Fast and Accurate Identification of Thermal Dynamics for Precision Motion Control: Exploiting Transient Data and Additional Disturbance Inputs

Enzo Evers\textsuperscript{a,1}, Niels van Tuijl\textsuperscript{a}, Ronald Lamers\textsuperscript{b}, Bram de Jager\textsuperscript{a}, Tom Oomen\textsuperscript{a}

\textsuperscript{a}Control System Technology group of the Department of Mechanical Engineering of the Eindhoven University of Technology, Eindhoven, PO Box 513, 5600 MB, The Netherlands.
\textsuperscript{b}Thermo Fisher Scientific, Eindhoven, The Netherlands.

Abstract
Thermally induced deformations are becoming increasingly important for the control performance of precision motion systems. The aim of this paper is to identify the underlying thermal dynamics in view of precision motion control. Identifying thermal systems is challenging due to strong transients, large time constants, excitation signal limitations, large environmental disturbances, and temperature dependent behavior. An approach for non-parametric identification is developed that is particularly suitable for thermal aspects in mechatronic systems. In particular 1) reduced experiment time is achieved by utilizing transient data in the identification procedure. 2) an approach is presented that exploits measured ambient air temperature fluctuations as additional inputs to the identification setup. 3) the non-parametric model, obtained through 1) and 2), is used as a basis for parameter estimation of a grey-box parametric model. The presented methods form a complete framework that facilitates the implementation of advanced control techniques and error compensation strategies by providing high-fidelity models, enabling increased accuracy and throughput in high precision motion control.

Keywords: Identification, Frequency Response Function, Thermo-mechanical, modeling

1. Introduction
Impressive progress in advanced motion control of precision mechatronics has led to a situation where thermally-induced deformations are a major error source\cite{1} in positioning accuracy and are no longer negligible on the overall system performance. These deformations are typically induced by heat dissipation from actuators and encoders or by environmental temperature fluctuations. Modern precision motion systems are capable of achieving positioning accuracy in the nano-meter range. These precise movements are essential in several industrial applications, e.g., the manipulation of the sample in an electron microscope and the manufacturing of integrated circuits.

To meet the ever increasing demands to enhance the throughput and positioning accuracy, thermal deformations must be analyzed and compensated for through advanced control approaches utilizing an appropriate thermo-mechanical model.

Ideally, using a limited amount of temperature measurements combined with an accurate thermo-mechanical model enables the use of advanced control and error-compensation techniques\cite{2,3}. An application of error-compensation is illustrated in\cite{4} where a thermo-mechanical actuator is employed to complement the closed-loop performance of a hard disk drive positioning system. Here, the small thermal mass of the hard disk drive magnetic head allowed the thermal actuator to obtain an extremely fast response rate. Unfortunately, the thermal mass of the systems considered in this paper, e.g., electron microscopes, are significantly larger, thereby drastically increasing the time constant of the thermo-mechanical response.

Earlier solutions to compensate for the deformations in electron microscopes, e.g., image-based
feedback in [5], can not always cope with large de-
formations and strongly depend on model quality [6, 7]. Therefore, an accurate parametric model is
desired for a model-based control approach. Accu-
rate modeling of precision thermo-mechanical sys-
tems is complex, resulting in large scale finite ele-
ment models that require significant effort to con-
struct due to, e.g., uncertainty in the parameters and contact resistances.

In sharp contrast, modeling for advanced motion
control, see, e.g., [1, 8] is fast, accurate and in-
expensive. Significant progress has been made in
Frequency Response Function (FRF) identification,
particular in addressing identification in transient
conditions. These recent developments include the
local parametric methods, see [9] for initial results
in this direction. The Local Polynomial Method
(LPM) [10] exploits the local smoothness of the
transient term that otherwise would cause a bias.
Both the transient contribution and system dynam-
ics are modeled with a polynomial in a small fre-
quency window. The local rational method (LRM)
[11], is an extension of the LPM that can lead to
improvements over the LPM [12]. However, the LRM
is non-convex due to the rational parameterization.
In addition, the variance is only accurately com-
puted for a high SNR, since measured output sig-
nals are included in the regression matrix.

