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Frequency Response Function Identification of LPV Systems: A 2D-LRM Approach with Application to a Medical X-ray System

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Abstract— **LPV control has emerged as a systematic approach in the design of gain-scheduled controllers. This requires the identification of LPV models. The aim of this paper is to develope a flexible and accurate 2D-LRM approach, to enable fast and accurate non-parametric system identification of a frequency response function. The scope of this paper is on the identification of open-loop SISO LPV systems. Smoothness between several frozen LTI conditions within the LPV system is exploited to enable accurate pre-testing for parametric LPV modeling. The proposed approach achieves smoother estimations of the LPV behavior without an increase in estimation errors and with reduced variances. Traditional LPM and LRM approaches can be recovered as a special case of the proposed approach. The potential of the approach is shown by virtue of a simulation of a medical X-ray system.**

I. INTRODUCTION

Increasing performance requirements on mechatronic systems have led to the development of many model-based control and observer techniques. Model-based techniques impose increasing demands on the quality of the models for both linear and non-linear systems.

The control of linear parameter varying (LPV) systems, which are often used to describe a specific class of nonlinear systems, has emerged as a systematic approach in the design of gain-scheduled controllers, [1], [2]. The controller of the system is then defined by the current operating conditions of the system. In an LPV system, one or multiple scheduling parameters lead to varying dynamics. However, for each given combination of frozen operating conditions, the system behaves as a linear time invariant (LTI) system. For LTI systems, many non-parametric identification methods have been developed, see e.g., [3] for an extensive overview.

In the literature, modeling of LPV systems can be separated in local and global approaches, see e.g., [4], [5], [6], and [7]. Global approaches are based on a single dedicated experiment in which the LPV behavior is modeled, [8], [9], and [10]. Local approaches first identify a set of "frozen" combinations of varying parameters, such as possible positions, followed by interpolation between the separate LTI models, [11], [12], [13]. The relation between the frozen experiments in the local approach is not made before the interpolation step and not based on physical relations. The LTI models are identified independently. Linear systems are often identified using an intermediate step of non-parametric frequency response function (FRF) measurements. In fact, this is also done in certain local LPV methods, including [14].

Over the recent years, local parametric methods, such as the local polynomial method (LPM) and the local rational method (LRM) have been developed for the non-parametric identification of systems to enhance accuracy of the FRF. The LPM exploits the smoothness of the transient effects and the systems dynamics over the frequencies to locally approximate the behavior accurately in an efficient manner, see e.g., [15], [3, Sec. 7.2] for measurements using non-periodic excitations and [3, Sec. 7.3], [16] and [17] for periodic excitations. The LRM is an extension of the LPM towards local approximations with rational functions enabling increased flexibilities in the estimations, see e.g., [18] and [19]. In [20], a 2-dimensional (2D) approach of the LPM has been presented, limited to non-periodic excitation signals. Similar to the exploitation of smoothness over the frequencies in the standard LPM, smoothness over the scheduling parameter of an LPV system is exploited.

Although the 2D-LPM approach shows promising results, it does not exploit the full potential of non-parametric identification. The aim of this paper is to incorporate the freedom in the design of experiments by using periodic excitation signals and, in addition, fit local rational models. The LRM is experienced in [18] to be a better local model compared to the LPM. These aspects aim to improve the accuracy and efficiency of the novel non-parametric identification method, by means of a periodic 2D-LRM approach introduced for single-input and single output (SISO) open-loop systems. The periodic excitation signals allow for a user-defined frequency grid, enabling a significant improvement in the signal-to-noise ratio (SNR) compared to non-periodic excitation signals, [21, Sec. 6.2]. Furthermore, the periodic excitations enable a distinction between noise and nonlinear contributions to the estimation.

The contributions in this paper are:

1) the development of a local rational identification method towards LPV systems,
2) a fast and efficient identification approach by using periodic excitation signals instead of the non-periodic excitations in the earlier 2D-LPM approach, and
3) the validation of the proposed method using a simulation example of a medical X-ray system.

The 1D-LRM can be recovered as a special case of the

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proposed 2D-LRM for periodic excitation signals. Potential applications of the proposed approach are i) manual controller tuning and FRF-based stability analysis using Nyquist type techniques for a range of fixed conditions, and ii) identification of parametric models for control design, e.g., future lightweight systems, [22], [23].

