Accurate Frequency Response Function Identification of LPV Systems: A 2D Local Parametric Modeling Approach

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***Abstract*— In the last decades, LPV control has emerged as a systematic approach to design gain-scheduled controllers. This development has led to the emergence of LPV identification techniques. The aim of this paper is to present a new approach that enables fast and accurate identification of nonparametric frequency response functions of LPV systems. Main applications of the proposed approach include pre-testing for global LPV modeling and the use of nonparametric models as a basis for local LPV modeling. The proposed method extents recent developments of local parametric modeling techniques to a 2D situation. Pre-existing LPM/LRM methods are recovered as a special case. The advantages of the proposed approach are illustrated on a simulation model.**

I. INTRODUCTION

Control of Linear Parameter Varying (LPV) systems has emerged in the last two decades as a systematic framework to design controllers for a class of nonlinear systems [1], [2]. Essentially, it enables the systematic design with guaranteed global performance of gain-scheduled controllers, which are currently one of the most commonly applied and popular design methodologies for nonlinear systems. In addition, synthesis algorithms have been further developed, see [3], [4], and [5].

The development of LPV control design methodologies has spurred the development of system identification techniques that deliver the required model, see [6], [7], for early results and [8], [9], [10] for an extensive overview of the present state of the art. The developments can be divided into two approaches. On the one hand, global approaches identify an LPV model in one shot, typically from a single global experiment. Examples of global LPV modeling approaches include [7], [11], [12]. On the other hand, local approaches first identify a set of LTI models for "frozen" parameter values, followed by an interpolation towards an LPV model [13], thereby closely resembling traditional gain-scheduling methods, which are applied on the system model instead of the controller. Examples of local LPV modeling include [14], [15], [16], [17] and [18].

Independent of the choice of a global or local LPV identification approach, well-designed preliminary tests are extremely valuable when attempting to represent real-life systems as LPV models. Indeed, many successful applications of system identification using linear time invariant models have been reported in the literature, while virtually any real-life system will be nonlinear and time varying to a certain extent. From this perspective, the additional complexity and modeling cost that is associated with modeling a system as LPV has to be justified from the accuracy requirements that originate from the modeling goal. Note that preliminary pre-testing is quite standard in applications of system identification [19, Chapter 10].

One obvious approach that is suitable for pre-testing in LPV systems is to estimate nonparametric LTI systems for certain frozen operating conditions, where the frozen parameters are the ones that are expected to induce the parameter dependent behavior. For instance, in mechanical positioning system applications, the position of the system is an immediate choice where various frozen behaviors can be analysed, see, e.g., [15], [16], and [20, Section 5.7] for a control-relevant perspective. In the case where the nonparametric models indicate a large position variation, then a full LPV model can be identified, possibly using the identified nonparametric models as a starting point as in, e.g., [15], [16].

The identification of nonparametric models, in particular frequency response functions, has been significantly advanced in recent years, see [21] for a recent overview of state-of-the-art methods. An important aspect to enhance accuracy and reduce measurement time has been the use of local parametric modeling methods and extensions, see [22], [23], [24] and [21, Chapter 7]. These methods exploit smoothness of the frequency response function (FRF) and the transients to reduce variance and bias errors compared to alternative methods.

Although both LPV identification and nonparametric FRF identification have been significantly developed in recent years, these developments have been made independently of each other and the LPV behavior of the system is not yet fully exploited during nonparametric FRF modeling. The aim of this paper is to develop an accurate and fast approach to identify nonparametric, frozen LTI models of LPV systems. The main idea is to exploit the fact that the LPV model is typically a smooth function of the frequency, as well as the scheduling variable. This leads to several advantages compared to pre-existing approaches, including the following.

- Increased time efficiency and/or accuracy.
- Enhances a key step in many LPV approaches, including pre-testing in case of global approaches, or as a basis for local LPV modeling approaches with subsequent interpolation.