Related work in the mechanical domain includes
a newly developed rational parametrization with
prescribed poles (LRMP), introduced in [8] where it
is applied to a simulation example featuring lightly
damped mechanics, and first applied to an experi-
mental thermal system in [13], that yields superior
estimation accuracy over the LPM while maintain-
ing linearity in the parameters. Although thermo-
mechanical interactions are increasingly relevant,
the modeling of this behavior in view of control is
a key challenge. And although estimating FRFs
using a local modeling approach shows promising
results by suppressing the transients, these tech-
niques are not yet applied to thermal dynamics in
precision motion systems. The methods are mainly
developed and applied on (lightly damped) mecha-
nical systems, see, e.g., [14, 15]. Important aspects
such as excitation signal design, transient measure-
ment conditions and ambient disturbance reduction
need to be re-evaluated.

To improve low frequency estimation quality of
the FRF, a measurement of the ambient air tem-
perature is used as an additional uncontrollable ex-
citation input in the identification procedure. It is
shown that by utilizing the additional excitation,
mainly present at lower frequencies, the estimation
quality of the FRF can be significantly improved.
Comparable approaches in the mechanical domain
have been explored in an active vibration isolation
application [16], using ground vibrations.

For some applications, e.g., direct feedback of a
fast thermal system in [4], models based on curve
fitting a frequency response function can be suf-
ficient. The application considered in this pa-
per requires a more extensive model. Therefore,
a first principles lumped-mass modeling approach
is adopted in conjunction with frequency response
function parameter calibration to obtain a predic-
tive model of sufficient complexity.

Although FRF identification is well-developed
for mechatronic systems from the electromechanical
perspective, at present these techniques are not tai-
lored towards identifying accurate thermal models
for precision control. The aim of this paper is to de-
velop a framework for advanced identification, par-
ticularly suitable for thermal-mechanical systems
and to experimentally validate this approach on a
representative experimental setup.

This paper builds on previous results reported in
[17] and expands on these results with additional
details and techniques providing a complete frame-
work. The main contributions of this paper are:

C1 An overview of the significant challenges in
thermal system identification, illustrated on a
representative experimental setup.

C2 Application of a new FRF identification ap-
proach that facilitates identification under
transient conditions.

C3 Exploiting additional temperature measure-
ment to reduce the low-frequency estimation
error by explicitly including ambient air tem-
perature fluctuations in the identification pro-
cess.

C4 Estimation of a temperature dependent con-
vection coefficient to improve model quality
over a large temperature range.

C5 An extensive case study, leveraging the im-
proved FRF identification results to calibrate
model parameters yielding a high fidelity para-
metric model.
2. Thermal system identification: challenges and problem formulation

In this section, an overview of the challenges in thermal system identification for precision motion systems is given. Moreover, the experimental setup that serves as a representative case study to illustrate the significance of these challenges is presented. The setup also provides a suitable experimental platform for the application of improved identification techniques.

2.1. Industrial challenges

Deformations induced by thermal gradients are increasingly relevant in several industrial applications, see, e.g., [18] for a selection. Examples include, warping and wafer edge deformation in lithography applications, thermal-induced drift in Transmission Electron Microscopy (TEM) [7], see Fig. 1, and frame deformations in machine tools [19, 2]. While the full temperature field is relevant for the prediction of thermal induced deformations, the expansion is often most relevant in a single Degree of Freedom (DOF). In this paper, it is assumed that many industrial applications can be considered in one dimension (1D) such that the geometry can be simplified without loss of generality. This simplification is valid for many industrial applications where the thermal behavior is often analysed in 1D, e.g., in the tool-path direction in machine tools [19] or perpendicular to the electron beam in TEM applications [18], e.g., the expansion loop shown in Fig. 1.