In Sec. II, the considered class of LPV systems is introduced given a single scheduling parameter. In Sec. III, the proposed approach using the LRM for LPV systems is introduced, including a brief overview of the 1D-LRM. In Sec. IV, the potential of the novel approach is demonstrated on a simulation example, in which it is compared to the 1D-LRM. This paper is concluded with an overview of the results and ongoing work in Sec. V.

II. PROBLEM STATEMENT

A. Linear Parameter Varying Systems

The class of nonlinear systems considered in this work is the class of linear parameter varying (LPV) systems, for which the continuous scheduling parameter can be frozen into local linear time invariant (LTI) systems. In state-space form, the dynamics of the system can be written as,

$$G(\Omega,\theta) := \left\{ \begin{array}{ll} \dot{x}(t) & = A(\theta)x(t) + B(\theta)u(t) \\ y(t) & = C(\theta)x(t) + D(\theta)u(t) \end{array} \right. , \quad (1)$$

with $G(\Omega, \theta)$ the frequency domain interpretation of the system dynamics, $x(t)$ the states, A, B, C, and D the system matrices depending on the scheduling parameter θ, and $u(t)$ the input signal, [24, Chap. 1].

The scheduling parameter θ may represent any behavior varying condition of the system. For mechanical systems, typically positions or temperature are parameters that can lead to parameter varying behavior. The domain θ is typically limited by physical boundaries, furthermore, the rate of variation $\dot{\theta}$ will be bounded by design, [7], [11]. The limitation of $\dot{\theta}$ is key for the implementation of the proposed approach, where smoothness over the scheduling parameter is assumed.

B. Problem Definition

The identification of an LPV system in a local approach is typically a time-consuming procedure, since each desired LTI system has to be identified individually. In many non-parametric identification approaches, such as averaging to obtain the best linear approximation (BLA), [3, Sec. 4.3] or windowing and overlapping methods, [25], the transient behavior is neglected in the processing step. The local parametric approaches, i.e., the LPM and LRM, estimate the transient behavior, allowing for utilization of the entire data set. With this increased efficiency, the measurement times have already been reduced by the length of the transient period for each LTI experiment.

In the previous section, it is defined that the rate of variation of the scheduling parameter $\dot{\theta}$ is limited to enable the smoothness assumption over the scheduling dimension. The smoothness over the scheduling parameter can be exploited by extending the function fitted through a local frequency window towards a complex 2D plane of functions, [20]. The aim of this paper is to enable fast and accurate identification of LPV systems, fully utilizing the smoothness property in frequency and scheduling parameter directions and by exploiting the advantageous properties of periodic excitation signals, [21, Sec. 6.2].

III. 2D-LRM FOR LPV SYSTEMS

In this section, the proposed approach of the local rational method in two dimensions, i.e., the frequency axis and the scheduling dimension, is introduced. To allow the introduction of the novel approach, the LRM using periodic excitation signals is discussed. In [3, Sec. 7.3], both a robust and a fast approach for the periodic LPM have been introduced. The transient behavior is defined as an LTI contribution, therefore, the robust method is not considered in this paper, see [20].

A. Local Rational Method for Periodic Excitations

The general idea behind the LRM, is based on the assumption that the systems dynamics are smooth over the frequency axis, which can be exploited to fit a rational function through a neighboring frequencies. The general output spectrum, in the frequency domain, of a system can be given by,

$$Y(k) = G(\Omega_k)U(k) + T(\Omega_k) + V(k), \quad (2)$$

with $G(\Omega_k)$ the plant dynamics, $T(\Omega_k)$ the transient effects of the plant and the noise filter, and where $Y(k)$, $U(k)$ and $V(k)$ are the frequency domain representations of the systems output, input and measurement noise respectively. The frequency domain signals are obtained through the discrete Fourier transform (DFT). The DFT $X(k)$ of a time domain signal $x(t)$ is defined by,

$$X(k) = \frac{1}{\sqrt{PN}} \sum_{n=0}^{PN-1} x(nT_s)e^{-i2\pi nk/PN}, \quad (3)$$

with P the number of periods measured, N the number of samples per period and T_s the sampling rate. For periodic signals without any external disturbances, only the frequencies for which $k = \alpha P$ contain signal content, all intermediate frequencies, i.e., $k = \alpha P + r$ with $r = -(P-1),\ -(P-2), \dots, -1, 1, \dots, P-2, P-1$, satisfy $X(k = \alpha P + r) = 0$, see [16] for a more detailed analysis.