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- Enabling more accurate analysis of frozen behavior of both LPV and gain-scheduled controllers. Indeed, nonparametric frequency response functions are standard in many control engineering applications, including mechanical systems, e.g., as in [25], [26], where model-based $\mathcal{H}_\infty$ optimal controllers are always evaluated using Bode and Nyquist diagrams using nonparametric frequency response functions prior to implementation.
- The proposed method incorporates the advantage of LPM methods to use transient responses, i.e., the experimenter does not have to wait until transients have disappeared and the system is in steady state. This is an important advantage in modeling LPV systems with frozen parameters, since the experimenter can vary the scheduling parameter without losing valuable experimental time waiting for transients to disappear for each frozen scheduling value.
- The approach is applicable to both multisine and noise excitation signals and a local quantification of the errors imposed by the frozen parameter compared to a true local LTI representation can be given by virtue of the Best Linear Approximation (BLA).

The main contribution of this paper is a new framework for nonparametric LPV modeling through an extended 2D local parametric modeling approach. The approach is validated on a simulation model of a wafer stage, which is used in lithographic processing, see [26], [27], [28]. Exactly the same model has been used in [16], [29] using a local approach, where in [29] both the simulation model and the real physical system have been identified and controlled. However, the nonparametric pre-processing approach in [16], [29] was done using classical noise excitation and windowing, using a massive amount of experimental effort in terms of measurement data and experiment time. The present paper aims to improve this pre-processing step, enabling enhanced time efficiency and accuracy compared to the earlier obtained results.

Finally, it is pointed out that several related identification techniques have been developed in the literature. First, the main step to obtain the frequency response function is transforming the data to the frequency domain with the Fourier transformation. In the proposed approach, the Fourier transform is taken for each data set separately, and subsequently post-processed using the proposed 2D-LPM/LRM approach. An alternative approach is to extend the Fourier transform as in [30], [31], higher dimensional Fourier transforms are applied to reconstruct spatial acoustic fields in mechanical systems. Although higher order Fourier transforms potentially enable the direct global identification of the frequency response function, the proposed frozen parameter approach avoid issues arising from dynamic scheduling. Second, in the line of research [32], [33], [34], time-varying systems are investigated using related problems. Essentially, this relates to the LPV case by taking only time as scheduling variable, see [5, Section 1]. As a result, the extrapolation properties of the model are much more restrictive, and the resulting model is not directly suitable for LPV controller synthesis algorithms.

In Sec. II, the considered class of LPV systems is described and the problem is formulated. In Section III, a novel approach is introduced for local parametric estimations of an LPV system by means of a 2D extension of the LPM. The potential of the proposed approach is illustrated on a case study, based on a wafer stage model, in Sec. IV. The work concludes with the obtained results and a discussion on ongoing work in Sec. V.

II. PROBLEM FORMULATION

A. Linear Parameter Varying Systems

Systems that exhibit parameter dependent behavior on non-stationary parameters, e.g., position or temperature, can be cast into an LPV framework. Consider the parameter depending state-space description [5, Chapter 1]

$$\dot{x}(t) = A\left(\theta(t)\right)x(t) + B\left(\theta(t)\right)u(t), \tag{1}$$

$$y(t) = C\left(\theta(t)\right)x(t), \tag{2}$$

with for single-input, single-output (SISO) systems, $u(t) \in \mathbb{R}^1$ an input vector, $y(t) \in \mathbb{R}^1$ a measured output, $x(t) \in \mathbb{R}^l$ the state vector with l the number of states and $\theta(t) \in \Omega_\theta$ the scheduling parameter restricted by the bounded set $\Omega_\theta \subset \mathbb{R}^p$ [5, Chapter 1]. This set limits the magnitude and rate of variation by $\theta_{\text{min}} \leq \theta(t) \leq \theta_{\text{max}}$ and $\dot{\theta}_{\text{min}} \leq \dot{\theta}(t) \leq \dot{\theta}_{\text{max}}$. By keeping $\dot{\theta}_{\text{max}}$ to be relatively small, second-order dynamics due to fast variations of the scheduling parameter variations are limited, [14], [10].

The trajectory of the scheduling parameter can also be a function of the state, $\theta(t) = f\left(x(t)\right)$, which implies that LPV systems can represent a specific class of nonlinear systems as presented in [5]. Throughout, it is assumed that θ can be frozen, which is a valid assumption in many local LPV approaches, including [14] [15] [16]. In this case, for each θ, (1), (2) generates an infinite family of LTI models.