2.2. Problem formulation

2.2.1. Experimental setup

In this section the experimental setup is presented. The setup consists of a round uniform cylinder with a length of 250 mm and a diameter of 25 mm. The system has two heat inputs \( u_1, u_2 \) and five temperature outputs \( T_{1, \ldots, 5} \), in the form of power resistors and negative thermal coefficient (NTC) thermistors respectively. A photograph of the experimental setup including its inputs and outputs is shown in Fig. 2. The experimental setup consists of two aluminium cylinders with a small piece polyoxymethylene (POM) in between that acts as a high thermal resistance, as displayed in Fig. 2. This setup represents a typical industrial use case, where commonly mixed-material system designs are used. Typically, thermal properties of aluminum are accurately known, however, the thermal conductivity of POM is often an uncertain parameter. The conductivity of POM varies between 0.22 to 0.39 W/mK at 20°C depending on the manufacturing process of the material. Accurately identifying the conductivity parameter value provides an excellent benchmark for a grey-box parameter estimation problem.

2.2.2. Transient response

Thermal actuators are often limited to positive input signals or to binary sequences [20], e.g., the power resistors in Fig. 2 can only apply a positive thermal flux. As a result of this positive flux, the temperature of the system increases, causing components in the system to expand. This could exceed the measurable temperature or deformation range. Especially with systems with multiple inputs, where
the applied heat input is cumulative. The design of thermal excitation signals should have a low mean input while the input spectra remains rich. Designing optimal excitation signals remains a challenge for accurate FRF estimation.

Conventional approaches to frequency response function identification are typically unable to cope with large transients present in the response. These transient are, e.g., a result of the offset in the input or initial system conditions.

Commonly, the measurement data obtained under transient conditions is discarded in the identification process leading to loss of usable data. Moreover, due to the slow dynamics of thermal systems, removing this initial transient leads to an unacceptable increase of experiment time. In this paper, a novel approach to FRF identification is applied, first described in [8], that is suitable for FRF identification under transient conditions. This facilitates a reduced experiment time by eliminating the need for transient data removal and improves the overall estimation quality by removing the biased caused by the transient contribution.

2.2.3. Environmental disturbances

The response of thermal systems is highly influenced by environmental disturbances, consequently the identification accuracy often deteriorates. Typically, these environmental disturbances are dominated by day/night cycles or fluctuations in thermal conditioning systems, e.g., water chillers, which have relatively slow dynamics. As a consequence, the disturbance spectrum is relatively large at low frequencies and converges to a typical flat spectrum at higher frequencies, this is sometimes referred to as $1/f$ noise [21]. This is illustrated in Fig. 3 where a measured spectrum of the ambient air temperature surrounding the experimental setup is shown.

Specific limited time frames in the day/night cycle, e.g., at night, have relatively little environmental disturbances, however performing experiments at these specific time frames is not always viable. Reducing the impact of environmental disturbances is needed to increase the estimation accuracy and to reduce the experiment time. In this paper, an approach is used where the ambient temperature measurement, taken by sensors shown as $T_{a1}$, $T_{a2}$ in Fig. 2 is taken as an additional system input. This provides additional information for the system identification problem, and can increase the estimation accuracy, specifically at low frequencies.

![Figure 3: A typical power spectral density of the temperature of the ambient air surrounding the experimental setup. Illustrating the low-frequency ambient disturbances and high frequency measurement noise.](image)

2.2.4. Parameter varying dynamics

Typically, the assumption of linear time invariance (LTI) for thermal systems is only valid around a certain temperature. Realistically, the convection is expected to vary even within a relatively small temperature range [22]. For larger temperature ranges, the system behavior is nonlinear, and can be modeled as a linear parameter varying (LPV) system [23]. Typically, the convection coefficient only significantly alters the low frequency response of the system, resulting in a change in the largest time constant. Typically, the relation between temperature and the convection coefficient is described empirically [24] based on the geometry of the setup and properties of the fluid medium, e.g., air. In this paper, an approach is shown where this empirical relation is estimated for different operating conditions, allowing it to be included in the model as a temperature dependent parameter.

2.2.5. Problem formulation

In precision motion control thermo-mechanical interactions are increasingly relevant, yet the modeling of this behavior in view of control is a key challenge. To obtain an accurate system model it is key to take into account 1) transient contributions 2) environmental disturbances and 3) temperature dependency of the parameters. In this paper, estimating the FRF is taken as a first step towards high fidelity modeling. The estimated FRF can be used directly, e.g., for controller tuning [25]. In this work, the FRF is used as a basis for a grey-box approach to calibrate model parameters based on experimental data gathered under transient conditions. This facilitates the use of high-fidelity models for advanced model based control, enabling further
advances in precision motion control.