This paper focuses on the LRM using periodic excitation signals, which is a two-step procedure. The first step is the estimation of the transient effects, followed by a second step in which the plant dynamics are estimated.

Step 1: Using the knowledge on the unexcited frequencies for periodic signals, it can be seen that from (2),

$$Y(\alpha P + r) = T(\Omega_{\alpha P + r}) + V(\alpha P + r), \quad (4)$$

since the input signal $U(\alpha P + r) = 0$.

The LRM uses the smoothness assumption over the frequencies by approximating the system dynamics and transient effects by a rational function,

$$T(\Omega_{\alpha P + r}) = \frac{M(r)}{D(r)}, \quad (5)$$

with M and D polynomial functions corresponding to a Taylor series expansion around the central frequency αP,

$$M(r) = m_0 + m_1 r + \ldots + m_{N_m} r^{N_m} \quad (6)$$

$$D(r) = 1 + d_1 r + \ldots + d_{N_d} r^{N_d} := 1 + \tilde{D}(r). \quad (7)$$

Using the polynomial functions, the best fit through the $2(P-1)$ unexcited frequencies around the excited frequency αP is obtained, and substitution of $r = 0$ results in the approximation of the transient $T(\Omega_{\alpha P})$. By substitution of (6) and (7) into (5) and (4), the output spectrum can be written as

$$Y(\alpha P + r) = M(r) - \tilde{D}(r) Y(\alpha P + r) + V(\alpha P + r), \quad (8)$$

$$= \Theta_t K_t(r, Y) + V(\alpha P + r), \quad (9)$$

where $\Theta_t \in \mathbb{C}^{1 \times N_m + N_d + 1}$ contains the unknown parameters m_0 through m_{N_m} and d_1 through d_{N_d}, and $K_t \in \mathbb{C}^{N_m + N_d + 1 \times 2(P-1)}$ all known variables from (8), i.e., all combinations of r and the outputs Y. Equation (9) is linear in Θ_t and is solved using a linear least-squares optimization problem with cost function,

$$\mathcal{J} = \arg\min_{\hat{\Theta}_t} \|Y(\alpha P + r) - \hat{\Theta}_t K_t(r, Y)\|_F^2. \quad (10)$$

Due to the presence of the measured output in $K_t(r, Y)$, a possible biased result can be obtained. These effects have shown to be relatively small and do not outweigh the benefits of the proposed methods. The obtained estimated parameters $\hat{\Theta}_t$ are used to obtain the transient estimation,

$$\hat{T}(\Omega_{\alpha P}) = \hat{\Theta}_t \begin{bmatrix} 1 \\ O_{N_m + N_d \times 1} \end{bmatrix}, \quad (11)$$

with O a vector of zeros of dimensions $(N_m + N_d) \times 1$.

The obtained transient estimation $\hat{T}$ is used to correct the output spectrum for the transient effects,

$$Y_c(\alpha P) = Y(\alpha P) - \hat{T}(\Omega_{\alpha P}). \quad (12)$$

Step 2: The corrected output spectrum from *Step 1* is used in the plant estimation. Substitution of (12) into (2) leads to,

$$Y_c(\alpha P) = G(\Omega_{\alpha P}) U(\alpha P) + \tilde{V}(\alpha P), \quad (13)$$

with $\tilde{V}(\alpha P)$ the remaining perturbations on $Y_c(\alpha P)$. By defining a frequency window of excited frequencies $(\alpha + w)P$ with $w = -n, -(n-1), \ldots, 0, \ldots, n-1, n$ with n a user-defined number of frequencies, a similar approach is used for the estimation of the plant dynamics. The plant dynamics $G(\Omega_{(\alpha+w)P})$ can be parametrized similar to the transient term,

$$G(\Omega_{(\alpha+w)P}) = \frac{N(w)}{D(w)}, \quad (14)$$

with $N(w)$ and $D(w)$ as in (6) and (7). Substitution of (14) in (13) and rewriting, the output equation is given by