B. Problem Definition

The main focus in this paper is on obtaining an improved frequency response estimation of systems that exhibit parameter dependent behavior using local LTI measurements. Local parametric estimation approaches, such as the LPM, allow for the use of transient measurements enabling fast measurements of multiple LTI systems. To achieve this aim, a 2D approach of the Local Polynomial Method (LPM) is proposed. Existing local approaches of obtaining nonparametric LPV models typically consist of multiple nonparametric LTI models, without any correlation between the independent models. Dedicated experiments are performed for specific values of $\theta(t)$, e.g., position or workpoint dependent behavior. As in typical (local) LPV identification approaches, the variation over the scheduling parameter is assumed smooth, which essentially relates to a bounded magnitude and rate of variation in LPV systems. By assuming smoothness over the scheduling band, as well as over the frequency axis, the LPM method can be extended over multiple dimensions exploiting the physical relation between the various LTI models. In Sec. III, a 2D-LPM approach is proposed based on the 2D Taylor series expansion. Nonparametric modeling based on local estimations including the scheduling direction, enforces a correlation between the various LTI measurements. For clarity of exposition, the presentation of the approach is limited to SISO systems and a single scheduling parameter, however this is no strict limitation as indicated in Sec. V.

III. APPROACH: 2D LOCAL ESTIMATION METHOD

In Sec. III-A, the LPM approach as developed in [35] is briefly presented, followed by the proposed extended 2D approach in Sec. III-B. Proper conditioning and dealing with edge effects in presented in Sec. III-C, followed by a variance analysis in Sec. III-D.

A. Local Polynomial Method

For a frozen scheduling parameter, i.e., $\theta(t) = \tilde{\theta}$, the LPM can be applied on the system for which the general approach is developed in [35] is presented in this section. The assumption is imposed that the dynamical behavior of the plant $G(k,\tilde{\theta})$ is smooth over the frequencies. By defining a local frequency band of $2n+1$ points,

$$r = -n, \dots, 0, \dots, n, \tag{3}$$

the output spectrum at the discrete Fourier transform (DFT) line $k+r$, i.e., the $(k+r)^{th}$ frequency bin with $(k+r) \in \mathbb{N}_+$, and scheduling parameter $\tilde{\theta}$ can be written as,

$$Y(k+r,\tilde{\theta}) = G(k+r,\tilde{\theta})U(k+r,\tilde{\theta})+T(k+r,\tilde{\theta})+V(k+r,\tilde{\theta}), \tag{4}$$

with $U(k,\tilde{\theta})$ and $Y(k,\tilde{\theta})$ the DFT of the input- and output signals respectively, with the DFT of signal x defined by,

$$X(k,\tilde{\theta}) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(nT_s,\tilde{\theta}) e^{-i2\pi nk/N}, \tag{5}$$

with N the total number of measured samples, and T_s the sample time in seconds with Nyquist frequency, $f_n = 1/2T_s$ in Hz.

The local plant is described by $G(k,\tilde{\theta})$, $T(k,\tilde{\theta})$ is a leakage term as a result of transient effects, and $V(k,\tilde{\theta})$ the DFT of a zero-mean (filtered) white measurement noise. Due to the use of arbitrary randomly distributed input signals, $U(k,\tilde{\theta})$, the dominant deterministic error source is typically a leakage term [35]. Hence, the inclusion of the estimation of $T(k,\tilde{\theta})$ in (4).