3. Improved thermal system identification

In this section, a framework for advanced system identification is presented.

3.1. Non-parametric frequency response function estimation

Consider a causal linear time invariant (LTI) system in an open-loop identification setting as shown in Fig. 4. Throughout, the excitation signal \( u(n) \) is assumed to be a random phase multisine. Commonly, multisine signals are being used as a periodic excitation since their spectrum is deterministic. To cope with data gathered under transient conditions, these transient into account during the identification procedure. This method is also applicable to a more general class of systems.

**Definition 3.1.** In this paper, a Random Phase Multisine (RPMS) signal is defined as

\[
    u(n) = \sum_{k=1}^{N} A_k \sin(2\pi f_k n/N + \phi_k) + \Delta, \quad (1)
\]

where, \( n \) is a specific discrete sample, \( N \) is the total number of samples, \( A_k \) is the amplitude of the sinusoidal signal at frequency \( f_k \), \( \phi_k \) is a uniformly distributed random phase on \([0, 2\pi)\) such that \( \mathbb{E}\{e^{i\phi_k}\} = 0 \) and \( \Delta \) is an offset to enforce \( u(t) \geq 0 \).

These multisine excitation signals offer the possibility to tune their frequency content by adjusting \( A_k \), and due their periodic nature spectral leakage due to the excitation input can be minimized. Different possible realizations of multisine signals are presented in [27], e.g., using a linear or logarithmic power distribution to maximize the amplitude spectrum of the excitation signal. The response \( y(n) \) to input \( u(n) \) of a discrete LTI system is as follows

\[
    y(n) = \sum_{m=-\infty}^{\infty} g(n - m)u(m) + \nu(n), \quad (2)
\]

with \( g(n) \) the impulse response of the system and \( \nu(n) \) the additive noise contribution. The Discrete Fourier Transform (DFT) of a signal is given by

\[
    X(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N}. \quad (3)
\]

Applying the DFT to (2) results in

\[
    Y(k) = G(\Omega_k)U(k) + T(\Omega_k) + V(k). \quad (4)
\]

Here, \( \Omega_k = e^{-j2\pi nk/N} \), \( Y(k) \), \( U(k) \), \( V(k) \) are the DFT of \( y(n) \), \( u(n) \), \( \nu(n) \) respectively, \( G(\Omega_k) \) is the frequency response function of the dynamic system, and \( k \) denotes the \( k \)-th frequency bin. Here, \( T(\Omega_k) \) represents the combination of the system and noise transient terms. These transients are the result of the truncation of an infinite response to a finite measurement interval.

Traditionally, the empirical transfer function estimation (ETFE) is used to derive the FRF [9, 26]. The ETFE is defined as

\[
    \hat{G}(\Omega_k) = Y(k)U(k)^{-1}, \quad \hat{T}(\Omega_k) = G(\Omega_k)U(k)^{-1}, \quad \hat{V}(k) = V(k)/U(k)^{-1}
\]

\[
    \begin{array}{c|c|c|c}
    \text{Estimation error} & \text{Transient} & \text{Noise} \\
    \end{array}
\]

For thermal systems, the transient contribution is significant. While the ETFE can often yield acceptable results on mechanical systems, since the transient is significantly shorter than the measurement period, for thermal systems the estimation accuracy is severely biased due to the transient component in the estimation error in (5). In view of existing system identification methods, this poses additional challenges, since these methods often assume the transient component to be negligible. In this paper, a method is proposed that explicitly takes these transient into account during the identification procedure. This method is also applicable to a more general class of systems.

