RESEARCH ARTICLE

Controlling Aliased Dynamics in Motion Systems?
An Identification for Sampled-Data Control Approach

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Sampled-data control systems occasionally exhibit aliased resonance phenomena within the control bandwidth. The aim of this paper is to investigate the aspect of these aliased dynamics with application to a high performance industrial nanopositioning machine. This necessitates a full sampled-data control design approach, since these aliased dynamics endanger both the at-sample performance and the intersample behavior. The proposed framework comprises both system identification and sampled-data control. In particular, the sampled-data control objective necessitates models that encompass the intersample behaviour, i.e., ideally continuous time models. Application of the proposed approach on an industrial wafer stage system provides a thorough insight and new control design guidelines for controlling aliased dynamics.

Keywords: Aliased dynamics, Continuous time system identification, Sampled-data control, Intersample behaviour, motion systems, mechatronics

1 Introduction

Despite the fact that modern control systems are generally implemented in a digital computer environment, the majority of such control sampled-data control systems are still being designed using approximations. These approximations deteriorate the achieved control performance. Two distinct approximate schemes can be roughly distinguished. In the first approach, a continuous time controller is designed for the continuous time system. The designed controller is then approximated by a discrete time controller for actual implementation. In the second approach, a discrete time model is constructed (either by discretising a continuous time model or identifying a discrete time system directly), followed by the design of a discrete time controller. The approximation in the second approach arises from ignoring the intersample behavior. Indeed, the performance of sampled-data control systems should generally be evaluated in the continuous time domain. In this respect, good discrete time performance only provides a necessary condition for good sampled-data performance, since it is not sufficient.

For feedback controlled systems with relatively high sampling frequencies, the approximation error can often be made negligible. However, there are important examples of high performance systems and control design methodologies where the approximation error is very large and leads to a severe deterioration of control performance. Examples include high-precision positioning systems (Oomen et al. 2007), repetitive control (Hara et al. 1990), and iterative learning control techniques, which are able to attenuate disturbances up to the Nyquist frequency (LeVoci and Longman 2004, Oomen et al. 2009).

The design of sampled-data controllers leads to several phenomena that are not encountered in traditional continuous time or discrete time control designs, including

i) sampling zeros,
ii) aliased disturbances, and

iii) aliased system dynamics.

System theoretic aspects of i) sampling zeros have been investigated in Åström et al. (1984), Weller et al. (2001). The potentially poor intersample behavior that may be induced by sampling zeros is investigated in Oomen et al. (2009). Poor intersample behavior induced by ii) aliased disturbances (Riggs and Bitmead 2013) is investigated in Oomen et al. (2007, 2011). However, the iii) aliasing of system dynamics, including aliased resonance phenomena in mechanical systems, has not been explicitly addressed in the literature. This is evidenced by, e.g., Preumont (2004, Page 278), where it is stated that "Aliasing is of course not acceptable and it is therefore essential to place an analog low-pass filter...". In the present paper, it will be shown that aliasing can in fact be effectively dealt with through an explicit sampled-data control. Note that this paper focuses on the aspect of aliasing in deterministic input-output data. The aspect of aliasing of stochastic noise that contaminates the measurements is described in, e.g., Goodwin et al. (2013).

Interestingly, recent results in Blachuta and Grygiel (2009, 2008) have revealed that classical anti-aliasing filters often have limited benefits from a stochastic perspective.

Direct sampled-data control design enables the design of a discrete time controller that performs optimally for a continuous time system by explicitly addressing the intersample behavior. In Yamamoto (1994), Bambi et al. (1991), Chen and Francis (1995), lifting techniques have been proposed that have lead to $\mathcal{H}_2$ and $\mathcal{H}_\infty$ optimal control designs. Despite their optimality in the sampled-data problem formulation, these systems norms are not directly suitable for control design purposes due to the underlying linear periodically time-varying nature of the underlying sampled-data system. This is evidenced by the non-unique definition of the frequency response function for such systems, see Lindgärde and Lennartson (1997) for an overview and Quinn and Williamson (1985), Goodwin and Salgado (1994), Freudenberg et al. (1995), Cantoni and Glover (1997), Yamamoto and Khargonekar (1996) for different definitions. In view of these design aspects, in Oomen et al. (2007) a design framework for sampled-data control is presented that iterates between sampled-data analysis and discrete time controller synthesis. However, such a design framework necessitates a model that encompasses the intersample behavior, ideally being a continuous time model (Rao and Unbehauen 2006, Garnier and Wang 2008).

Although recently important developments have been made in the theory and design of sampled-data control systems, the developed techniques require a model of the intersample behaviour. This necessitates an increased complexity of the models and hence imposes additional requirements on the identification techniques. Indeed, many systems, including mechatronic systems, are already equipped with actuators, sensors, and control hardware. In this case, system identification is typically the preferred method to obtain models since it is fast, inexpensive, and accurate. These models are often identified using measured signals from the feedback control loop, where a preliminary controller is implemented to stabilize the system or for safety regulations (Van den Hof and Schrama 1995). As a result, the model typically operates at the same sampling frequency as the discrete time controller and hence a fully discrete time problem formulation is obtained that neglects the important intersample response. Hence, typical identification for control methods do not deliver models that are suitable for sampled-data control.

The key contribution of this paper is a framework for system identification and sampled-data control that is applied to an industrial wafer stage system with aliased resonance phenomena. The presented techniques provide guidelines to controlling aliased resonance phenomena in sampled-data systems that goes beyond the present state of the art. The proposed approach employs models of the intersample behavior, i.e., ideally identified using a direct continuous time system identification approach (Rao and Unbehauen 2006, Garnier and Wang 2008, Pintelon and Schoukens 1997). In the present paper, such direct continuous time modeling techniques are also adopted, revealing that these methods correctly address the aliasing of resonance phenomena. However, significant discrepancies between the direct continuous time model and the discrete time frequency response function are observed for the considered application (see Sec. 6.3). Therefore, an alternative multirate approach is developed in this paper. The obtained model in
the multirate approach enhances correspondence to the slow sampled discrete time frequency response function, at the expense of a reduced resolution in the intersample response. Note that presented sampled-data control framework can be extended straightforwardly for use with direct continuous time identification methods.

Throughout, \( t_c \in \mathbb{R} \) denotes continuous time, whereas \( t \in \mathbb{Z} \) denotes discrete time. Continuous time signals are indicated by solid lines, whereas discrete time signals with sampling frequency \( f^l \) and \( f^h \) are indicated by dashed and dotted lines, respectively. Here, \( f^h = \frac{1}{h^h} F f^l, F \in \mathbb{N} \), and \( h^h \) denotes sampling time. In addition, \( \omega^h = 2\pi f^h \). Similarly, \( h^l := \frac{1}{F^l} \) and \( \omega^l = 2\pi f^l \).

Throughout, it is assumed that sampling is non-pathological (Chen and Francis 1995, Definition 3.2.1). Finally, to facilitate the presentation many results and examples are presented for SISO systems. Generalisation to the multivariable situation is conceptually straightforward.