The plant $G(k,\tilde{\theta})$ and the transient $T(k,\tilde{\theta})$ are polynomial functions with continuous derivatives up to any order from which the Taylor series expansions are given by,

$$G(k+r,\tilde{\theta}) = G(k,\tilde{\theta}) + \sum_{l=1}^{Q} g_l(k,\tilde{\theta}) r^l, \tag{6}$$

$$T(k+r,\tilde{\theta}) = T(k,\tilde{\theta}) + \sum_{l=1}^{Q} t_l(k,\tilde{\theta}) r^l, \tag{7}$$

with g_l and t_l indicating the l^{th} derivative of the plant and transient term respectively and Q the order of the Taylor series approximation. Note that $Q \leq 2n$ to be able to describe a local estimation of the plant within the frequency band r as defined in (3). Substitution of the Taylor approximations in (6) and (7) respectively in (4), leads to the linear set of equations for each evaluated k,

$$Y(k+r,\tilde{\theta}) = \Theta(k,\tilde{\theta})K(k+r,\tilde{\theta}) + V(k+r,\tilde{\theta}), \tag{8}$$

where $Y(k+r,\tilde{\theta})$ represent row vector of dimensions $1 \times (2n+1)$ including the DFT of the measured output and Θ a $1 \times 2(Q+1)$ vector including the estimated parameters

$$\Theta = \big[\ G(k,\tilde{\theta}) \quad g_1(k,\tilde{\theta}) \quad \dots \quad g_Q(k,\tilde{\theta}), \\ T(k,\tilde{\theta}) \quad t_1(k,\tilde{\theta}) \quad \dots \quad t_Q(k,\tilde{\theta})\ \big]. \tag{9}$$

The regression matrix $K(k,\tilde{\theta})$ is constructed using the input data vector, which is in detail described in [35]. Based on this fundamental approach, an 2D extension is proposed in Sec. III-B.

B. 2D Local Polynomial Method

When multiple measurements are performed using multiple frozen scheduling parameters $\theta(t)$, a nonparametric LPV plant estimation can be obtained by repeating the approach of Sec. III-A multiple times. In this section, a novel approach that includes correlation between measured scheduling parameters is developed, based on the assumption that the changing dynamics over the trajectory of $\theta(t)$ are smooth.

An additional band is defined of dimension $2m+1$ over the scheduling parameter θ, according to

$$z = -m, \dots, 0, \dots, m. \tag{10}$$

A local 2-dimensional domain is defined for each frequency k and scheduling parameter θ by,

$$Y_{k,\theta}(r,z) = G_{k,\theta}(r,z)U_{k,\theta}(r,z) + T_{k,\theta}(r,z) + V_{k,\theta}(r,z), \tag{11}$$

describing a plane. Similar to the approach in the previous section, a Taylor expansion is computed for the plant and transient term. Since the plant is a function of a varying frequency and scheduling parameter, the 2D Taylor expansion is required, given by,

$$G_{k,\theta}(r,z) = \\ G(k,\theta) + \sum_{q=1}^{Q} \frac{1}{q!} \left(D_{\mathrm{r}} r^q + \sum_{i=0}^{q-1} D_{\mathrm{rz}} r^{q-i} z^i + D_{\mathrm{z}} z^q \right), \tag{12}$$

with,

$$D_{\mathrm{rz}} = q \frac{\partial^q G(k,\theta)}{\partial r^{q-i} \partial z^i}, \quad D_{\mathrm{r}} = \frac{\partial^q G(k,\theta)}{\partial r^q}, \quad D_{\mathrm{z}} = \frac{\partial^q G(k,\theta)}{\partial z^q}. \tag{13}$$

Note that $G(k,\theta)$ represents the parametric fit of the plant, D_{r} and D_{z} a derivative term over the frequency and scheduling direction respectively, and D_{rz} a derivative term over the cross-terms, i.e., the frequency and scheduling parameter.

In contrast to the plant estimation, the transient is estimated using a 1D Taylor series expansion as in (7). Since there is no correlation between the leakage components of the individual LTI measurements, smoothness can not be guaranteed over the scheduling parameter. Hence, for each individual LTI experiment, i.e., each frozen $\tilde{\theta}$ in the evaluated scheduling band $\theta + z$, an independent transient is estimated.

The order of the Taylor series expansion can chosen differently for the systems transfer function $G(k,\theta)$ and the leakage term $T(k,\theta)$, by Q and L respectively. Note that $L \leq 2n$, similar to the LPM approach. The selection of the order of Q is less intuitive since it involves the approximation of a plane, however the inequality,

$$\sum_{i=1}^{Q+1} i \leq (2n+1)(2m+1)-1, \tag{14}$$

should be satisfied. The reasoning behind the separation of the orders L and Q follows from the number of the to be estimated parameters leading from the Taylor series expansion, i.e., the leakage term only expands in the frequency direction due to the assumption that there is no correlation between the transients of different measurements. The plant approximation on the other hand is based on a plane and a function of both frequency and the scheduling parameter.