$$Y_c((\alpha + w)P) = N(w) U((\alpha + w)P) + \quad (15)$$

$$\ldots + \tilde{D}(w) Y((\alpha + w)P) + V((\alpha + w)P)$$

$$= \Theta_g K_g(w, U, Y) + V((\alpha + w)P), \quad (16)$$

equivalent to (9), with the unknown parameters in $\Theta_g \in \mathbb{C}^{1 \times N_n + N_d + 1}$ and the regression matrix $K_g(w, U, Y) \in \mathbb{C}^{N_n + N_d + 1 \times 2n+1}$ containing all known coefficients from (15). The linear set of equations can be solved using a least-squares optimization problem as defined by a similar cost function as in (10). The corresponding plant dynamics are defined by,

$$\hat{G}(\Omega_{(\alpha+w)P}) = \hat{\Theta}_g \begin{bmatrix} 1 \\ O_{N_n + N_d} \end{bmatrix}, \quad (17)$$

concluding the plant estimation using the fast LRM.

B. Local rational method for LPV systems

The general idea behind a 2D local parametric approach is to exploit the smoothness between different LTI FRFs of an LPV system, [20]. The same two-step procedure as in Sec. III-A is used for the 2D-LRM. One of the main contributions to the transient term $T(\Omega)$ is related to the initial conditions of the measurements, therefore, no smoothness can be guaranteed for the transient term between different LTI measurements. For each individual LTI measurement, *Step 1* is performed, resulting in a corrected output spectrum $Y_c(\alpha P, \theta)$. *Step 2* for the 2D-LRM includes the local parametric estimation over both the frequency axis and the scheduling parameter. For LPV systems, the general output equation from (13) for *Step 2* is given by

$$Y_c(\alpha P, \theta) = G(\Omega_{\alpha P}, \theta) U(\alpha P, \theta) + V(\alpha P, \theta), \quad (18)$$

for all $\alpha = 1, 2, \ldots, N$ and $\theta = \theta(1), \theta(2), \ldots, \theta(M)$ the measured poses of the system. The scheduling parameter θ is assumed to be a continuous function over the scheduling domain, however, only M poses are measured, similar to traditional local LPV experiments, [13]. The method exploits the smoothness to obtain a better estimate in terms of a lower variance at each measured frequency bin.

The proposed approach uses the local frequency window from *Step 2* in Sec. III-B for each LTI system in combination with a local plane in the frequency-scheduling domain. The plane is spanned by the vectors

$$\begin{array}{lcccccccc} w = & -n & -(n-1) & \ldots & 0 & \ldots & n-1 & n, \\ z = & -m & -(m-1) & \ldots & 0 & \ldots & m-1 & m, \end{array} \quad (19)$$

with n and m the user-defined widths of the window in frequency domain w and scheduling domain z respectively. The plant dynamics are parametrized by a rational function of polynomials,

$$G(\Omega_{(\alpha+w)P}, \theta + z) = \frac{N(w, z)}{D(w, z)}, \quad (20)$$

where both N and D can be functions of the frequency and scheduling variations, however, this is not a strict requirement. The plant dynamics can be approximated by a 2D Taylor series expansion, i.e.,

$$G(\Omega_{(\alpha+w)P}, \theta + z) = G(\Omega_{\alpha P}, \theta) + \frac{\partial G(\Omega_{\alpha P}, \theta)}{\partial w} w$$

$$\ldots + \frac{\partial G(\Omega_{\alpha P}, \theta)}{\partial z} z + \frac{1}{2!} \frac{\partial^2 G(\Omega_{\alpha P}, \theta)}{\partial w^2} w^2 + \ldots$$

or more general,

$$G(\Omega_{(\alpha+w)P}, \theta + z) = \sum_{i=0}^{Q} \sum_{j=0}^{Q-i} g_{i,j} w^j z^i, \tag{21}$$

$$g_{i,j} = \frac{\partial^{i+j} G(\Omega_{\alpha P}, \theta)}{\partial w^j \partial z^i}, \tag{22}$$

with Q the order of the polynomials. It should be noted that the orders in the frequency direction and the scheduling domain do not necessarily have to be identical. The approximation of $G(\Omega_{(\alpha+w)P}, \theta + z)$ is extended to the numerator and denominator polynomials, i.e.,

$$N(w, z) = \sum_{i=0}^{N_N} \sum_{j=0}^{N_N-i} n_{i,j} w^j z^i, \tag{23}$$

for the numerator polynomial. The denominator D is parametrized identical with typically $d_{0,0} = 1$ in line with the traditional LRM, without loss of generality [19]. The orders N_N and N_D are for the numerator and the denominator respectively.