2 Case study: Aliasing of dynamics

2.1 General problem setting

Consider the setup in Fig. 1. Herein, \( P \) denotes the continuous time system. The ideal sampler \( S \) is defined by

\[
S^l : y(t_c) \rightarrow \psi^l(t), \psi^l(t_i) = y(t_i h^l) \tag{1}
\]

In addition, the ideal zero-order-hold is defined by

\[
H^l : \nu^l(t) \rightarrow u(t_c), u(t_i h^l + \tau) = \nu^l(t_i), \tau = [0, h^l]. \tag{2}
\]

The discrete time system \( P^l \) is given by

\[
P^l = S^l P H^l. \tag{3}
\]

It is emphasized that (3) is not an approximation of \( P \) in the sense that it represents the relation \( \nu^l \rightarrow \psi^l \) exactly at the sampling instants.

2.2 Motivating example

In many motion and vibration control applications, resonance phenomena in \( P \) endanger closed-loop stability of the system. A well-known solution to designing controllers for resonant systems is by using so-called notch filters, see Preumont (2004, Sec. 4.5). Importantly, the design of such notch filters is often performed in continuous time (Preumont 2004, Sec. 4.5).

To introduce the concept of aliasing of resonance phenomena, the following example is considered.

Example 2.1 Consider the continuous time system

\[
P(s) = \frac{1}{\frac{1}{5^2} s^2 + \frac{2 \cdot 0.02}{2} s + 1} + 0.5 \frac{1}{\frac{1}{5^2} s^2 + \frac{2 \cdot 0.01}{5} s + 1}. \tag{4}
\]
which is implemented in the setup of Fig. 1 with sampling time $h^l = 1$. This leads to the system $P^l$ in (3). Bode diagrams of the systems $P$ and $P^l$ are depicted in Fig. 2. Interestingly, an aliased resonance phenomenon appears at approximately $0.2$ Hz that is not present in the underlying continuous time system $P$ at that particular frequency.

These observations are confirmed in Fig. 3, where the poles of $P$ in (4) and $P^l$ in (3) are plotted. Note that the poles of $P$ indicated by the diamond ($\diamond$) appear at a higher frequency compared to the ones indicated by a circle ($\circ$). In contrast, after discretisation in view of (3), the poles of $P^l$ indicated with the diamond appear at a lower frequency. Indeed, the frequency response of $P(s)$ is obtained by substituting $s = j\omega$, whereas for $P^l(z)$ this is obtained by setting $z = e^{j\omega h^l}$.

Example 2.1 reveals that aliasing arises in systems under sampled-data control. Traditional continuous time and discrete time control design only provide a partial perspective on the aspect of controlling these aliased dynamics:

- if a continuous time control design is pursued (based on either the frequency response function of $P$ or $P_d$ in Fig. 2), it is not clear how to deal with the aliased resonance at 0.2 Hz. Common guidelines include implementing anti-aliasing filters, e.g., Preumont (2004, Page 278). Note that the implementation of such filters is not always feasible in practice.
and the use of anti-aliasing filters often leads to a phase delay that is disadvantageous for achieving high control performance.

- if a discrete time control design is pursued, i.e., the direct design of a discrete time controller $C^l$ based on $P^l$, then standard results on discrete time systems reveal that the aliased resonance should be taken into account in a similar manner as the other resonance. For instance, the discrete time Nyquist test resorts to the frequency response function of $P_d$ in Fig. 2. However, this only provides a partial and approximate result, since the intersample response is completely ignored. In fact, cases have been reported where the intersample behavior is extremely poor despite of a perfect at-sample performance (De Souza and Goodwin 1984, Hara et al. 1990, Oomen et al. 2007).

2.3 Problem formulation and outline

The key question that this paper aims to address is how to deal with these aliased resonance phenomena in sampled-data feedback systems. Addressing this question necessitates a full sampled-data framework, i.e., by analyzing the interconnection of the continuous time system $P$ and the discrete time controller $C^l$.

This paper is organised as follows. In Sec. 3, the underlying mechanism of aliasing of system dynamics in Example 2.1 is presented. Next, in Sec. 4, a framework for system identification for sampled-data control is presented. Next, the sampled-data control design framework is presented in Sec. 5. The identification for sampled-data control framework is subsequently applied to a case study in Sec. 6 that addresses the aliasing of resonance phenomena in an industrial nano-positioning system. Finally, conclusions are presented in Sec. 7.

3 System-theoretic analysis of aliased system dynamics

To understand the underlying mechanism of aliased system dynamics in Example 2.1, let

$$P = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

be a minimal state-space realization of $P$. The following result provides a state-space description of $P^l$.

**Lemma 3.1:** Given a continuous time model $P$ with state-space realization $(A, B, C, D)$ in (5). Then, the ZOH discretisation $P^l$ in (3) is given by

$$P^l = \begin{bmatrix} A^l & B^l \\ C^l & D^l \end{bmatrix} = \begin{bmatrix} e^{Ah} & \int_0^{h} e^{\tau A} d\tau B \\ C & D \end{bmatrix},$$

The proof of Lemma 3.1 is well-known and can be found in, e.g., Chen and Francis (1995, Theorem 3.1.1). Next, Lemma 3.1 reveals that

$$\lambda(A^l) = e^{\lambda(A)h}$$

Next, for a certain $i$ let $\lambda_i(A) = \sigma_i + j\omega_i$. Then,

$$\lambda(A^l) = e^{h\sigma_i} e^{jh\omega_i} = e^{h\sigma_i} e^{j\omega_i(\omega_i + \frac{2\pi p}{h})}, p \in \mathbb{Z}.$$  

The aliasing of dynamics is now revealed by (6). To explain this, assume that $\lambda_i(A) = \sigma_i + j\omega_i$ is
an eigenvalue of \( A \) that is mapped to \( \lambda_i(A^t) \). Then, all eigenvalues \( \lambda_k(A) = \sigma_k + j\omega_k \) satisfying

\[
\sigma_k = \sigma_i \text{ and } \omega_k = \omega_i + \frac{2\pi}{h}p, \ p \in \mathbb{Z}
\]

lead to \( \lambda_i(A^t) = \lambda_k(A^t) \). Hence, the mapping from \( P \) to \( P^t \) in (5) and Lemma 3.1 is not injective and no unique inverse exists.

Returning to Example 2.1 reveals that a minimal state-space realization \( (A, B, C, D) \) of \( P \) leads to \( A \in \mathbb{R}^{4 \times 4} \). Next, the resulting eigenvalues of \( A \), i.e., \( \lambda_i(A), i = 1, 2, 3, 4 \) exactly correspond to the poles of \( P \) in (5) and Fig. 3 (left). Note that indeed two of the eigenvalues are strictly inside the primary frequency band, i.e., \( \Im(\lambda_i) < \frac{\pi}{h} = \pi \), whereas the other two eigenvalues are outside the primary frequency band. Next, observe from (6) that the relation between values in the primary frequency band (indicated in the left-hand plot of Fig. 3 by the magenta strip) map uniquely onto the unit disc in the discrete time plane (the right-hand plot in Fig. 3). However, the higher frequency bands (indicated in the left-hand plot of Fig. 3 by the green strips) also map onto the unit disk. Since there are eigenvalues inside the green strip, aliasing occurs.

