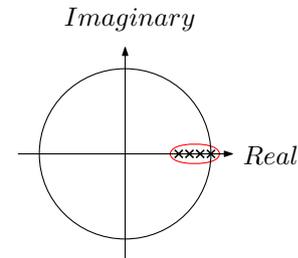
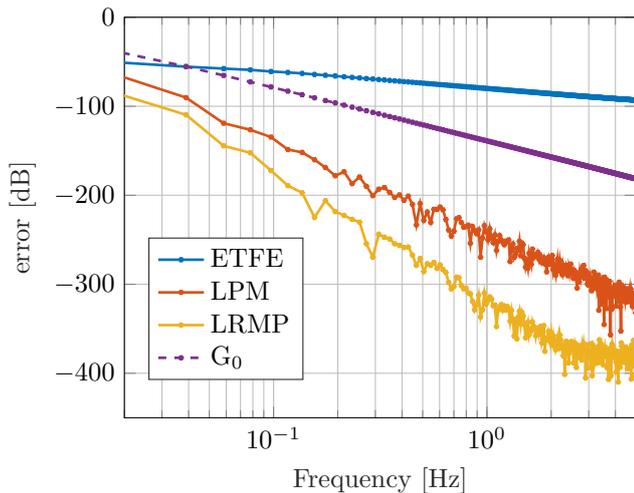


Fig. 5. Complex plane with the assumed region of  $\zeta(G_0)$ , since it can be considered reasonable prior knowledge when considering thermal systems.

*Poles on real axis* Possible prior information regarding the thermal system (21) is the location of the poles on the real-axis. Moreover, the thermal dynamics are dominated by low-frequent content such that a region indicated in Fig. 5 can be considered as reasonable assumption. Therefore, the OBFs are formed using poles distributed over a range of  $[10^{-3} : 10^{-2}]$  [Hz]. The results from the thermal system estimation is shown in Fig. 4.2.1, where simulation parameters are used as described in Sec. 4.1. It shows that the transients dominate the estimation error since the ETFE estimate even exceeds the plant  $G_0$  in gain level at higher frequencies. It also shows that both the LPM and LRMP methods successfully reduce the transient in the data and provide a more accurate estimation. Moreover, it shows that the pole locations used in the OBFs is a reasonable prior and results in less estimation error when compared to the basis used in LPM.

## 5. CONCLUSION

A different parameterization of the LRM is proposed using rational orthonormal basis functions. The parameterization directly facilitates the inclusion of prior system knowledge into the estimation problem. By explicitly estimating and removing the transients from the measurement data, the proposed method achieves an improved estimation of the frequency response function.



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