Background

Developments in robust control design methodologies have resulted in identification techniques that deliver
- control-relevant models \([1][2]\)
- (normalized) coprime factors \([3][4]\).

The only reason to consider coprime factors is to represent model uncertainty. However, present model uncertainty coordinate frames do not transparently connect to the control criterion \(\Rightarrow\) not control-relevant \(\Rightarrow\) conservative control design.

Aim

Transparencyly connect:
- identification of uncertain plant set \(\mathcal{P}(\hat{P}, \Delta)\):
  - consisting of model \(\hat{P}\) and model uncertainty \(\Delta\)
  - such that true plant \(P_\circ \in \mathcal{P}(\hat{P}, \Delta)\).
- control criterion \(J(P, C) = \|WT(P, C)V\|_\infty\):
  - \(T(P, C) = [P] [I (C + CP)^{-1} |C - I]\)
  - \(W, V\) control weighting filters.

Key idea:
- construct a suitable coordinate frame for representing model uncertainty \(\Delta\)
- by exploiting the unexplored freedom in coprime factor realizations \(\{\bar{N}, \bar{D}\}\), where \(P = ND^{-1}\).

Nominal model identification

Lightly damped flexible structures with parasitic dynamics:
- identification of full-order models illusive
- control-relevant identification \(\Rightarrow\) reliable models.

However, involved control-relevant identification criterion

\[
\| W \left( T(P_\circ, C_{\exp}) - T(\hat{P}, C_{\exp}) \right) V \|_\infty, \quad (1)
\]

invariant under varying coprime factor realization \(\{\bar{N}, \bar{D}\}\)!

Result: 4-block problem (1) can be recast as 2-block problem

\[
\| W \left( D_{\bar{N}} N_{\bar{D}} - \bar{N} \bar{D} \right) \|_\infty, \quad (2)
\]

where \(\{N_\circ, D_\circ\}\) and \(\{\bar{N}, \bar{D}\}\) are
- novel, unique, and control-relevant coprime factors
- generally not normalized.

Robust-control-relevant coordinate frame

For some \(P \in \mathcal{P}(\hat{P}, \Delta)\) and any uncertainty structure
- \(J(P, C_{\exp}) = \|M_{22} + M_{21} \Delta (I - M_{11} \Delta)^{-1} M_{12}\|_\infty\).

For any Dual-Youla-Kučera uncertainty structure \([5]\)
- \(J(P, C_{\exp}) = \|M_{22} + M_{21} \Delta_{12}\|_\infty\).

Stronger result: using control-relevant coprime factors
- \(J(P, C_{\exp}) \leq J(P, C_{\exp}) + \|\Delta\|_\infty \forall P \in \mathcal{P}(\hat{P}, \Delta)\)

\(\Rightarrow\) Unique coordinates connect size \(\Delta\) and \(J(P, C)\)!

Flexible beam experimental results

Coprime factor identification (2), see Fig. 1.

Steps: Coprime factors \(\bar{N}, \bar{D} \Rightarrow\) construct model set \(\mathcal{P}\) by estimating size of \(\Delta \Rightarrow\) robust control design.
- control-relevant \(\{\bar{N}, \bar{D}\} \Rightarrow\) \(\|\Delta\|_\infty = 1.9 \Rightarrow\) \(\mathcal{P}_{\text{RCR}}\)
- normalized \(\{\bar{N}, \bar{D}\} \Rightarrow\) \(\|\Delta\|_\infty = 0.3 \Rightarrow\) \(\mathcal{P}_{\text{NORM}}\)

Results

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Initial } & \text{Opt } & \text{Opt } & \text{Opt } \\
\text{C}_{\exp}(\mathcal{P}_{\text{RCR}}) & \text{C}_{\exp}(\mathcal{P}_{\text{RCR}}) & \text{C}_{\exp}(\mathcal{P}_{\text{NORM}}) & \text{C}_{\exp}(\mathcal{P}_{\text{NORM}}) \\
\hline
10.2 & 5.2 & 9.8 & 5.6 \\
12.1 & 5.6 & 50.6 & 33.5 \\
\hline
\end{array}
\]

New control-relevant coordinate frame \(\mathcal{P}_{\text{RCR}}\) leads to tight bound and improved performance compared to prior results \(\mathcal{P}_{\text{NORM}}\), see also Fig. 2!

Conclusions

Coprime factor identification has been extended to deliver new realizations
- directly identifiable from data
- orthonormal vector polynomials \(\Rightarrow\) numerically reliable \([6]\)
- transparent connection model uncertainty coordinate frame and control criterion
- suitable for any uncertainty modeling procedure
- generalization to inferential control in \([7]\).

References


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