Using the parametrization in two directions, the output equation in (18) is written as a linear combination of the unknown parameters $d_{i,j}$ and $n_{i,j}$. The cost function is defined by

$$\mathcal{J} = \arg\min_{\hat{\Theta}_g} \|Y_c(\alpha P, \theta) - \hat{\Theta}_g K_g(w, z, U, Y)\|_2^2, \tag{24}$$

with $\hat{\Theta}_g$ the to be estimated parameters and $K_g(w, z, U, Y)$ the regression matrix for the least-squares problem. By minimizing the cost function, the plant estimation $\hat{G}(\Omega_{\alpha P}, \theta)$ is obtained based on the information of the $(2n+1)(2m+1)$ measured bins directly neighboring grid point $(\Omega_{\alpha P}, \theta)$, where $\hat{n}_{0,0} = \hat{G}(\Omega_{\alpha P}, \theta)$.

Variance analysis: Variance analysis provides insight in the estimation quality. The use of periodic excitation signals enables a distinction between a noise variance and a variance as a consequence of nonlinearities in the system. The noise variance is fully based on the estimation of the transient effects, since the unexcited frequencies only contain transient terms and non-periodic perturbations. An estimate for the total covariance is obtained from the plant estimation, since all perturbations, both periodic and non-periodic, are present on the excited frequencies. The nonlinear variance is defined by the difference between the total covariance and the noise variance.

The noise variance is defined by,

$$^{\text{noise}}\hat{\sigma}_y^2(\alpha P) = \frac{\mu}{q} \mathcal{R}_t \mathcal{R}_t^H, \tag{25}$$

$$\mathcal{R}_t = Y(\alpha P + r) - \hat{\Theta}_t K_t(r, Y), \tag{26}$$

with $\mu = 1 + \|\Sigma \mathcal{V}^H\|_2^2$ from $K_t^H = \mathcal{U} \Sigma \mathcal{V}^H$, and q the degrees of freedom of the least-squares problem,

$$q = 2(P-1) - (N_m + 1 + N_d) = 2(P-1) - \text{rank}(K_t).$$

The total variance is defined by,

$$^{\text{total}}\hat{\sigma}_y^2(\alpha P) = \frac{1}{q} \mathcal{R}_g \mathcal{R}_g^H \tag{27}$$

$$\mathcal{R}_g = Y_c((\alpha + w)P) - \hat{\Theta}_g K_g(w, z, U, Y), \tag{28}$$

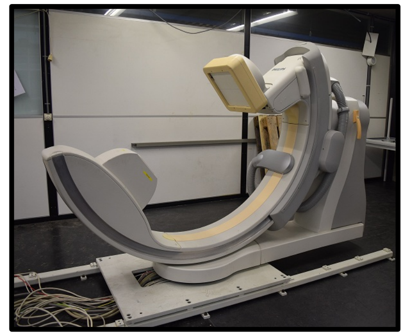


Fig. 1: Philips Allura Centron C-arc system floor-mounted

with $q = (2n+1)(2m+1) - \text{rank}(K_g)$. Note that, due to the noisy regression matrices K_t and K_g, the covariances can be biased.

The variances described above are related to the output spectrum. Typically it is desired to have an estimate of the variance compared to the plant dynamics, therefore,

$$\hat{\sigma}_G^2(\Omega_{\alpha P}) = \overline{S^H S} \otimes \hat{\sigma}_y^2(\alpha P), \tag{29}$$

$$S = K^H (K K^H)^{-1} \begin{bmatrix} 1 \\ O \end{bmatrix}, \tag{30}$$

for both the noise and the total variance, with $K = K_t$ and $K = K_g$ respectively. A detailed analysis of the variances for the fast method of the LPM, which is a specific case of the LRM, can be found in [3, Sec. 7.3.7] and [16]. The variances are applicable to each frequency bin individually, however, it exploits the correlation between adjacent bins.