3.2. Transient elimination

To cope with data gathered under transient conditions, a local modeling method is adopted. This method [3] uses a local rational parameterization of \( G(\Omega_k) \) and \( T(\Omega_k) \) in (4) on a subset of points

\[
    w = \{\{k - l, \ldots, k + l\}|w \neq k\} \subset \mathbb{N}, \quad (6)
\]
i.e., a local window of width \( l \in \mathbb{N} \), in the complex plane. Consider again (5) and let

\[
G(\Omega_w) = \sum_{b=0}^{N_b} \theta_{G}^b(b) \Psi(b, w)
\]

(7)

\[
T(\Omega_w) = \sum_{b=0}^{N_b} \theta_{T}^b(b) \Psi(b, w),
\]

(8)

such that locally the terms in (5) are approximated in a local window \( w \) by an expansion of degree \( N_b \) using general basis functions \( \Psi(w) \in \mathbb{C} \) where \( \theta_{G}^b(b) \in \mathbb{C} \) and \( \theta_{T}^b(b) \in \mathbb{C} \) are the local coefficients for the plant and transient respectively.

An identical basis function is used for the system and transient estimation as the dynamics are the same. This parametrization is linear in the parameters, i.e., \( \theta \) can be obtained in a closed-form solution, while having the advantages of using a general basis \( \Psi \) that allows for user-chosen parameterizations.

For instance, the basis \( \Psi \) can be chosen to be a polynomial, rational, or fractional function of the window parameter \( w \). Moreover, the basis straightforwardly allows for the inclusion of prior knowledge on the system dynamics through pre-scribed poles in \( \Psi \).

Generally, thermal systems are of first order with inherently stable poles. Commonly, a first estimation of the time constants of the system can be made and included as prior knowledge through \( \Psi \) to improve the estimation error of the FRF, see, e.g., [13].

### 3.3. Incorporating additional inputs

One of the main environmental disturbances for thermal systems is ambient air temperature fluctuations. To reduce the effect of these functions on the FRF identification, measurements of the ambient temperature are incorporated as an additional input [20]. Since it is difficult to excite the ambient air temperature in this paper it is considered an uncontrollable additional input. Measuring the environment and treating it like an input is commonly done for identifying vibration isolation systems, e.g., using ground vibrations as additional excitation sources [10].

The temperature of the environment is spatially dependent, and therefore ambient temperature measurements are only valid under the assumption that the ambient temperature surrounding the experimental setup is relatively uniform. The measured ambient air temperature is incorporated as an additional input in the identification procedure, yielding

\[
Y(w) = \tilde{G}(\Omega_w) [U(w)] + T(\Omega_w) + V(w)
\]

(9)

where \( \tilde{D}(w) \) is the DFT of the measured environmental disturbance and \( w \) denotes the local window [6]. Here, \( \tilde{G} \) is now a \( 1 \times 2 \) multi-variable system model due to the additional system input. The augmented plant \( \tilde{G} \) can now straightforwardly be estimated through the procedure described in Sec. 3.2.

### 4. Thermal modeling: parametric model applications

In this section, an approach to lumped-parameter parametric modeling for thermal systems is presented. Moreover, it is shown that the improved FRF obtained by applying the framework presented in Sec. 3 facilitates a grey-box parameter estimation approach. Lastly, a preliminary approach for identification of a temperature dependent convection coefficient is described.

#### 4.1. Thermal modeling

Consider a thermal system, e.g., the setup in Sec. 2.2.1, where the thermal dynamics are described by the heat equation

\[
c_p(x)\rho(x) \frac{\partial T(x, t)}{\partial t} = \kappa(x) \frac{\partial^2 T(x, t)}{\partial x^2} + h\left(T(x, t) - T_\infty(t)\right) + Q(x, t).
\]

(10)

With \( T(x, t) \) the temperature at position \( x \), \( T_\infty(t) \) the ambient temperature, \( Q(x, t) \) the heat flux, \( h \) the convection coefficient, \( \kappa(x) \) the thermal conductivity, \( \rho(x) \) the material density, and \( c_p(x) \) the specific heat capacity. The heat transfer due to radiation is linearized and combined in the convection coefficient \( h \). By employing spatial discretization, shown in Fig. 5, the partial differential equation (10) is transformed into a set of ordinary differential equations and the parameterized model can be represented in state space form by

\[
\begin{align*}
\dot{T}(t) &= A(\varphi)T(t) + B(\varphi)u(t) \\
y(t) &= C(\varphi)T(t)
\end{align*}
\]

