Estimating the $\mathscr{H}_\infty$ norm with the Local Rational Method takes less data

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1 Introduction

The model error Δ is of vital importance to guarantee robustness of feedback controlled systems. In robust control, a model set $\mathscr{P}$ is used to design a controller: its 'location' is set by a nominal model $\hat{P}$, while its 'size' can e.g. be determined by the model uncertainty $\|\Delta\|_\infty$. On the one hand, $\mathscr{P}$ has to include the actual plant behavior P_0 for performance and stability guarantees, but on the other hand, an overly large set results in a conservative controller.

Different model-error-modeling methods exist, but each has its own pitfalls. A full parametric approach entails a model selection step that is difficult for complex plants. Frequency-response-based methods don't need such a step but can underestimate Δ significantly [1]. This is mainly due to the presence of sharp peaks in $\Delta(\omega)$, which can easily be missed when the frequency resolution is limited (see e.g. Figure 1). Such an underestimated $\|\Delta\|_\infty$ results in overly optimistic controller designs.

2 The Local Rational Method (LRM)

The Local Rational Method (LRM) [2] estimates a Frequency Response Function (FRF) by exploiting the smoothness of the FRF. The smooth FRF $P(\omega_k)$ and the leakage/transient contribution $T(\omega_k)$ are captured as low-order rational functions around every measured frequency ω_k:

$$P(\omega_k+\delta) = \frac{\sum_{i=0}^{N_B} b_i(k)\delta^i}{1+\sum_{i=1}^{N_A} a_i(k)\delta^i}, \tag{1a}$$

$$T(\omega_k+\delta) = \frac{\sum_{i=0}^{N_T} t_i(k)\delta^i}{1+\sum_{i=1}^{N_A} a_i(k)\delta^i}. \tag{1b}$$

The coefficients $b_i(k)$, $a_i(k)$ and $t_i(k)$ are fitted to the known system input $U(\omega_k)$ and measured output $Y(\omega_k)$ spectra by considering $Y(\omega) = P(\omega)U(\omega) + T(\omega)$ in a small bandwidth around each frequency ω_k. The more widespread Local Polynomial Method (LPM) [3] can be regarded as a specific case of the LRM with $a_i = 0$ imposed.

3 Application

While the LRM can estimate the FRF of a plant $P(\omega_k)$ and an error system $\Delta(\omega_k)$, with reduced leakage, its local models (1) form an untapped resource. Here, we focus on the error system Δ and its $\mathscr{H}_\infty$ norm, measured from an industrial Active Vibration Isolation System (AVIS). By interpolating neighboring local models (1), $\Delta(\omega)$ can be estimated in-between the observed frequency bins ω_k. For estimating $\|\Delta(\omega)\|_\infty$ in specific, this scheme amounts to retaining the largest amplitude of the relevant local models.

This LRM-interpolation [4] approach yields similar results on a sparse frequency grid as the LPM estimated on denser frequency grids. $\Delta(\omega)$ is captured very well by the interpolation as shown in Figure 1. Moreover, the estimate of $\|\Delta\|_\infty$ is improved significantly. This allows us to design robust controllers that are more reliable, without increasing the measurement time.

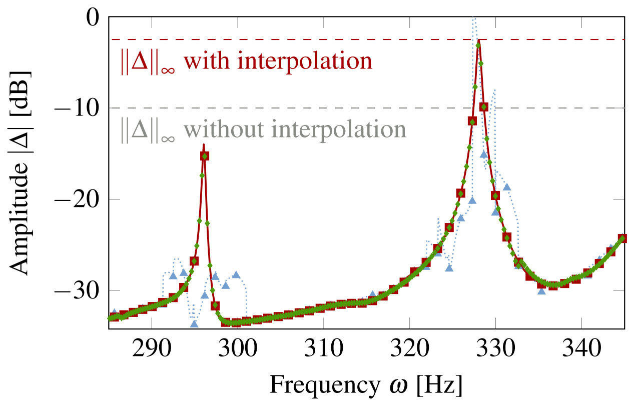

Figure 1: Contrary to the LPM (▲), the LRM (■) is able to capture sharp resonances well in the measured Δ of an AVIS. Interpolating the LPM (⋯⋯), obviously, cannot mend this. The interpolated LRM (——) agrees with a much denser frequency grid (•). Consequently, the estimated $\|\Delta\|_\infty$ (- - -) is improved by 7.5 dB.

References

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