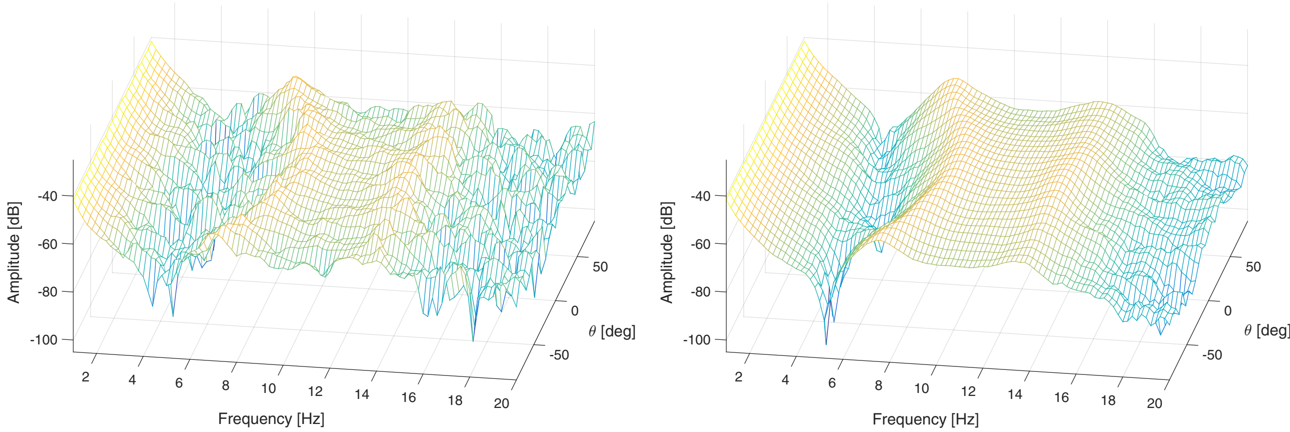
The main advantage of the current approach compared to Sec. III-A is the parametric approximation of a single point based on an increased number of data points, directly surrounding the point of interest. Compared to the 2D-LPM approach in [20], the proposed approach has an increased flexibility in the estimations due to the denominator polynomial. Additionally, the periodic excitation signal allows for a more efficient experiment design and therefore increased efficiency in the identification of a system. The extension to multivariable systems is outside the scope of this paper.

IV. CASE STUDY

In this section, the potential of the proposed approach is illustrated using a simulation example on a medical X-ray system as depicted in Fig. 1.

A. System Description

As a simulation example in this paper, the C-arc based medical X-ray system in Fig. 1 and 3 is considered. The system has a C-shaped element with a diameter of approximately 2 meters and a mass of 600 kg, which moves around the patient in approximately 5 seconds, the generation of high-definition 3D reconstructions of the interior of the human body. The C-shaped element supports the X-ray source and the detector and is connected to the floor. The detector and source have the freedom to rotate around the patient over the angles θ_1 and θ_2 as indicated in Fig. 3. The rotational movements are made by a rotation around the lateral axis of the patient and a sliding motion through the support sleeve respectively.



(a) Result of the LRM as in III-A performed for each frozen θ.

(b) Full 2D-LRM as described in Sec. III-B.

Fig. 2: Plant estimations of the model including a simulated 10% measurement noise.

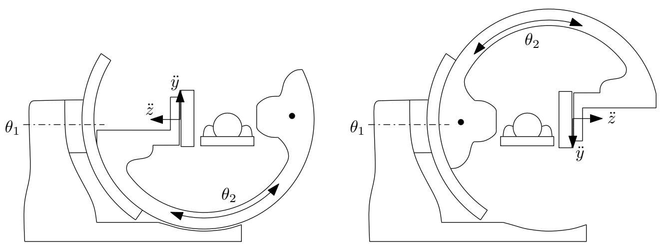


Fig. 3: Schemetic representation of the Philips Allura Centron in two positions of the roll-rotations, i.e., $\theta_2 = -90$ and $\theta_2 = 90$ degrees.

The motions of the system can be measured using collocated incremental encoders on the motor axis. Additionally, external absolute acceleration measurements are available at the detector position with respect to the local coordinate frame as indicated in Fig. 3. The degrees of freedom of the system make it a multivariable system, with the applied motor currents $u = [u_1 \ u_2]^T$ as inputs and outputs $y = [\theta_1 \ \theta_2 \ \ddot{x} \ \ddot{y} \ \ddot{z}]^T$ both the collocated encoder measurements and the accelerations. This paper only focuses on SISO systems, therefore, only the collocated measurement with u_2 as both scheduling parameter and input signal and θ_2 as measured output is considered.