Using the approximations in (12) and (7), the local output DFT in (11) can be written as a linear function of the plant estimate $G(k,\theta)$, the leakage term $T(k,\theta)$, and the corresponding higher-order derivatives. As a result, for each combination of k and θ, a linear set of equations can be written in the form,

$$Y_{k,\theta}(r,z) = \Theta(k,\theta)K_{k,\theta}(r,z) + V_{k,\theta}(r,z), \tag{15}$$

with $Y_{k,\theta}(r,z)$ and $V_{k,\theta}(r,z)$ the output- and noise DFT as defined in (4), $\Theta(k,\theta)$ a vector including the unknown values $G(k,\theta)$, $T(k,\theta)$ and the corresponding derivatives following from the Taylor series expansion, and $K_{k,\theta}(r,z)$ the regression matrix. Note that the linear set of equations for each k and θ is similar to the result in (8). The possible combinations for r and z, as defined in (3) and (10), up to order Q are included in the left-upper triangular of,

$$\kappa(r,z) = \underline{r}\underline{z}^T = \begin{bmatrix} 1 & z^1 & \cdots & z^Q \\ r^1 & rz & & rz^Q \\ \vdots & & \ddots & \vdots \\ r^Q & r^Q z^1 & \cdots & r^Q z^Q \end{bmatrix}, \tag{16}$$

with, $\underline{r} = \begin{bmatrix} r^0 \ r^1 \ \cdots \ r^Q \end{bmatrix}^T$, and $\underline{z} = \begin{bmatrix} z^0 \ z^1 \ \cdots \ z^Q \end{bmatrix}^T$. Note that the order Q is chosen equal over the frequency and scheduling band, however this is not a strict requirement.

By stacking the left-upper triangular elements of the matrix $\kappa(r,z)$ in a single vector, i.e.,

$$\bar{\kappa} = \text{vec}(\kappa_{i,j}) \quad \forall\ i+j \leq Q+1, \tag{17}$$

with i and j indicating a row and column of κ respectively, the used stacking as used in matrix $K(\omega,\varphi)$ and subsequently $Y(\omega,\varphi)$, with $\omega = k+r$ and $\varphi = \theta + z$ is defined. By defining an additional vector,

$$\bar{\kappa}_L(r) = \begin{bmatrix} r^0 & r^1 & \cdots & r^L \end{bmatrix}^T, \tag{18}$$

corresponding to the leakage term, the regression matrix is defined by,

$$K(\omega,\varphi) = \begin{bmatrix} \bar{\kappa}(r,z) & \otimes & U(\omega,\varphi) \\ I_{(2m+1)} & \otimes & \bar{\kappa}_L(r) \end{bmatrix}, \tag{19}$$

with $I_{(2m+1)}$ indicating a identity matrix of dimensions $(2m+1)\times(2m+1)$.

Solving (15) for $\Theta(k,\theta)$ in a least square sense according to,

$$\hat{\Theta} = \arg\min_{\Theta} |Y(\omega,\varphi) - G(\omega,\varphi)U(\omega,\varphi) - T(\omega,\varphi)|^2, \tag{20}$$

and selecting the appropriate element, results in an accurate estimation of $G(k,\theta)$, which should be repeated for every k and θ.

As a final remark, it should be noted that the LPM approach described in Sec. III-A is a special case of the method developed in the current section. For the situation $m=0$, the exact approach as in [35] is recovered.

C. Least-Square Conditioning and Edges

In order to ensure a full rank of the regression matrix K, the number of rows should be smaller or equal to the number of columns, hence,

$$(2n+1)(2m+1) > L + 1 + \sum_{i=1}^{Q+1} i. \tag{21}$$

Note that the inequality sign $>$ is used, which is required for the computation of the output variances in (23) in Sec. III-D.