(11)

where \( \varphi \in \mathbb{R}^{N_p \times 1} \) is a parameter set with \( N_p \) number of parameters including, but not limited to, material constants and contact resistances.
Figure 5: An illustration of the lumped-mass discretization of the experimental setup. The setup is divided into lumps, represented as capacitances $C$, that are interconnected to each other and the ambient temperature $T_a$ by resistances $R$. The actuator inputs are represented as a heat flux $Q_{u1,2}$ entering the appropriate lump. The dashed lines indicate a repeating pattern of the appropriate lumps, the illustration is simplified to facilitate the presentation.

4.2. Grey box identification

The parameterized model (11) contains uncertain parameters $\varphi$ that limit the prediction accuracy and suitability for advanced control. The aim of grey box identification is to calibrate the parameter set $\varphi$ such that the model (11) yields an accurate representation of the real system. The grey-box approach is based on minimizing the discrepancy between the measured non-parametric FRF and the FRF of the parametric model with the following cost function

$$J = \min_{\varphi} \left\{ \| W(\Omega_k) \left( G(\Omega_k) - \hat{G}(\Omega_k, \varphi) \right) \|_2^2 \right\}. \quad (12)$$

Here, $\hat{G}(\Omega_k)$ is the measured non-parametric FRF obtained by applying the approach presented in Sec. 3, $G(\Omega_k, \varphi) = C(\varphi)(\Omega_k I - A(\varphi))^{-1} B(\varphi)$ is the FRF of the parametric model (11), $W(\Omega_k) \in \mathbb{C}^{N_y \times N_x}$ is a dynamic weighting filter depending on the variance of the FRF at each frequency, and $N_u$ and $N_y$ are the number of inputs and outputs, respectively. By minimizing (12) the parameter set $\varphi$ is calibrated such that the model (11) best describes the experimental system. This facilitates the use of the calibrated model for advanced control and error compensation techniques.

4.3. Non-linear convection coefficient estimation

In this section, a procedure is proposed to estimate the temperature dependent convection coefficient. The convection coefficient is often assumed to be constant in a small temperature range. However, realistically, the convection coefficient depends, amongst others, on the temperature difference between the aluminium bar and its surroundings, $\Delta T = T(x, t) - T_a(t)$ [22] and therefore varies with temperature. To accurately capture larger temperature variations, this coefficient needs to be estimated for a wide temperature range. The empirical relation between the convection coefficient and $\Delta T$ can be estimated, under the assumptions that the Rayleigh number is linear in terms of $\Delta T$ and the Prandtl number is constant, by

$$h(\Delta T) = a + b\Delta T$$

where $a, b, c$ are model parameters. Here, it is proposed to apply a staircase function to the input. Due to the constant input, the temperature converges to a steady state for various $\Delta T$. Next, for each steady state temperature, the convection coefficient is estimated by minimizing the error between the measured and simulated output.

5. Case study: from measurement to model

In this section, the proposed identification methodology is applied to the experimental setup presented in Sec. 2.2.1 to yield a high fidelity parametric model.

5.1. Measurement: obtaining the FRF

In this section, the FRF of the experimental setup is estimated by using 1) transient suppression techniques as presented in Sec. 3.2 and 2) incorporation of additional inputs, presented in Sec. 3.3 to improve low-frequency estimation.

5.1.1. Transient suppression

Since the input to the system is constrained to be positive, the excitation input $u_1(t)$ is selected as a RPMS with offset, see Def. 3.1 for a definition. The temperature response $T_1$ to the input $u_1$ is shown in Fig. 6. Initially, the temperature response consist of a first order step response due to the offset $\Delta$ in the excitation signal. After the initial transient has settled, the output consist of the response of the excitation signal and environmental disturbances.

Two sub-records of the same dataset are considered, one includes the first two periods, which consist of a significant initial transient, and environmental disturbances. The second sub-record consist of the last two periods with a reduced transient and environmental disturbance and is used as a validation dataset.