Hence, Lemma 3.1 provides insight in the mechanism of aliasing based on state-space realizations. To connect this to the frequency response function results in Example 2.1, note that the Fourier transforms of \( \Psi^t(e^{j\omega h^t}) \) of \( \psi^t \) and \( Y(j\omega) \) of \( y \) in (1) and Fig. 1 are related by (Chen and Francis 1995, Lemma 3.3.1)

\[
\Psi^t(e^{j\omega h^t}) = \frac{1}{h^t} \sum_{k=-\infty}^{\infty} Y(j\omega - j\omega^t k).
\]

In addition, the Fourier transforms \( U(j\omega) \) of \( u \) and \( N^t(e^{j\omega h^t}) \) of \( \nu^t \) in (2) and Fig. 1 are related by (Chen and Francis 1995, Lemma 3.3.2)

\[
U(j\omega) = \mathcal{H}^t(j\omega)N^t(e^{j\omega h^t}), \quad \mathcal{H}^t(j\omega) = \frac{1 - e^{-j\omega h^t}}{j\omega}.
\]

The function \( \mathcal{H}^t(j\omega) \) in (8) is illustrated in Fig. 4. The following result connects the frequency response functions of \( P^t(e^{j\omega h^t}) \) and \( P(j\omega) \).

**Lemma 3.2:** The frequency response functions \( P^t(e^{j\omega h^t}) \) and \( P(j\omega) \) of the systems in Fig. 1 are related by

\[
P^t(e^{j\omega h^t}) = \frac{1}{h^t} \sum_{k=-\infty}^{\infty} P(j\omega - j\omega^t k)\mathcal{H}^t(j\omega - j\omega^t k).
\]

**Proof** First, observe that \( P^t(e^{j\omega h^t}) = \frac{\Psi^t(e^{j\omega h^t})}{N^t(e^{j\omega h^t})} \). Next, note that \( Y(j\omega) = P(j\omega)U(j\omega) \) and \( U(j\omega) = \mathcal{H}^t(j\omega)N^t(e^{j\omega h^t}) \). Combining this with (7) immediately yields (9). \( \square \)

Lemma 9 reveals how aliasing occurs. In particular, note that if \( \nu^t \neq 0 \), then \( u \) contains frequency components outside of the primary frequency band, since \( N^t(e^{j\omega h^t}) \) is periodic with period \( \frac{1}{h^t} \) and \( |\mathcal{H}^t(j\omega)| > 0 \) for \( \omega > \frac{\pi}{h^t} \). The continuous time system is thus excited at frequencies \( \omega > \frac{\pi}{h^t} \). Since \( P \) is assumed LTI, the same frequency components are present in the output \( y \). These high-frequency components in \( y \) are subsequently observed in the primary frequency band \( \omega < \frac{\pi}{h^t} \) through (7). Thus, if \( P \) has a large amplification at high frequencies, e.g., due to a resonance frequency, then the result will be observed at lower frequencies.
4 Identification for sampled-data control

4.1 Problem setting: towards a multirate approach

The goal in this paper is to investigate the control design for systems having aliased dynamics as is illustrated in Example 2.1. This investigation will be performed for an industrial nanopositioning system that immediately confirms practical relevance of the presented results. This necessitates a full analysis that comprises both the at-sampled response, i.e., using sampled signals available to the discrete time controller, and the intersample behavior. To address the full intersample behavior, such an approach requires a continuous time model of the system that has to be identified. Although such a continuous time model is highly desirable for the approach in this paper, an approximation is introduced to obtain a tractable and validatable approach, since

- measurement limitations prohibit the use of continuous time signals, e.g., for performance validation of the implemented controller. In this paper, a fast sampled process output will be used to validate the results, as these signals are readily available for the considered system.
- the identification of continuous time models, as is required by a full sampled-data control design approach, is not straightforward for certain classes of systems, including the experimental setup that is considered in this paper. Indeed, these systems generally have many high-frequency resonance phenomena (Hughes 1987) and the modeling of such systems is commonly performed using an intermediate step of frequency domain system identification (Pintelon et al. 1994), as is also considered in the related applications in Van de Wal et al. (2002), De Callafon and Van den Hof (2001), Oomen et al. (2013). As a result, frequency domain approaches to continuous time system identification, see Pintelon et al. (1994, 2000), Pintelon and Schoukens (2012) are considered.

In a frequency domain formulation, a continuous time model can be identified directly if band-limited excitation signals (Pintelon and Schoukens 2012) are employed or if access is available to the continuous time input and output and anti-aliasing filters can be implemented (Pintelon and Schoukens 1997). This is only approximately the case in the considered experimental setup in Sec. 6, since all continuous time signals are constructed using zero-order-hold interpolation and the implemented anti-aliasing filters will be insufficient to completely eliminate aliasing. This is further experimentally investigated in Sec. 6.3.2.

In view of the above considerations, the experimental validation and control design will be performed in a multirate setup. Therefore, the multirate control setup of Fig. 5 is considered throughout, which constitutes the best approximation of the sampled-data control problem Yamamoto (1994), Bamieh et al. (1991), Chen and Francis (1995) in view of measurement limitations. Herein, $\omega^h$ and $\zeta^h$ denote all exogenous inputs and outputs, respectively, and are sampled

\(^1\)The notation $\omega$ is used to represent both radial sampling frequencies and exogenous inputs, it follows from the context which one is referred to.
at the high sampling frequency $f^h$. The measured output that is available for the controller is given by $\psi^l$ and is sampled at the slow sampling frequency $f^l$. In addition, the manipulated control input is given by $\nu^l$ and also operates at the low sampling frequency $f^l$. The discrete time controller $C^l$ operates at the low sampling frequency $f^l$, whereas the standard plant $G^h$ operates at the high sampling frequency $f^h$. In addition, $C^l$ and $G^h$ are interconnected through the downsampler

$$S_d : e^h(t) \rightarrow e^l(t), \ e^l(t_i) = e^h(Ft_i), \ t_i \in t. \quad (10)$$

and multirate zero-order-hold

$$\mathcal{H}_u = I^F(z)S_u,$$

where the upsampler $S_u$ and zero-order-hold interpolator $I^F(z)$ are given by

$$S_u : u^l(t) \rightarrow \tilde{u}^h(t), \ \tilde{u}^h(t_i) := \begin{cases} u^l(\frac{t_i}{p}) & \text{for } \frac{t_i}{p} \in \mathbb{Z} \\ 0 & \text{for } \frac{t_i}{p} \notin \mathbb{Z}. \end{cases} \quad (11)$$

$$I^F(z) = \sum_{j=0}^{F-1} z^{-j}, \quad (12)$$

see also Oomen et al. (2007) for more details.

Note that the setup of Fig. 5 encompasses common control structures, including the feedback interconnection in Fig. 1. This is illustrated in Fig. 6, where all signals corresponding to Fig. 5 are indicated. Here, $G^h$ contains a model $P^h = S^h \mathcal{P} \mathcal{H}_u$. This motivates the identification of a fast sampled system. It is emphasized that the results in this paper apply equally to the sampled-data case, i.e., if the exogenous input $\omega^h$ and output $\zeta^h$ are continuous time signals. In fact, the sampled-data setup is recovered if $h^h \rightarrow 0$. 

Figure 5. Multirate standard plant setup.