For the medical X-ray system in Fig. 1, the dynamical behavior is a function of the scan angles, i.e., variations of θ_1 and/or θ_2 lead to changes in the mechanical structure as indicated in Fig. 3. Hence, the pole and zero locations of the system are strongly correlated to the pose and vary during a scan operation, leading to an intuitive choice of θ_1 and θ_2 as scheduling parameters for an LPV model. In this paper a simplified model based on physical insight of the system is used where θ_2 is used as scheduling parameter, ranging from -90 to 90 degrees with 25 frozen experiments, equally distributed over θ. Validation measurements have been performed confirming accurate dynamical behavior for typical control design requirements.

B. Simulation Results

The potential of the proposed method is shown in the current section based on a simulation of the system described in the previous section. The comparison is made between the fast method for the LRM as described in Sec. III-A and the 2D-LRM approach as discussed in Sec. III-B.

In the simulations in this section, a multisine signal with a period length of 5 seconds, repeated for 6 periods has been used as an input signal for each frozen parameter measurement. A single LTI experiment takes up to 30 seconds measurements, leading to a total of 12.5 minutes for the full scheduling domain. An additive measurement noise has been included to achieve an SNR of 20dB. The simulated outputs have been processed, resulting in the plant estimations as depicted in Fig. 2, with the results using the LRM for each LTI system in Fig. 2a, and the proposed approach in Fig. 2b. It can clearly be observed that the plant dynamics have been estimated more smoothly than using the individual LRM estimations. The least-squares optimization enables efficient processing of the FRF measurements in the order of magnitude of 20 seconds for both the traditional and the proposed method. The orders of the polynomials have been tuned manually and are typically small. In [18] the rules of thumb $N_n = N_m = 2$ and $N_d = 1$ are given, which yielded the results in this paper. For the plant estimations using the LRM, 23 frequency bins are used, while for the 2D-LRM, this is increased to 5 scheduling parameters to $23 \times 5 = 115$ bins in the neighboring measured bins.

To enable a quantitative comparison between the two approaches, the estimation errors should be observed. A smoother estimation does not necessarily mean a better performance of the method. The absolute estimation error is only defined in simulation. It is determined for each individual grid point in the frequency-scheduling domain. The root mean square of the error has been calculated for each frequency point over the different poses of the system, which is shown in Fig. 4. It can clearly be seen that the estimation error using the newly proposed approach is lower

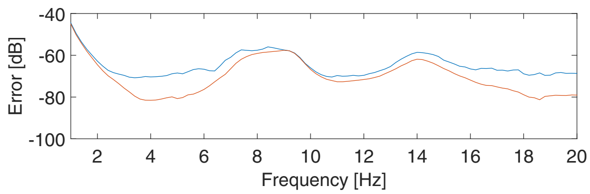


Fig. 4: Comparison between the simulation errors of the LRM for each LTI measurement (blue) and using the 2D approach (red). The errors are the root mean square errors over the scheduling parameters.

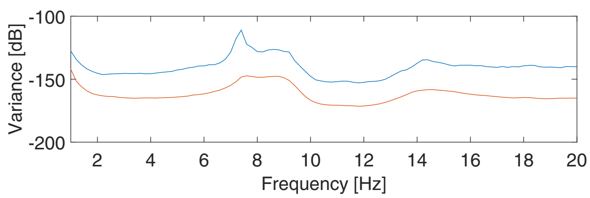


Fig. 5: Comparison of the total variances of the LRM for each LTI measurement (blue) and using the 2D-LRM approach (red). The variances have been averaged over the identified LTI positions.

for all frequencies compared to the individual estimations. This leads to the conclusion that the 2D-LRM approach enables an accurate and efficient estimation of the plant dynamics without introducing additional modeling errors.

The confidence in the method is not only determined by the absolute estimation errors, also the variance on the estimation is important. In Fig. 5, the total variances on the plant estimation, averaged over the LTI systems, is shown. The variance of the 2D-LRM is on average 20 dB lower than using the individual LRM.

V. CONCLUSIONS

In this paper, a local rational method for non-parametric system identification of open-loop single-input, single-output LPV systems is presented. The potential of the proposed approach, which assumes the smoothness between individual LTI systems, has been shown on a simulation example of an interventional X-ray system. The simulations show an increased smoothness of the system estimation without an increase in modeling errors and with decreased variance.

The current paper focuses on systems with a single degree of freedom, i.e., a single scheduling parameter, operating in an open-loop measurement setup for SISO systems. Ongoing research focuses on extensions to the multivariable case by employing suitable parameterizations.

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