At the edges of the frequency grid, i.e., $k \leq n$ or $k \geq \frac{N}{2} - n$, the locally evaluated frequency band index r degenerates. By using an asymmetric frequency band around DFT line k, this problem can be overcome, see [35]. An alternative approach is by making use of the periodicity of the DFT over the frequency, as is done in [23]. The latter approach can not be used for the edges of the scheduling parameter since there is not periodicity, however an asymmetric windowing approach can be applied. In this work, the traditional LPM approach as described in [35], [21] is applied near the edges of the scheduling direction while the periodicity is exploited near the edges of the frequency axis.

D. Variance Analysis

Similar to the LPM methods, the output variance for SISO systems can be computed using the residuals of the least-square estimation as in (20), for each evaluated frequency and scheduling band, *i.e.*,

$$V(\omega,\varphi) = Y(\omega,\varphi) - \Theta(k,\theta)K(\omega,\varphi), \tag{22}$$

leading to the output variance,

$$\sigma_Y^2 = \frac{1}{q}VV^H, \tag{23}$$

with superscript H indicating the hermitian transpose and q a scaling following from (21),

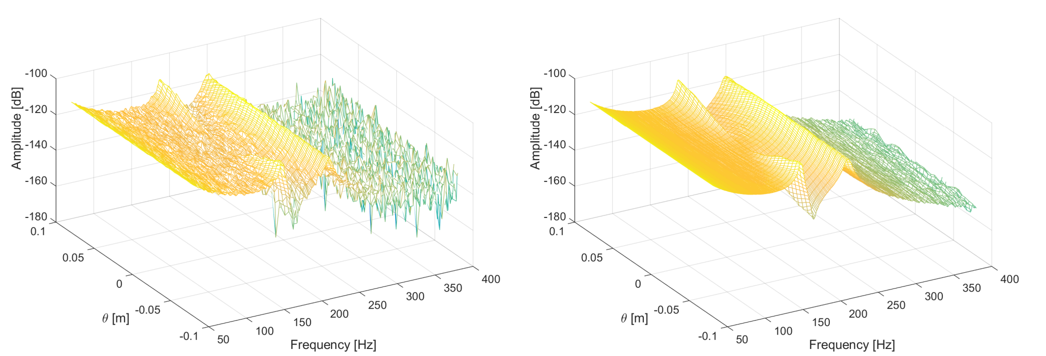
$$q = (2n+1)(2m+1) - \left(L + 1 + \sum_{i=1}^{Q+1} i\right). \tag{24}$$

Note that the inequality sign in (21) is required to guarantee a strictly positive scaling q to avoid infinite or negative variances. Assuming a noise free input signal $U(k,\theta)$, the general notation for the $\text{cov}\left(\text{vec}\left(G(k,\theta)\right)\right)$ is given by,

$$\sigma^2 = (\overline{UU^H})^{-1} \otimes \sigma_Y^2, \tag{25}$$

from which the variance is given by $\sigma_G^2 = \text{diag}(\sigma^2)$. For SISO systems, the computation of the variance reduces to,

$$\sigma_G^2(k,\theta) = \frac{\sigma_Y^2}{|UU^H|}. \tag{26}$$



(a) Result of the LPM as in [35] performed for each frozen θ.

(b) Full 2D-LPM as described in Sec. III.

Fig. 1: Plant estimations of the true model including 10% noise.

The variance is typically used as a quality measure for the plant estimation. In Sec. IV, a case-study is given for a system that satisfies the properties introduced in Sec. II, enabling the use of the 2D-LPM approach.

IV. CASE STUDY: A WAFER STAGE MODEL

A. System Description

To illustrate the potential of the proposed 2D-LPM approach, a numerical simulation is presented based on a wafer stage model, which is used for lithographic processing. The model is also used in [16] and [29] for local LPV modeling, and has also served as an application for the work in [26]. Given the application relevance, the modeling aim here is to shown enhanced nonparametric FRF quality with respect to the pre-existing methods. The system can be characterized by an actuated long- and short actuation stroke to position wafers using linear Lorentz motors, see, e.g., [36]. Based on the relative positions, the dynamical behavior of the actuated system varies, as is elaborately presented in [29]. The system is modeled based on the equations of motion in state-space as described in (1) and (2). The position dependency is modeled as a varying parameter θ in the system matrix $A(\theta)$ and the output matrix $C(\theta)$, while the input matrix B is independent of θ. Multiple frozen LTI FRF measurements are performed using random excitation signals, with a sample rate of 1 kHz. A zero mean, Gaussian distributed measurement noise is added with a variance of approximately 10% of the variance of the output signal.