Figure 6. Feedback interconnection of Fig. 1 in a multirate setting.
4.2 System identification approach

The key idea in this paper is to identify the model $P^h$ using measured data obtained at the sampling frequency $f^h$. This is indeed feasible in many practical applications, where the sampling frequency $f^l$ is upper bounded due to the constraint that the feedback controller has to compute a new control input in real time. In contrast, in many control applications it is possible to measure and store the error signal at a higher sampling frequency $f^h$, or implement a low-complexity controller that requires less computational effort and thus can be implemented at a high sampling frequency $f^h$. As a result, the measured signals at the high sampling frequency can be processed off-line by the identification algorithm.

The approach to identify $P^h$ consists of two steps.

i) Identify a frequency response function of $P^h_o(z_i)$ at sampling frequency $f^h$, $z_i = e^{j\omega_i^h}$. This will be described in detail in Sec. 6.2, see also Pintelon and Schoukens (2012) for further background information.

ii) Next, a parametric model is estimated using the criterion

$$J(\theta) = \sum_{i=1}^{N} \left\| W_i \left( P_o(z_i) - \hat{P}(z_i, \theta) \right) \right\|_2^2,$$

where $W_i$ denotes a user-chosen weighting, and provides freedom to identify e.g. a maximum-likelihood estimate (Pintelon and Schoukens 2012) or a control-relevant model (Schrama 1992, Gevers 1993, Oomen and Bosgra 2012, Oomen et al. 2013). See also De Callafon et al. (1996) for multivariable extensions. The model is parameterized as

$$\hat{P}(z, \theta) = \frac{b(z, \theta)}{a(z, \theta)} = \frac{b_{nb} z^{n_b} + b_{n_b-1} z^{n_b-1} + \ldots + b_0}{z^{n_a} + a_{n_a-1} z^{n_a-1} + \ldots + a_0},$$

where $n_a$ denotes the order and $n_b = n_a$ for a bi-proper model. The actual minimisation of (13) is then performed using the algorithm in, e.g., Oomen and Steinbuch (2014).

The result of this two-step procedure is a model $P^h$ that is suitable for control design. The aspect of controller design is investigated next.

Remark 1: In the case where an anti-aliasing filter $G_{AA}$ is implemented before the sampler $S$, then this is automatically incorporated in the model as $S^h G_{AA} P^h$. The resulting procedure automatically takes this into account. However, such a direct implementation of an anti-aliasing filter cannot eliminate the aspect of aliasing completely and often comes at the expense of a phase delay. Note that besides the input-output dynamics $S^h G_{AA} P^h$, the anti-aliasing filter $G_{AA}$ also affects the noise characteristics before sampling, see Goodwin et al. (2013), Blachuta and Grygiel (2009, 2008). Furthermore, the introduction of phase delay by anti-aliasing filter can be avoided by more general implementations, see Goodwin et al. (2013), Delgado et al. (2011).

5 Sampled-data control design: A frequency domain multirate approach

In Sec. 4.2, an approach to identify a model $P^h$ is estimated that will be an essential part of the generalised plant $G^h$ in Fig. 5. In this section, the standard plant $G^h$ will be further completed by specifying control design objectives. The controller design for many systems, including the experimental setup in Fig. 6, is performed in the frequency domain. Such controller designs can be either based on manual tuning, e.g., loop-shaping design of PID controllers, or using optimization algorithms, e.g., $H_\infty$-optimization. Note that frequency domain controller designs can equally well be done in the continuous time and discrete time domain, see Oomen et al. (2007). However, for the sampled-data and multi-rate setup of Fig. 5, pursuing a frequency
domain approach is not straightforward. This is investigated in detail next, followed by a control design procedure.

5.1 The setup in Fig. 5 is LPTV

The key aspect in controller design for the setup of Fig. 5 is the fact that the notion of frequency response function is not defined uniquely. This is confirmed by the following lemma.

**Lemma 5.1:** Let $\omega^h$ in Fig. 5 have discrete time Fourier transform $\Omega^h(e^{j\omega^h})$. Then, $\zeta^h$ has a discrete time Fourier transform given by

\[
Z^h(e^{j\omega^h}) = G^h_{11}(e^{j\omega^h})\Omega^h(e^{j\omega^h}) + G^h_{12}(e^{j\omega^h})\mathcal{I}_{ZOH}(e^{j\omega^h})Q^l(e^{j\omega^l}) + \frac{1}{F} \sum_{f=0}^{F-1} G^h_{21}(e^{j\omega^h(\omega - \frac{f}{F} \omega^h)})\Omega^h(e^{j\omega^h(\omega - \frac{f}{F} \omega^h)})
\]

where

\[
Q^l(e^{j\omega^l}) = (I - C^l(e^{j\omega^l})G^l_{22}(e^{j\omega^l})^{-1})C^l(e^{j\omega^l})
\]

and $G^l_{22} = S_d G^h_{22} \mathcal{H}_u$ as in Lemma 6.1 and $G^h$ is partitioned appropriately, i.e.,

\[
G^h = \begin{pmatrix} G^h_{11} & G^h_{12} \\ G^h_{21} & G^h_{22} \end{pmatrix}.
\]

**Proof** The result (14) follows directly from the fact that the Fourier transforms corresponding to (10) is given by $E^l(e^{j\omega^h}) = \frac{1}{F} \sum_{f=0}^{F-1} E^h(e^{jh^h(\omega - \frac{f}{F} \omega^h)})$. In addition, the Fourier transform corresponding to (11) is given by $\tilde{U}^h(e^{j\omega^h}) = U(e^{j\omega^l})$. Finally, (12) is used. $\Box$

Lemma 5.1 reveals that the setup in Fig. 5 has significantly different properties compared to standard discrete time or continuous time system interconnections. To see this, note that application of a single sinusoid in (14) reveals that $Z^h(e^{j\omega^h})$ contains multiple frequency components. This is due to the fact that the multirate system $\mathcal{L}_l(G^h, \mathcal{H}_u C^l S_d)$ in Fig. 5 is linear periodically time varying (LPTV). In the next section, new notions for frequency response functions for the setup of Fig. 5 are presented.

5.2 Frequency response functions for sampled-data systems

Lemma 5.1 reveals that the multirate setup in Fig. 5 is LPTV and hence the notion of frequency response function, as is defined for LTI systems, does not directly apply. In fact, several generalisations of frequency response functions for the setup of Fig. 5 can be defined. The signal spaces

\[
\mathcal{W}_D = \left\{ w(t) \left| w(t) = ce^{j\omega^h t}, \|c\|_2 < \infty \right. \right\}
\]

\[
\mathcal{W}_{MR} = \left\{ w(t) \left| w(t) = \sum_{f=0}^{F-1} cf e^{j(\omega - f\omega^h) h^h t}, \|c\|_2 < \infty \right. \right\}
\]
are considered. This leads to the following generalisations of frequency response functions.

