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On Identification-Related Uncertainty Structures

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Background

A key step in identification for robust control involves the selection of the uncertainty structure. LFT-based structures

$$\mathcal{P} = \left\{ P \middle| P = \mathcal{F}_u(\hat{H}(\hat{P}), \Delta_u), \Delta_u \in \mathbf{\Delta}_u \right\} \quad (1)$$

lead to the closed-loop performance bound

$$\mathcal{J}_{\mathrm{WC}}(\mathcal{P}, C) \leq \mathcal{J}(\hat{P}, C) + \underbrace{\sup_{\Delta_u \in \mathbf{\Delta}_u} \left\| \hat{M}_{21}\Delta_u (I - \hat{M}_{11}\Delta_u)^{-1}\hat{M}_{12} \right\|_\infty}_{\text{depends on uncertainty structure}} \quad (2)$$

Connecting identification and robust control

Robust control goal: determine

$$C^{\mathrm{RP}}(\mathcal{P}) = \arg\min_{C} \mathcal{J}_{\mathrm{WC}}(\mathcal{P}, C)$$

Connecting control and identification criterion: determine

$$\mathcal{P} = \arg\min_{\mathcal{P}} \mathcal{J}_{\mathrm{WC}}(\mathcal{P}, C^{\mathrm{exp}})$$

Controller C^{exp}:

- initial stabilizing controller
- iteratively update C^{exp} $\Rightarrow$ monotonic convergence

Key results

General LFT uncertainty structures

- including additive $\mathcal{P}^{\mathrm{ADD}}$, multiplicative, ...
- $\mathcal{J}_{\mathrm{WC}}(\mathcal{P}, C^{\mathrm{exp}})$ in (2) may be unbounded

Earlier result: Dual-Youla uncertainty structure $\mathcal{P}^{\mathrm{DY}}$ [1]

- leads to bounded $\mathcal{J}_{\mathrm{WC}}(\mathcal{P}, C^{\mathrm{exp}})$:

$$\mathcal{J}_{\mathrm{WC}}(\mathcal{P}^{\mathrm{DY}}, C^{\mathrm{exp}}) \leq \mathcal{J}(\hat{P}, C) + \sup_{\Delta_u \in \mathbf{\Delta}_u} \left\| \hat{M}_{21}\Delta_u \hat{M}_{12} \right\|_\infty$$

- Key problem: $\hat{M}_{21}$ and $\hat{M}_{12}$:
  - frequency dependent
  - multivariable

  } structured Δ_u necessary

Robust-control-relevant uncertainty structure $\mathcal{P}^{\mathrm{RCR}}$ [2]:

- connects size of Δ_u and criterion $\mathcal{J}_{\mathrm{WC}}(\mathcal{P}, C^{\mathrm{exp}})$

$$\mathcal{J}_{\mathrm{WC}}(\mathcal{P}^{\mathrm{RCR}}, C^{\mathrm{exp}}) \leq \mathcal{J}(\hat{P}, C) + \gamma, \quad \gamma = \sup_{\Delta_u \in \mathbf{\Delta}_u} \|\Delta_u\|_\infty$$

- scaling of uncertainty channels w.r.t. control criterion
  - frequency dependent
  - multivariable

  } unstructured Δ_u
- based on new connection between control-relevant and coprime factor identification

Performance guarantees:

1. $\mathcal{J}_{\mathrm{WC}}(\mathcal{P}^{\mathrm{ADD}}, C^{\mathrm{RP}}(\mathcal{P}^{\mathrm{ADD}})) \leq \mathcal{J}_{\mathrm{WC}}(\mathcal{P}^{\mathrm{ADD}}, C^{\mathrm{exp}})$
2. $\mathcal{J}_{\mathrm{WC}}(\mathcal{P}^{\mathrm{DY}}, C^{\mathrm{RP}}(\mathcal{P}^{\mathrm{DY}})) \leq \mathcal{J}_{\mathrm{WC}}(\mathcal{P}^{\mathrm{DY}}, C^{\mathrm{exp}})$
3. $\mathcal{J}_{\mathrm{WC}}(\mathcal{P}^{\mathrm{RCR}}, C^{\mathrm{RP}}(\mathcal{P}^{\mathrm{RCR}})) \leq \mathcal{J}_{\mathrm{WC}}(\mathcal{P}^{\mathrm{RCR}}, C^{\mathrm{exp}})$
   $\Rightarrow$ smallest upper bound

