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Inferential Motion Control: Identification and Robust Control with Unmeasured Performance Variables

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Abstract— **Next-generation motion systems are expected to exhibit dominant flexible dynamical behavior. As a result, a dynamic relation between the measured variables and the performance variables is inevitable. The aim of the present paper is i) to develop a control framework to deal with unmeasured performance variables and ii) to implement the proposed methodology on a prototype experimental setup. A system identification for robust inferential control approach is pursued. Indeed, in the case that the performance variables cannot be measured directly, then these can be inferred from the measurements by means of a model. Experimental results i) confirm that prior approaches that aim at an improved response in terms of the measured variables can result in a deteriorated performance and ii) reveal that the proposed framework enables high performance robust inferential control.**

I. Introduction

Increasing speed and accuracy demands in high precision positioning systems, e.g., used in integrated circuit (IC) manufacturing, require next-generation motion control to deal with flexible dynamical behavior. On the one hand, these systems are expected to be lightweight in the near future, since by virtue of Newton's law, reducing the mass of the system implies increasing accelerations and thus increasing the speed. As a consequence of these lightweight constructions, these systems exhibit flexible dynamical behavior at lower frequencies. On the other hand, due to increasing accuracy requirements, control has to be effective at higher frequencies. Combining these developments reveals that control has to deal with flexible dynamical behavior in the cross-over frequency region, see also [1] for a related problem in vibration control.

An important consequence of the presence of flexible dynamical behavior in positioning systems is the inevitable situation of unmeasured performance variables. Indeed, it is generally impossible to measure exactly on the location where performance is required, since at this location a product is being processed. Due to the presence of flexible dynamical behavior, a dynamic relation between the measured variables and performance variables is present. The presence of this dynamic relation complicates the controller design compared to traditional situations. Specifically, the systems can be considered rigid in traditional situations. As a result, in this traditional situation, a static relation between the performance variables and measured variables exists, which can, e.g., be determined from the system geometry.

The explicit distinction between performance variables and measured variables can be dealt with by means of model-based control design. In this case, a dynamic model is used to infer the performance variables from the measured variables, either implicitly, e.g., in model-based optimal control, or explicitly by constructing an observer. Such an approach is referred to as inferential control. For the class of positioning systems, system identification provides an inexpensive and fast methodology to obtain the required models.

The observation that any model is a simplification of reality has led to the development of system identification for control, see, e.g., [2], [3], [4], [5], [6] for an overview. However, these system identification for control methodologies are restricted to the case where the performance variables are also measured variables. Although an exception is reported in [7], the considered identification approach is less suitable for motion systems, since it cannot deal with closed-loop operation and is tailored towards model predictive control. Recently, in [8], a framework has been presented that enables the identification of models in view of robust inferential control. The inferential identification and control framework in [8] resolves three important deficiencies of earlier system identification approaches for control: 1) the controller structure and control goal are extended, 2) control-relevant system identification approaches for nominal model identification are proposed, and 3) new model uncertainty structures are developed. The use of robust control techniques is especially crucial in the case of inferential control, since the inferred performance variables hinges on the model quality.

Although system identification for control methodologies have recently been extended in [8] to deal with the inferential control situation, the potential hazards of unmeasured performance variables and benefits of the inferential control framework for a realistic system have not yet been investigated. Indeed, applications of inferential control are mainly limited to the area of process control, see, e.g., [9], [10].

The main contribution of the present paper is to present and apply a framework for system identification for robust inferential control that can deal with unmeasured performance variables and to compare the results with standard system identification for robust control approaches. Hereto, a prototype next-generation motion system is used that represents a realistic scenario in next-generation motion control. A key advantage of this system is that it enables access to the performance variables for model building and performance validation, which is obviously required at a certain point in the control design. In the next section, the experimental system, problem formulation, and approach in the paper are discussed in more detail.

II. Problem definition

A. Experimental setup

The prototype experimental system in Figure 1 is considered, which is specifically designed to exhibit dominant

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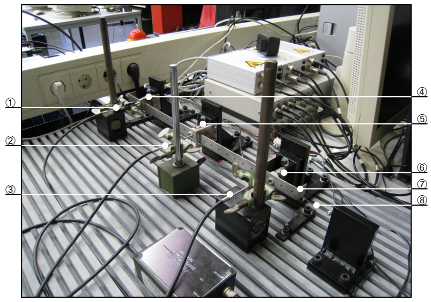

Fig. 1. Photograph of the experimental flexible beam setup, where ①: sensor s_1, ②: sensor s_2, ③: sensor s_3, ④: actuator a_1, ⑤: actuator a_2, ⑥: actuator a_3, ⑦: beam, ⑧: leaf spring.

flexible dynamical behavior that is expected to arise in next-generation positioning systems. The movable part of the system consists of a steel beam of $500 \times 20 \times 2$ mm. Four motion degrees-of-freedom (DOFs) are fixed by means of leaf springs. Hence, two motion DOFs remain, in addition to flexible dynamical behavior, as is illustrated in Figure 2.

To enable the evaluation of newly developed control strategies, the system is equipped with three inputs and three outputs. The inputs are current-driven voice-coil actuators. The outputs are contactless fiberoptic sensors with an accuracy of approximately $1\ \mu\mathrm{m}$. The control system is implemented in a PowerDAQ rapid prototyping environment in conjunction with Matlab/Simulink at a sampling frequency of 1 kHz.