Definition 5.2 Consider the setup of Fig. 5. Then,

- \( \mathcal{F}(e^{i\omega h}) = \left. \frac{\text{proj}_{W_D} \zeta^h}{\omega^h} \right|_{\omega^h \in W_D \setminus 0} \)
  
  \[ = G_{11}^h(e^{i\omega h}) + \left( \frac{1}{P} G_{12}^h(e^{i\omega h}) I_{ZOH}(e^{i\omega h}) \right) Q^r(e^{i\omega h}) G_{21}^h(e^{i\omega h}) \]

- \( \mathcal{P}(e^{i\omega h}) = \sup_{\omega^h \in W_D \setminus 0} \frac{\|\zeta^h\|_P}{\|\omega^h\|_P} \)

- \( \mathcal{R}(e^{i\omega h}) = \sup_{\omega^h \in W_{MR} \setminus 0} \frac{\|\zeta^h\|_P}{\|\omega^h\|_P} \)

Definition 5.2 represents different notions for frequency response functions for multirate systems. Herein, \( \mathcal{F} \) only reflects the fundamental frequency component of the LPTV system in Fig. 5 when a sinusoidal input is applied, hence it only partially reflects the intersample behavior. In contrast, \( \mathcal{P} \) reflects the power norm of the system response to sinusoidal inputs and hence captures the full intersample behavior. Finally, \( \mathcal{R} \) also encompasses the full intersample behavior but allows the input to be in a larger set \( W_{MR} \), i.e., the input is allowed to contain the same frequencies as the output \( \zeta^h \). Although \( \mathcal{R} \) provides the closest connection to the \( H_\infty \)-norm, i.e., its peak value equals the \( H_\infty \)-norm as in the LTI case, it may be of less value for control design, since the frequency response to an input signal in the class \( W_{MR} \) is generally hard to interpret.

Remark 2: The frequency response functions in (5.2) deal with the linear, periodically time varying nature of the system in Fig. 5. In contrast, the generalisations in, e.g., Rijlaarsdam et al. (2012), address non-linear time-invariant behavior of systems.

5.3 Proposed control design approach

In this paper, an \( H_\infty \)-optimal control design approach is considered, since for LTI systems (Skogestad and Postlethwaite 2005)

- the \( H_\infty \)-norm facilitates loop-shaping based control designs, and
- the \( H_\infty \)-norm is an induced norm and hence can be used to design robust controllers.

The advantages of those approaches are evidenced by many successful control designs for the considered class of systems in Sec. 6, see e.g., Schönhoff and Nordmann (2002), Steinbuch and Norg (1998).

In view of the frequency response analysis of Fig. 5 in Sec. 5.1 and Sec. 5.2, a direct multirate controller synthesis is not straightforward. On the one hand, for quantifying performance, e.g., through a loop-shaping based approach, the use of \( \mathcal{F} \) and especially \( \mathcal{P} \) appears useful. However, these functions are not exactly related to the \( H_\infty \)-norm, see also Cantoni and Glover (1997). On the other hand, the \( \mathcal{R} \) provides a connection to the \( H_\infty \)-norm but shaping this function is less useful from a performance perspective. In addition, dealing with model uncertainty is not straightforward in an LPTV setting. Indeed, using LTI model uncertainty, which appears to be most useful from an uncertainty modeling perspective, already introduces a significantly higher computational burden in the multirate framework of Fig. 5 (Dullerud 1996).

These observations motivate a iterative scheme that alternates between discrete time controller synthesis and multirate analysis, as suggested in Oomen et al. (2007) and summarized next.

Procedure 5.3 Alternate between the following steps until convergence

1. design a discrete time \( H_\infty \)-optimal controller, i.e., downsample the setup of Fig. 5 such that \( F = 1 \) and all signals operate at \( \omega^l \), and
2. analyse the intersample behavior using \( \mathcal{F} \) and \( \mathcal{P} \) and modify the weighting function design parameters in Step 1.
In Step 1, a downsampled model of $P^h$ is used, i.e., $P^l = S_d P^h H_u$, see also Lemma 6.1. In this case, all functions $F$, $P$, and $R$ are equal and hence (14) becomes the standard frequency response function for discrete time systems. In contrast, the intersample analysis in Step 2 resorts to a model $P^h$. Note that if the intersample behavior is not satisfactory in Step 2, the weighting function design parameters regarding the function $Q^l$ in (15) should be adjusted in Step 1.

Procedure 5.3 is employed in the forthcoming sections to design a controller that deals with aliased resonance phenomena. First, the modeling of $P^h$ and hence $P^l$ is considered, since these models are required for Procedure 5.3.

6 Application to controlling aliased resonance phenomena of a wafer stage

6.1 Experimental setup

The considered experimental setup in this paper is a wafer stage, see Fig. 7. Wafer stages are nano-positioning systems that are used in semiconductor manufacturing. These systems are controlled in all six motion degrees-of-freedom (i.e., three rotations and three translations), and have an operating range in the order of 1 m in the horizontal plane, and a typical accuracy in the order of 1 nm. This is achieved by laser interferometers that have sub-nanometer accuracy, in conjunction with a moving coil permanent magnet planar motor that provides contactless operation (Oomen et al. 2013). Due to the high accuracy of the laser interferometers, almost no pre-processing is done in the sense of anti-aliasing filters. Indeed, common implementations of such filters generally lead to phase lag.

The controller $C^l$ that will be designed in this section operates at a sampling frequency of $f^l = 1$ kHz. In contrast, since identification can be performed off-line and does not impose any real-time computational requirements, this can be performed at much higher sampling frequencies. In the used software implementation, it is possible to identify $P^h$ at a sampling frequency of $f^h = 5$ kHz, hence $F = 5$.

To investigate the aspect of controlling aliased dynamics, attention is restricted to the vertical translational direction, i.e., the identification and control design will be done for a SISO system. The remaining degrees-of-freedom are controlled using low-performance PID controllers. For a MIMO control design along the same lines, the reader is referred to Oomen et al. (2013). In the MIMO case, a high performance controller is designed for all six motion degrees-of-freedom, which is significantly more complex during both system identification and control design. Indeed, the resonance phenomena are inherently multivariable. This implies that a direct multivariable model has to be estimated, which is numerically and computationally significantly more challenging. Furthermore, the multivariable resonance phenomena have to be controlled using a multivariable controller, which significantly increases the complexity during the design and possible uncertainty modeling.

6.2 Frequency response identification: aliasing of dynamics

The first step in the system identification procedure is the identification of a frequency response function. Two frequency response functions are identified, both at the low sampling frequency $f^l$ and at the high sampling frequency $f^h$. An external excitation $\phi$ is applied at the system input, see Fig. 8 for the case with $f^l$. The measured signals are $\nu^l$ and $\psi^l$. Despite the advantages of periodic input signals (Pintelon and Schoukens 2012, Chapter 2), the signal $\phi$ is automatically selected in the industrial application as a realization of a stochastic process, being normally distributed white noise. Next, frequency response function estimates of the closed-loop transfer functions

$$
\begin{bmatrix}
P^l S^l \\
S^l
\end{bmatrix} = \begin{bmatrix}
P^l (1 + C^l P^l)^{-1} \\
(1 + C^l P^l)^{-1}
\end{bmatrix}
$$
are computed using spectral analysis, see, e.g., Ljung (1999, Section 6.4). It is well-known that these are essentially open-loop identification problems, since $\psi$ is noise-free. Since abundant data is available with large signal-to-noise ratio, frequency response function estimates with very small bias and variance are obtained. Next, a frequency response estimate of $P_l$ is obtained by performing the division $(P_lS_l)(S_l)^{-1}$. An analogous approach is applied for $P_h$. The results are depicted in Fig. 9. Note that these results correspond to the systems $P_l$ and $P_h$ in Fig. 6, respectively.

