Transparently Connecting Model Uncertainty Identification and Robust Control Design by Adopting a New Coordinate Frame

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Background

Developments in robust control design methodologies have resulted in identification techniques that deliver

- control-relevant models [1] [2]
- (normalized) coprime factors [3] [4].

The only reason to consider coprime factors is to represent model uncertainty. However, present model uncertainty coordinate frames do not transparently connect to the control criterion $\Rightarrow$ not control-relevant $\Rightarrow$ conservative control design.

Aim

Transparently connect:

- identification of uncertain plant set $\mathcal{P}(\hat{P}, \Delta)$:
  - consisting of model $\hat{P}$ and model uncertainty Δ
  - such that true plant $P_o \in \mathcal{P}(\hat{P}, \Delta)$.
- control criterion $J(P,C) = \|WT(P,C)V\|_\infty$:
  - $T(P,C) = \begin{bmatrix} P \\ I \end{bmatrix} (I + CP)^{-1} \begin{bmatrix} C & I \end{bmatrix}$
  - W, V control weighting filters.

Key idea:

- construct a suitable coordinate frame for representing model uncertainty Δ
- by exploiting the unexplored freedom in coprime factor realizations $\{N, D\}$, where $P = ND^{-1}$.

Nominal model identification

Lightly damped flexible structures with parasitic dynamics:

- identification of full-order models illusive
- control-relevant identification $\Rightarrow$ reliable models.

However, involved control-relevant identification criterion

$$\left\| W \left(T(P_o, C^{\exp}) - T(\hat{P}, C^{\exp}) \right) V \right\|_\infty , \qquad (1)$$

invariant under varying coprime factor realization $\{\hat{N}, \hat{D}\}$!

Result: 4-block problem (1) can be recast as 2-block problem

$$\left\| W \left(\begin{bmatrix} N_o \\ D_o \end{bmatrix} - \begin{bmatrix} \hat{N} \\ \hat{D} \end{bmatrix} \right) \right\|_\infty , \qquad (2)$$

where $\{N_o, D_o\}$ and $\{\hat{N}, \hat{D}\}$ are

- novel, unique, and control-relevant coprime factors
- generally not normalized.

Robust-control-relevant coordinate frame

For some $P \in \mathcal{P}(\hat{P}, \Delta)$ and any uncertainty structure

- $J(P, C^{\exp}) = \|\hat{M}_{22} + \hat{M}_{21}\Delta(I - \hat{M}_{11}\Delta)^{-1}\hat{M}_{12}\|_\infty$.

For any Dual-Youla-Kučera uncertainty structure [5]

- $J(P, C^{\exp}) = \|\hat{M}_{22} + \hat{M}_{21}\Delta\hat{M}_{12}\|_\infty$.

Stronger result: using control-relevant coprime factors

- $J(P, C^{\exp}) \leq J(\hat{P}, C^{\exp}) + \|\Delta\|_\infty \ \forall P \in \mathcal{P}(\hat{P}, \Delta)$

$\Rightarrow$ Unique coordinates connect size Δ and $J(P,C)$!

Flexible beam experimental results

Coprime factor identification (2), see Fig. 1.

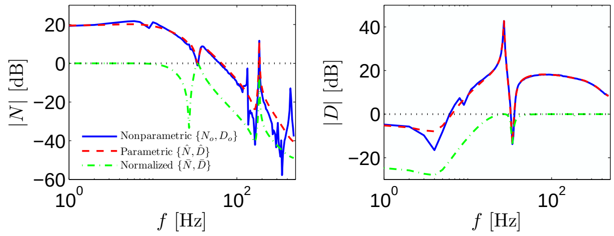

Figure 1: Control-relevant identification of coprime factors.

Steps: Coprime factors $\hat{N}, \hat{D} \Rightarrow$ construct model set $\mathcal{P}$ by estimating size of $\Delta \Rightarrow$ robust control design.

- control-relevant $\{\hat{N}, \hat{D}\} \Rightarrow \|\Delta\|_\infty = 1.9 \Rightarrow \mathcal{P}^{\mathrm{RCR}}$
- normalized $\{\bar{N}, \bar{D}\} \Rightarrow \|\Delta\|_\infty = 0.3 \Rightarrow \mathcal{P}^{\mathrm{NORM}}$

Results	$J(\hat{P}, C)$	$\sup_{P \in \mathcal{P}^{\mathrm{RCR}}} J(P,C)$	$\sup_{P \in \mathcal{P}^{\mathrm{NORM}}} J(P,C)$
Initial $C^{\exp}$	10.2	12.1	34.6
$C^{\mathrm{opt}}(\mathcal{P}^{\mathrm{RCR}})$	5.2	5.6	∞
$C^{\mathrm{opt}}(\mathcal{P}^{\mathrm{NORM}})$	9.8	50.6	33.5

New control-relevant coordinate frame ($\mathcal{P}^{\mathrm{RCR}}$) leads to tight bound and improved performance compared to prior results ($\mathcal{P}^{\mathrm{NORM}}$), see also Fig. 2!

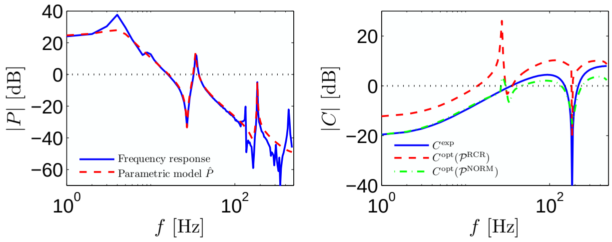

Figure 2: Control-relevant plant model (left) and optimal controllers (right).

Conclusions

Coprime factor identification has been extended to deliver new realizations

- directly identifiable from data
- orthonormal vector polynomials$\Rightarrow$numerically reliable [6]
- transparent connection model uncertainty coordinate frame and control criterion
- suitable for any uncertainty modeling procedure
- generalization to inferential control in [7].

References

[1] R. J. P. Schrama. Accurate identification for control: The necessity of an iterative scheme. *IEEE Trans. Automat. Contr.*, 37(7):991–994, 1992.
[2] M. Gevers. Towards a joint design of identification and control ? In H. L. Trentelman and J. C. Willems, editors, *Essays on Control : Perspectives in the Theory and its Applications*, chapter 5, pages 111–151. Birkhäuser, Boston, MA, USA, 1993.
[3] P. M. J. Van den Hof, R. J. P. Schrama, R. A. de Callafon, and O. H. Bosgra. Identification of normalised coprime plant factors from closed-loop experimental data. *Eur. J. Contr.*, 1(1):62–74, 1995.
[4] G. Gu. Modeling of normalized coprime factors with ν-metric uncertainty. *IEEE Trans. Automat. Contr.*, 44(8):1498–1511, 1999.
[5] S. G. Douma and P. M. J. Van den Hof. Relations between uncertainty structures in identification for robust control. *Automatica*, 41:439–457, 2005.
[6] A. Bultheel, M. van Barel, Y. Rolain, and R. Pintelon, Numerically robust transfer function modeling from noisy frequency domain data, *IEEE Trans. Automat. Contr.*, 50(11):1835–1839, 2005.
[7] T. Oomen, O. Bosgra, and M. van de Wal. Identification for robust inferential control. In *Proc. 48th Conf. Dec. Contr.*, Accepted, Shanghai, China, 2009.