In Fig. 7 the FRF estimation using sub-record 1 is shown. Clearly, the ETFE is unable to estimate the dynamics correctly using sub-record 1 since the transient contribution is relatively large. The LRMP estimates and suppresses the transient...
function between $T_\infty$ and $T_1$, where the latter can be used for a disturbance sensitivity analysis.

In Fig. 5 the amplitude estimation of the FRF $G(\Omega_k)_{u_1 \rightarrow T_1}$ and the $3\sigma$ uncertainty bounds are shown. The results show the estimation using only the input $u_1$, shown in red (a), and using $u_1$ and the ambient temperature measurements as an additional input, shown in blue (b). The amplitude estimation and variance at medium to high frequencies are similar for both estimations. However, using only the input $u_1$ a large variance is obtained in the low frequency region. By then incorporating the ambient measurement as an additional input, the variance of the estimation of the FRF is reduced significantly.

5.2. Model: parametric modeling and simulation

In this section, facilitated by the improved FRF obtained in Sec. 5.1, a high fidelity parametric model of the experimental setup is constructed.

5.2.1. Parameter estimation

In this section, the parameters of a Multi-Input Multi-Output (MIMO) lumped mass model, i.e., a model in the form (11), are calibrated by minimizing the discrepancy between the parametric model and the non-parametric FRF estimation using (12). The parameters that are optimized include, but are not limited to, the conduction coefficient, contact resistances and material properties of the aluminum and POM. In Fig. 9 the estimated non-parametric FRF, estimated at a temperature of approximately 300 [K], and the calibrated parametric model, is shown. Clearly, the estimated parametric model is within the $3\sigma$ uncertainty of the FRF estimation.
The conductivity of the slice of POM material is estimated at 0.32 W/mK, which is well within the range supplied by the manufacturer. The procedure yields a MIMO high fidelity parametric model of the experimental system.

5.2.2. Estimating the convection coefficient

The method proposed in Section 4.3 is applied to the experimental setup. The system is excited using a staircase function with a step gain of 0.05 W and a step time of 5 hours. The response and input signal are shown in Fig. 10. Clearly, the temperature response suffers from ambient temperature fluctuations. At each steady state, in terms of the heat input, the error between temperature simulation and measurement is minimized by tuning the convection coefficient. The convection coefficient is found for various $\Delta T$. An temperature dependent function is derived by fitting (13) onto the evaluated points from the experiment in a least squares manner, yielding results as shown in Fig. 11.

5.2.3. Time domain validation

By applying the framework presented in this paper a high fidelity parametric model is obtained. This model can be used for various objectives, e.g., controller design or predictive simulation studies. The predictive accuracy of the model is validated by using a step response measurement using heater input $u_1$. A step response of 0.5 [W] is applied to $u_1$, at $t = 1$[h], and the simulated response of

![Graphs showing input $u_1$ and $u_2$ with corresponding output $T_1$ and $T_2$.](image)

Figure 9: Non-parametric FRF estimate (●) of the experimental setup and the 3σ estimation uncertainty ( []). The FRF is used for a grey-box parameter estimation, yielding a high fidelity parametric model (—).

![Graph showing experimental data and calculated convection coefficients.](image)

Figure 10: Experimental data used to estimate the temperature dependent convection coefficient. The heater input (—) is applied in a stair sequence. The resulting temperature (—) is then evaluated at specific points (●). The ambient temperature (—) is recorded and taken into account.

![Graph showing estimated empirical function and calculated convection coefficients.](image)

Figure 11: Estimated empirical function (⋯) and calculated convection coefficients (●) from Fig. 10 describing the temperature dependency of the convection coefficient.
$T_1$ and $T_2$ is compared to the measured response as shown in Fig. [12](top). The results clearly show the high thermal resistance of the POM, located between $u_1$ and $T_2$, indicated by a slower response when compared to $T_1$. It is seen that by utilizing the procedure presented in this paper a high fidelity model is obtained as illustrated by the small prediction error shown in Fig. [12](bottom). While the results for the experimental setup are not yet in the millikelvin range, it is expected that the presented approach scales well to more sophisticated hardware.