Inspection of Fig. 9 confirms that the influence of noise on the estimated frequency response function is small in the frequency range 10 – 5000 Hz. Below 10 Hz, the influence of the noise is large due to the fact that $S_l$ has a small magnitude at low frequencies. This induces errors in the estimation of $P_l$ due to the presence of a feedback controller, as is explained in detail in Heath (2001) and Pintelon and Schoukens (2001).

When comparing the results in Fig. 9, it appears that the 1 kHz identification contains a resonance phenomenon around 347 Hz. Interestingly, the identification result at 5 kHz does not contain a resonance phenomenon at 347 Hz. Instead, it contains a resonance at 653 Hz that ‘folds’ around the Nyquist frequency of 500 Hz. Hence, the identified result at 1 kHz contains aliased dynamics, which clearly resembles the situation in Example 2.1.

6.3 Parametric identification for sampled-data control

6.3.1 Identification at high sampling frequency $f^h = 5$ kHz

The sampled-data approach pursued in this paper necessitates a fast sampled model at 5 kHz to investigate the intersample behavior. Visual inspection of the frequency response function in Fig. 9 reveals the presence of many resonance phenomena. In fact, more resonance phenomena will become visible as the sampling frequency increases (Hughes 1987). Recent research (Oomen et al. 2013) has revealed that only a small number of resonance phenomena are relevant for control design. If a control-relevant identification criterion is formulated (Gevers 1993, Schrama 1992), then the identification approach automatically selects these dynamics and incorporates these in the identified model. However, in view of Lemma 5.1, the sampled-data system is LPTV. Hence, posing and solving a control-relevant identification criterion for sampled-data systems
is not immediate. This is also reflected by the fact that the sampled-data design involves an
iterative procedure, see Procedure 5.3, which is in sharp contrast to the approaches in (Gevers
1993, Schrama 1992) where the control design problem is posed as a single optimisation problem.

In view of the difficulties associated with control-relevant identification for sampled-data con-
trol, a high-order model is estimated using the criterion (13). Recall from Sec. 6.2 that due to
a high signal-to-noise ratio and long experiment time, the variance of the identified frequency
response function is negligible in the frequency range 10−5000 Hz. In a first step, the frequencies
below 10 Hz are excluded from the criterion (13). Next, to ensure a good fit that matches the
identified frequency response functions for all frequencies, i.e., both poles and zeros, a relative
error weighting is adopted, i.e., $W_i = \frac{1}{|\hat{P}_h(e^{j\omega_i})|}$ in (13). By iteratively increasing the model
order, a 60th order model appears to be sufficiently flexible to accurately fit all the resonance
phenomena, including the frequency range beyond 1 kHz, where very many resonance phenom-
ena are present. Due to the high signal-to-noise ratio and long experiment time, the variance of
the estimated parameters is sufficiently small despite the high order of the estimated model. The
resulting model is depicted in Fig. 10 and denoted $\hat{P}_{h60}$. From visual inspection it is observed
that the model accurately matches the high-frequency resonance dynamics. It is emphasised
that such a high model order may not be necessary for high performance control. In fact, it will
be shown in Sec. 6.3.2 that a much lower order is required for an accurate model at the low
sampling frequency $f^l$. However, as is argued above, at present no systematic control-relevant
identification approach exists for sampled-data control.

6.3.2 Identification at low sampling frequency $f^l = 1$ kHz

Downsampling approach

Besides the identification of a model of $\hat{P}^h_{60}$ at $f^h = 5$ kHz in Sec. 6.3.1, the actual controller
synthesis, i.e., Step 1 in Procedure 5.3, relies on a model of $P^l$ that operates at a sampling
frequency $f^l = 1$ kHz. Two approaches are considered in this section.

First, the identified model $P^h_{60}$ in Sec. 6.3.1 is used and downsampled to a lower sampling
frequency. The rationale behind this approach is that the identification is performed over a larger
frequency grid. As a result, more data is used in the identification, which is typically useful from
a statistical perspective. In addition, the resonance phenomena have larger magnitude and are
less densely spaced prior to downsampling, which generally leads to improved fitting accuracy.
Furthermore, this does not affect the variance, see Pintelon et al. (1996).
Figure 10. Identified frequency response function in z-direction (solid blue) and identified 60th order model $\hat{P}_{h60}$ (dashed red). Both models have sampling frequency 5 kHz.

**Lemma 6.1:** Let a model $P^h$ operate at sampling frequency $f^h$ and have state space realization

$$P^h = \begin{bmatrix} A^h & B^h \\ C^h & D^h \end{bmatrix}. $$

Then a state-space realization of the downsampled system $P^l$, see Fig. 6 and (3), is given by

$$\begin{bmatrix} A^l & B^l \\ \star & \star \end{bmatrix} = \begin{bmatrix} A_{d,h}^p & B_{d,h}^p \\ 0 & I \end{bmatrix}^F, \quad \begin{bmatrix} C^p_d \\ D^p_d \end{bmatrix} = \begin{bmatrix} C_{d,h}^p \\ D_{d,h}^p \end{bmatrix},$$

where $\star$ denotes a matrix entry that is not used in further computations.

**Proof** Follows directly from successive substitution of the state equation $\xi^h(t+1) = A^h\xi^h(t) + B^h\nu^h(t)$ and $\nu^h(t+f) = \nu^l(t/F)$, $f = 0, \ldots, F-1$.

Lemma 6.1 is applied to the model $\hat{P}_{h60}$, leading to a 60th order model denoted $\hat{P}_{l60}$. The results are depicted in Fig. 11. Interestingly, the downsampling model $\hat{P}_{l60}$ also has a resonance at approximately 349 Hz that was not present in the original model $\hat{P}_{h60}$. Hence, Lemma 6.1 takes sampling of system dynamics into account as is required.

Direct continuous time approach

Unfortunately, the fit of the aliased resonance is less accurately modeled at sampling frequency $f^l$ when compared to the original model $\hat{P}_{h60}$, as can clearly be observed from Fig. 11. A possible explanation is that the system exhibits small parasitic nonlinear effects. Indeed, note that if the discrete time frequency response function $P^l$ is identified, a signal $\nu^l(t)$ in Fig. 6 is generated. This signal is constructed using $H^l$ in Fig. 6. The frequency response of $H^l$ resembles the Bode diagram in Fig. 4. Hence, high-frequency components are reduced in amplitude when compared to the direct excitation at the sampling frequency $f^h$. Interestingly, in Smith (1998), similar observations have been reported in a comparison between open-loop and closed-loop identification of a related system. Note that an LTI anti-aliasing filter has not caused the discrepancy in Fig. 11, since from an identification perspective the anti-aliasing filter is identified as part of the LTI system.

To further investigate (1) the discrepancy in Fig. 11, and (2) the high model order equal to 60, a direct continuous time approach is adopted. Here, a frequency domain continuous time identification approach is employed, where a frequency response function with band-limited excitation signals is approximated. The resulting frequency response function is depicted in
Fig. 11. Identified frequency response function in z-direction (solid blue), downsampling 60th order model $\hat{P}_l^{60}$ (dashed red), and identified 6th order model $\hat{P}_l^6$ (dash-dotted green). All models have sampling frequency 1 kHz.