Note that due to $A(\theta)$ and $C(\theta)$ being a function of the scheduling parameter θ, both the poles and zeros are parameter dependent, i.e., both mechanical resonances and anti-resonances are influenced by θ.

B. Simulation Results

Nonparametric FRF measurements are performed for all values of the scheduling parameter described by, $\theta = [0.1\ 0.105\ \cdots\ 0.3]\ [m]$, leading to a total of 41 local models. Only short measurements of 5 second are used, leading to 5000 samples for each measurement and a total measurement time of 205 seconds. The method proposed in Sec. III is performed and compared to the Local Polynomial Method as presented in [35]. The parameters used for both methods are provided in Table I. Note that Q, L and n can be given the same interpretation for both methods, however, the order of Q is chosen higher for the 2D-LPM approach in order to accurately estimate the full 2D-plane. The parameter m only appears for the 2D-LPM approach since the LPM approach identificaties each frozen LTI model independently without any correlation. Hence, the order L is chosen lower then Q. For a fair comparison, n and L are chosen equal for both methods. In Fig. 1, the resulting plant estimations for both approaches are provided. Both estimation approaches lead to plant estimates, however, close observation shows that the 2D-LPM approach leads to a more smooth surface. In Fig. 2, the average error from a Monte-Carlo simulation, i.e., 200 repetitions, is shown. This average error is used as a more reliable quantity, since the results of each simulation will be slightly different due to the stochastic nature of the excitation signal and the measurement noise. The resulting errors show that the 2D-LPM approach results in overall smaller bias errors than the normal LPM approach. It should be noted that additive measurement noise is added to the output. Observation of the variance, obtained from one single simulation, in Fig. 3 shows that the variance on the plant estimation is significantly lower then for the traditional LPM.

TABLE I: Identification parameters

Tuning parameters	Q	L	n	m
LPM	2	2	3	–
2D-LPM	5	2	3	4

V. CONCLUSIONS AND ONGOING RESEARCH

In this paper, a local parametric estimation approach is presented for fast and accurate nonparametric FRF identification for LPV systems. Simulations have shown an improved estimation compared to pre-existing results. The main method to enhance accuracy and speed is to fit local parametric models in higher dimensions. As a special case, in 1D the pre-existing LPM techniques are recovered.

The extension of this work towards vector valued scheduling parameters is immediate by an nD-LPM approach with n-1 scheduling parameters. In a research extensions for MIMO systems, nD-LPM/LRM approaches, multisine excitation signals, and physical applications will be considered.

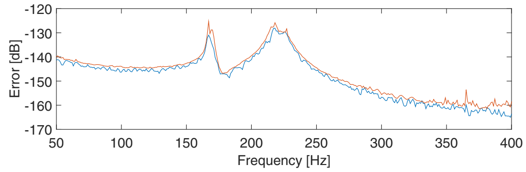


Fig. 2: Estimation errors of the estimated plant $\hat{G}$ with respect to the true plant G_{LPV}, averaged over all θ; the traditional LPM as in [35] (red), and the proposed 2D-LPM (blue).

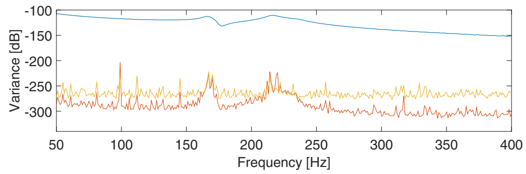


Fig. 3: Variance on the plant estimation using the 2D-LPM; the plant averaged over all θ (blue); the plant variance computed using (26) (red), and the LPM variance (yellow).

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