6. Conclusion

The identification framework presented in this paper enables fast and accurate identification of thermal dynamics in view of precision motion control. By applying the local parametric method an improved non-parametric FRF estimate is obtained. Furthermore, by explicitly taking into account the transient contributions a significant reduction in measurement time is achieved when compared to classical methods. Moreover, the estimation error for low-frequencies is significantly reduced by incorporating additional sensor data that makes use of environmental disturbances to provide additional excitation input. Building on the improved FRF estimation, a grey-box parameter calibration approach is presented that yields high fidelity parametric models of the thermo-dynamical system. The proposed methodology is applied to a multi-variable experimental setup. The method achieves significant improvements in estimation accuracy, and a reduced experimentation time by incorporating the transient and disturbance contributions. The presented methods form a framework that facilitates the implementation of advanced control techniques and error compensation strategies, enabling increased accuracy and throughput in high precision motion control.

References


Figure 12: Time domain validation experiment on the parametric model obtained using the framework presented in this paper. In the top figure, the simulated (blue dashed) is compared to the measured (black dotted) output. In the bottom figure, the simulation error is shown for both outputs.


Enzo Evers received the B.Sc. degree (‘14) and M.Sc. degree (cum laude) (‘16) in Mechanical Engineering from the Eindhoven University of Technology, Eindhoven, The Netherlands. He is currently pursuing the Ph.D. degree in the Control Systems Technology group within the department of Mechanical Engineering of the Eindhoven University of Technology. His research interest is centered on advanced identification and control for thermal-mechanical systems.

Niels van Tuijl received the M.Sc. degree (with great appreciation) (‘19) in Mechanical Engineering with a specialization in control systems technology from the Eindhoven University of Technology, Eindhoven, The Netherlands. Currently, he is a Mechatronics Design Engineer at ASML, Veldhoven, The Netherlands. His research interests are system identification, and control, with applications in precision mechatronics.

Ronald Lamers received the B.Eng. degree in Mechanical Engineering (‘99) from the Fontys University of Applied Sciences, Eindhoven, The Netherlands, and the M.Sc. degree in Mechatronics Systems Engineering (with distinction) (‘07) from Lancaster University, Lancaster, United Kingdom. He has held positions at Philips and MI-Partners, both located in Eindhoven, The Netherlands. He currently works as a thermo-mechanical engineer at Thermo Fisher Scientific, Eindhoven, The Netherlands. His main research interests are in the field of thermal aspects within electron microscopes, such as cryogenic cooling and sub-nanometer position stability.
Bram de Jager received the M.Sc. degree in mechanical engineering from Delft University of Technology, Delft, The Netherlands, and the Ph.D. degree from Eindhoven University of Technology, Eindhoven, The Netherlands. He was with Delft University of Technology and with Stork Boilers BV, Hengelo, The Netherlands. He is currently with the Eindhoven University of Technology. His research interests include robust control of (nonlinear) mechanical systems, integrated control and structural design, control of fluidic systems, control structure design, and applications of (nonlinear) optimal control.

Tom Oomen received the M.Sc. degree (cum laude) and Ph.D. degree from the Eindhoven University of Technology, Eindhoven, The Netherlands. He held visiting positions at KTH, Stockholm, Sweden, and at The University of Newcastle, Australia. Presently, he is associate professor with the Department of Mechanical Engineering at the Eindhoven University of Technology. He is a recipient of the Corus Young Talent Graduation Award, the IFAC 2019 TC 4.2 Mechatronics Young Research Award, the 2015 IEEE Transactions on Control Systems Technology Outstanding Paper Award, the 2017 IFAC Mechatronics Best Paper Award, the 2019 IEEJ Journal of Industry Applications Best Paper Award, and recipient of a Veni and Vidi personal grant. He is Associate Editor of the IEEE Control Systems Letters (L-CSS), IFAC Mechatronics, and IEEE Transactions on Control Systems Technology. He is a member of the Eindhoven Young Academy of Engineering. His research interests are in the field of data-driven modeling, learning, and control, with applications in precision mechatronics.