Fig. 12. Notice the large difference in phase when comparing these results to Fig. 10. Next, a 12th order continuous time model $\hat{P}(s)$ is identified that fits the dominant resonance phenomena. These resonance phenomena are the physical resonance modes and are known to contain no aliasing phenomena. The identified 12th order continuous time model $\hat{P}(s)$ is extended by a computational delay of $1.6 \cdot 10^{-4}$ s, which has been identified separately, see also Pintelon and Schoukens (2012, Section 8.5) for details. The discretised model at $f_l = 1$ kHz that includes the computational delay is finite dimensional and of 13th order, see Åström and Wittenmark (1990, Sec. 3.2) for an explanation on the resulting model order. The resulting discretised model is depicted in Fig. 13.

The following conclusions are drawn from the resulting discretised model in Fig. 13. First, the direct continuous time approach correctly predicts the aliasing of resonance phenomena and results in a slightly better fit of the resonance at approximately 349 Hz. However, the improvement compared to the results using the downsampling approach in Fig. 11 is only marginal. Note that this aliased resonance phenomenon is of key importance, since it clearly endangers stability as is shown in Fig. 15 in Sec. 6.4.1. Note also that the continuous time fit is less accurate in other frequency ranges and misses several other resonance phenomena when compared to the 60th order model in Fig. 11. Partially, this is caused by a significantly lower order of the model. Although alternative direct continuous time models may significantly enhance the quality, the aim in the present paper is to investigate the role of aliased resonance phenomena. Therefore, for analysing the intersample behavior, the 60th order in Fig. 11 is used.

Next, a discrete time approach is investigated for obtaining an accurate model for controller synthesis.

**Discrete time identification**

Motivated by the discrepancy in Fig. 11 and Fig. 13, and the relatively high order of the model $\hat{P}_l^{60}$, a second approach is pursued to obtain a model of $P_l$. Herein, the identification approach in Sec. 4.2 is used to directly fit a model to the identified frequency response function at $f_l = 1$ kHz in Fig. 11. The resulting 6th order model is also depicted in Fig. 11 and is denoted $\hat{P}_l^6$. From visual inspection of 11, it appears that the model $\hat{P}_l^6$ is a significantly better fit of the aliased resonance phenomenon at 347 Hz compared to the model $P_l^{60}$. 
6.4 Sampled-data controller design and analysis

6.4.1 Step 1: Controller design and at-sample analysis

Next, a controller is designed using Procedure 5.3. In Step 1, an $H_\infty$-optimal controller is synthesized. The controller is designed using the loop-shaping weighting functions as proposed in Steinbuch and Norg (1998) and Van de Wal et al. (2002), in conjunction with the $w$-plane approach in Oomen et al. (2007, Sec. 5.1).

The resulting $H_\infty$-optimal controller is denoted $C_1^f$ and is shown in Fig. 14. The resulting controller roughly has the shape of a PID controller with high-frequency roll-off and achieves a bandwidth of approximately 65 Hz. A Bode diagram is shown in Fig. 14. Interestingly, the resulting controller has a notch filter at approximately 347 Hz, which is exactly the frequency where the aliased resonance phenomenon appears.

To investigate the aspect of aliased resonance dynamics, it may be argued that the resonance phenomenon at 347 Hz is not present in the physical system and hence does not have to be attenuated. In fact, typical continuous time control design approach as in Preumont (2004) will not include a notch at that frequency, since in the underlying system the resonance is at 653 Hz. To illustrate the effect of not suppressing the aliased dynamics by means of a notch filter, the
Figure 14. Controller $C_l^1$ ($\mathcal{H}_\infty$-optimal controller) (solid blue), Controller $C_l^2$ ($\mathcal{H}_\infty$-optimal controller with notch filter removed) (dashed red). Both controllers have sampling frequency 1 kHz.

Figure 15. Nyquist diagram corresponding to loop-gain $PC$ with Controller $C_l^1$ (solid blue), and $C_l^2$ (dashed red). All models and controllers have sampling frequency 1 kHz.

local notch filter is removed using a modal reduction technique. The resulting controller is called $C_l^2$ and is depicted in Fig. 14.

An discrete time analysis, i.e., solely at the sampling frequency $f^l$ leads to the Nyquist diagram of the loop-gains in Fig. 15. Interestingly, both controllers have a phase margin of $29^\circ$. However, the gain margin reduces from 7.58 dB for $C_l^1$ to 0.593 dB for $C_l^2$. This implies poor robustness properties for $C_l^2$. These observations are corroborated by the sensitivity function $\frac{1}{1+P_lC}$ in Fig. 16, revealing a large peak for $C_l^2$, implying a poor modulus margin. In addition, the sensitivity function exactly describes the at-sample performance. This implies that controller $C_l^2$ is already expected to have a significantly worse at-sample behaviour compared to $C_l^1$. Finally, the process sensitivity functions $\frac{P_l}{1+P_lC}$ are depicted in Fig. 18. These results confirm the earlier conclusions in the sense that $C_l^2$ leads to worse at-sample attenuation of disturbances at the system input when compared to $C_l^1$. However, these observations do not provide any guarantees whether $C_l^1$ actually leads to better performance, since the intersample behaviour has been completely ignored so far.
6.4.2 Step 2: Intersample analysis

The controllers designed in Sec. 6.4.1 have been designed at a sampling frequency \( f^I \), thereby completely ignoring the intersample behaviour. This step is of crucial importance, since several cases have been reported (see Sec. 1) where controllers deteriorate the performance despite of an almost perfect at-sample response. Therefore, Step 2 in Procedure 5.3 is performed.

To anticipate on the experiments that will be performed in Sec. 6.5, the function \( \mathcal{P} \) is computed using the setup in Fig. 17. In particular, the reference signal is set to zero and disturbances on the fast sampled system input are considered. The exogenous output is selected as the system output \( \psi^h \).

Figure 16. Bode magnitude diagram corresponding to Sensitivity function \( \frac{1}{1 + P_l C_1} \) (solid blue), and \( \frac{1}{1 + P_l C_2} \) (dashed red). All models and controllers have sampling frequency 1 kHz.

Figure 17. Setup for computation of \( \mathcal{P} \) in Definition 5.2.

Figure 18. Intersample analysis: \( \mathcal{P} \) (see Definition 5.2) with \( C_1 \) (solid blue), \( \mathcal{P} \) with \( C_2 \) (dashed red).
The resulting multirate frequency response function $P$ is depicted in Fig. 18. Of particular interest is the function $P$ evaluated at frequencies 347 Hz and 653 Hz. Note that at 347 Hz, the system output does not show a significant increase in terms of power of the output compared to neighbouring frequencies. This is in sharp contrast to the function $\frac{P_{l6}}{1+P_{l6}C}$ that has a large peak. Instead, $P$ has a peak around 653 Hz. Interestingly, controller $C_{l1}$ has a significantly smaller peak compared to $C_{l2}$. This implies that a sinusoidal excitation of 653 Hz results in a smaller power of the output for $C_{l1}$ compared to $C_{l2}$, and hence better intersample behavior. This implies that the improved at-sample behavior of $C_{l1}$ in Sec. 6.4.1 also leads to improved intersample behavior.

6.5 Implementation

To confirm the observations in Sec. 6.4, both controllers are implemented on the wafer stage system. During the experiment, stand-still errors are measured, i.e., the reference $r$ in Fig. 1 is set to zero and the control system should attenuate environmental disturbances that affect the system.