CVT experimental results

- 2 inputs u_1, u_2
- 2 outputs y_1, y_2
- control-relevant nominal model $\hat{P}$
- control goal: bandwidth of 6 [Hz]

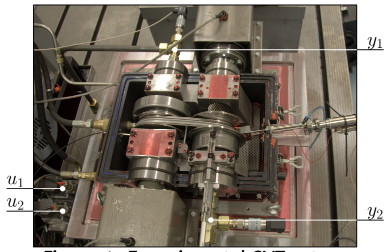

Figure 1: Experimental CVT setup.

Comparing uncertainty structures:

Results	$\mathcal{P}^{\mathrm{ADD}}$	$\mathcal{P}^{\mathrm{DY}}$	$\mathcal{P}^{\mathrm{RCR}}$
$\mathcal{J}(\hat{P}, C^{\mathrm{exp}})$	6.14	6.14	6.14
γ	1.73	0.55	0.60
$\mathcal{J}_{\mathrm{WC}}(\mathcal{P}, C^{\mathrm{exp}})$	∞	11.06	6.73
$\min_C \mathcal{J}_{\mathrm{WC}}(\mathcal{P}, C)$	3.63	3.20	2.50

Visualization

Visualization procedure:

- SISO: (1) is a Möbius transformation [1]
- MIMO: new approach using extensions of μ, see [3]

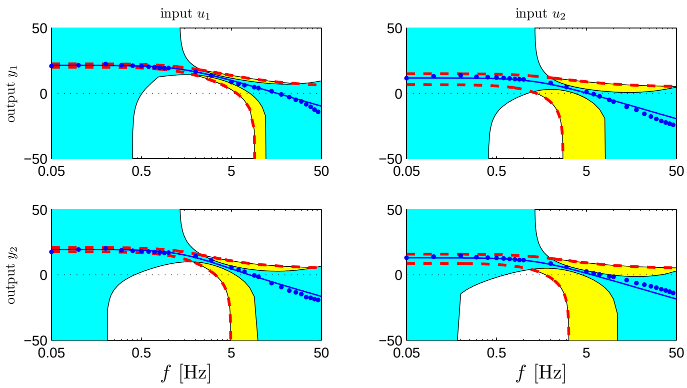

Figure 2: Uncertain CVT models. Blue dots: frequency response function estimate. Blue solid: $\hat{P}$. Dashed red and yellow colored: $\mathcal{P}^{\mathrm{ADD}}$. Cyan: $\mathcal{P}^{\mathrm{RCR}}$.

Observations:

- $\mathcal{P}^{\mathrm{ADD}}$ emphasizes low frequencies and $u_1 \mapsto y_1$
- $\mathcal{P}^{\mathrm{RCR}}$ emphasizes bandwidth region ($\approx$ 6 [Hz]) and balances uncertainty for different inputs and outputs

Conclusions and outlook

New uncertainty structure

- connects identification and robust control
- enables use of unstructured Δ_u
  - frequency-dependent and multivariable scaling
  - large-scale multivariable implementations
  - use of nonparametric uncertainty modeling [4] in view of robust performance

References

[1] S. G. Douma and P. M. J. Van den Hof. Relations between uncertainty structures in identification for robust control. *Automatica*, 41:439–457, 2005.
[2] T. Oomen and O. Bosgra. System identification for achieving robust performance, 2011.
[3] T. Oomen, S. Quist, R. van Herpen, and O. Bosgra, "Identification and visualization of robust-control-relevant model sets with application to an industrial wafer stage," in *Proc. CDC*, pp. 5530–5535, Atlanta, GA, USA, 2010.
[4] T. Oomen, C. R. Rojas, H. Hjalmarsson, and B. Wahlberg. Analyzing iterations in identification with application to nonparametric $\mathcal{H}_\infty$-norm estimation. In *Proc. IFAC WC*, pp. 9972 – 9977, Milan, Italy, 2011.