The results are depicted in Fig 19 and Fig. 20. It can directly be observed that $C_{l1}$ achieves high performance, both in terms of the at-sample response and the intersample behavior, with an amplitude of the measured error smaller than 6 nm. In contrast, controller $C_{l2}$ leads to a significantly worse performance, leading to a maximum error of 25 nm. Note that the intersample response using controller $C_{l2}$ does not contain a 347 Hz component as the discrete time analysis in Sec. 6.4.1 (in particular Fig. 16) might indicate. Instead, it contains a dominant frequency component at 653 Hz, which is the frequency where the underlying resonance phenomena occurs in the continuous time system. This aspect is especially clear from the cumulative power spectral densities in Fig. 20. Note that this is difficult to predict from the resulting $P$ in Fig. 18, since this function only considers the power norm of the system response. One approach to investigate the frequency content of the output using the techniques in this paper is to also generate the function $\mathcal{F}$.

6.6 Discussion on the experimental results

Concluding, both from a discrete-time (i.e., at-sample) and sampled-data (i.e., both at-sample and intersample) perspective aliased system dynamics should be considered as usual during control design, i.e., a for motion system a notch filter should be placed in a similar fashion as non-aliased dynamics. Note that a continuous time control design with a posteriori controller
discretisation is generally inferior to both sampled-data and discrete time control designs, as these will not automatically attenuate the aliased resonance phenomenon. In other words, these will deliver poor performance that is similar to controller $C_{l2}$. The enhanced performance of controller $C_{l1}$ compared to $C_{l2}$ can be understood by analyzing (14). Indeed, note that $Q_{l}$ in the setup of Fig. 17 corresponds to $Q_{l} = \frac{C_{l}'}{1 + P_{l}C_{l}}$. Hence $Q_{l}$ is the only term that actually depends on $C_{l}$ in the computation of the multirate frequency response functions in Definition 5.2. Clearly, due to the presence of the notch filter in $C_{l1}$, $Q_{l1}$ is smaller at 347 Hz, and since $G_{h11} = 0$, this leads to a smaller intersample response compared to $C_{l2}$.

Remark 3: In certain situations, resonance phenomena in motion control are dealt with by ‘inverting’ the notch filter, i.e., increasing the locally gain of the controller instead of decreasing it. This can only be done if the phase of the resulting loop-gain has positive real part, in which case $\left|\frac{1}{1 + P_{l}C_{l}}\right| < 1$ (see, e.g., Oomen et al. (2013, Fig. 16-17) for an example). However, from the perspective of the results in this paper, such an approach should be followed with caution. Indeed, the above analysis regarding $Q_{l1}$ reveals that this may potentially lead to poor intersample behavior, e.g., as in Oomen et al. (2007).

Remark 4: The intersample analysis in Fig. 18 clearly reveals the poor intersample behavior associated with $C_{l2}$. However, it is remarked that the actual results may be more severe than predicted by $P$ due to the mismatch in Fig. 11. Hence, the accuracy of $P$ may be improved by enhancing the correspondence between the downsampled model and the identified frequency response function and model at the low sampling frequency $f_{l}$.

Remark 5: The obtained results shed light on the required order of the models that are identified in Sec. 6.3.1 and Sec. 6.3.2. Indeed, a 6th order model is obtained at the sampling frequency $f_{l}$, which accurately represents the aliased resonance phenomenon at 349 Hz. This resonance is essential for obtaining the notch in $C_{l1}$, see Fig. 14. Indeed, otherwise the standard PID type controller $C_{l2}$ would have been obtained. This immediately reveals the advantages of the proposed control design framework, since such a PID controller would lead to the disastrous performance associated with $C_{l2}$ in Fig. 19 (bottom). Note that the 60th order model only is used for intersample behavior analysis, and does not directly affect the order of the designed controller. Still, a lower order of the fast sampled model is preferable, provided that it can be identified in a control-relevant manner. See also Sec. 6.3.1 and Sec. 7 for a further explanation.
7 Conclusions

This paper has addressed a case study in sampled-data control design, revealing the importance of a full sampled-data framework instead of continuous time or discrete time approximations. A new system identification and sampled-data control framework is proposed that effectively deals with the digital controller implementation, including aliasing and intersample behavior. Thus, the application motivates the need for a specific continuous time system identification approach.

Although a full sampled-data approach is essential for high control performance, the approach induces severe requirements on the underlying system identification approach. In particular, dealing with sampled-data aspects requires dealing with the system dynamics over a much larger frequency range and this typically leads to complex high-order dynamics. In this paper, a multirate approach has been adopted to deal with measurement limitations and deal with the resulting complexity of the model. The approach also enables the identification of continuous time models from frequency response data, provided that an appropriate frequency response function is available, e.g. using a band-limited assumption on the excitation signals (Pintelon et al. 1994).

The case study in this paper has lead to important guidelines for control system design. It has been shown both theoretically and by means of experiments that aliased resonance phenomena can effectively be controlled using an appropriate sampled-data control design approach. In particular, notch filters can be used effectively to locally reduce the controller gain, hence from a discrete time and sampled-data control perspective no distinction should be made for aliased dynamics. From standard discrete time system theory, it can be easily understood that the proposed control approach leads to good discrete time at-sample behavior. The key contribution lies in a solid multirate analysis of the intersample behavior using new notions of frequency response functions. These results have shown that controlling the aliased resonance phenomena as proposed also enhances the intersample response. Note that this is by no means trivial, since many cases have been reported in the literature where pursuing good at-sample behavior goes at the expense of a poor intersample behavior.

Regarding continuous time controller designs, it is remarked that opposed to, e.g., (Preumont 2004, p. 278), aliasing can effectively be dealt with provided that a suitable sampled-data control design is pursued as is done in the present paper. Indeed, a continuous time control design with a posteriori discretisation generally is incapable of generating a notch at the alias frequency, and hence is clearly outperformed by the proposed sampled-data control design approach.

Ongoing research focuses on the following aspects:

- Combining the iterative steps in Procedure 5.3 to a single non-iterative step.
Refining the identification criterion (13) to address sampled-data control-relevant objectives to extend the results of Oomen et al. (2013), Oomen and Bosgra (2012) to the multirate case, see Sec. 6.3.1 for a discussion on this aspect.

Addressing the aspect of model quality (including variance) in the multirate frequency response functions in Definition 5.2.

Enhancing the continuous time model identification step and address the full continuous-time intersample behavior in the presented framework. In particular, in this paper, discrepancies between fast sampled/continuous time models and their slow sampled discretisation were found. The cause of these discrepancies has to be investigated and alternative identification methods may be considered, see, e.g., Young (1998, Sec. 3), where accurate results for the vibration modes of a cantilever beam are obtained through an alternative time domain continuous time identification approach. An additional important advantage associated with the use of continuous time models is that the model can be discretised at different sampling frequencies. In particular, using the results presented here, this allows to choose the best sampling frequency in the case of aliased resonance phenomena.

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Experimental investigation of the model discrepancies in Sec. 6.3.2.

Dealing with the complexity induced by the sampled-data problem formulation. In particular, the increased sampling frequency reveals higher order dynamics. This leads to a numerically challenging identification problem. In particular, to obtain the high order model in Sec. 6.3.1, the numerically reliable approach in Oomen and Steinbuch (2014), see also Bulthoo et al. (2005), is adopted. Ongoing work focuses on extending these results to enhance efficiency, see Van Herpen et al. (2012).

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