To mimic the inferential control situation, a specific input-output selection is performed. For clarity of exposition, scalar measurement and performance variables are considered in the translational x-direction, as is defined in Figure 2. The goal is to control the performance variable z_p at the middle of the beam, i.e., at sensor location s_2. Hence, in the case of noise-free measurements,

$$z_\mathrm{p} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}^T.$$

To mimic the inferential control situation, s_2 is unavailable for the feedback controller. Instead, regarding the measured variable y_p, the response at the middle of the beam is determined by averaging the outer sensors s_1 and s_3, i.e.,

$$y_\mathrm{p} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}^T, \quad (1)$$

which in fact corresponds to a sensor transformation based on geometric relations as is indicated in Figure 2. Consequently, a discrepancy between the measured variable y_p and performance variable z_p may exist due to internal deformations of the beam. The outer actuators are used to translate the beam:

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}^T u_\mathrm{p} \quad (2)$$

where u_p is the manipulated input. Comparing (1), (2), and Figure 1 reveals that u_p and y_p are collocated, whereas u_p and z_p are non-collocated. The resulting system is given by

$$\begin{bmatrix} z_\mathrm{p} \\ y_\mathrm{p} \end{bmatrix} = \begin{bmatrix} P_\mathrm{z} \\ P_\mathrm{y} \end{bmatrix} u_\mathrm{p} = P u_\mathrm{p}. \quad (3)$$

The considered setup has close connections to many positioning systems. Specifically, a product typically is processed in the middle and hence this is the location where performance is required. On the other hand, measurements are performed at the edges of the system. The goal of the inferential servo problem is to minimize $z_\mathrm{ref} - z_\mathrm{p}$, where z_ref is a predefined reference trajectory, by the design of a controller

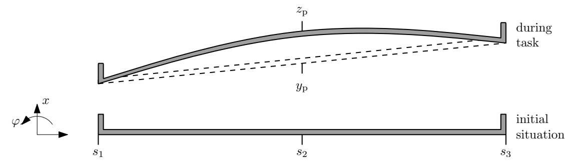

Fig. 2. Schematic top view illustration of flexible beam setup.

$$u_\mathrm{p} = C(z_\mathrm{ref}, y_\mathrm{p}). \quad (4)$$

Throughout, the focus is on the design of a feedback controller through optimization that is formalized next, where $\mathcal{J}^9$ denotes the control criterion that is defined more precisely in Section III.

Definition 1 (Inferential control goal): Given P_o with the interconnection (3) - (4), determine

$$C^\mathrm{opt} = \arg\min_C \mathcal{J}^9(P_o, C). \quad (5)$$

B. System identification for robust inferential control

1) Towards model-based robust control: The optimization (5) cannot be solved directly since P_o is unknown. To perform the actual optimization, model knowledge is exploited. For the considered class of electromechanical systems, this knowledge can be obtained in a fast, accurate, and inexpensive manner through system identification. Since the model is used for subsequent control design, a low-order model description is essential.

The choice for a specific system identification and robust control approach hinges on the (expected) main error sources. For the considered system, the system behavior is expected to be mainly linear and time invariant. With respect to expected error sources, two aspects are important. On the one hand, systematic errors are expected to be dominant. Firstly, the considered flexible system generally has infinitely many flexible modes, as is discussed in [11]. Hence, the model cannot be parameterized such that there exists a finite dimensional parameter vector that corresponds to the true system. Consequently, undermodeling is always present. Additionally, smooth nonlinearities, e.g., introduced by nonlinear damping effects, typically are present, see [12]. On the other hand, if the model is identified from data, then errors are introduced by finite time noisy observations. For the considered system that has a high signal-to-noise ratio and a large experimental freedom, these errors can be made arbitrarily small by an appropriate experiment design.

2) Towards system identification for robust control: Due to the presence of model inaccuracies, the model quality should be evaluated in terms of its goal. In addition, a single nominal model is inadequate to represent the full system behavior. Thereto, a model set is considered, i.e.,

$$\mathcal{P} = \left\{ P | P = \mathcal{F}_u(\hat{H}, \Delta), \Delta \in \mathbf{\Delta} \right\},$$

where $\mathcal{F}_u$ denotes the upper linear fractional transformation (LFT), $\mathbf{\Delta} \subset \mathcal{RH}_\infty$, and $\hat{H}$ contains $\hat{P}$ and the uncertainty structure, see Section V. Such $\mathcal{H}_\infty$-norm-bounded perturbations encompass a large class of errors, including dynamic uncertainty due to undermodeling and certain nonlinearities.

Associated with $\mathcal{P}$ is the worst-case performance

$$\mathcal{J}^9_{\mathrm{WC}}(\mathcal{P}, C) = \sup_{P \in \mathcal{P}} \mathcal{J}^9(P, C)$$

Control design aiming at robust performance then involves

$$C^{\mathrm{RP9}} = \arg\min_{C} \mathcal{J}^9_{\mathrm{WC}}(\mathcal{P}, C). \tag{6}$$

Two requirements are imposed with respect to $\mathcal{P}$. Firstly, the constraint

$$P_o \in \mathcal{P}(\hat{P}, \Delta). \tag{7}$$

should be satisfied to guarantee that C^{RP9} stabilizes P_o. Secondly, C^{RP9} should achieve good robust performance. Thus, a system identification for robust control procedure should deliver a model set $\mathcal{P}$ such that it delivers a minimum value of $\mathcal{J}^9_{\mathrm{WC}}(\mathcal{P}, C^{\mathrm{RP9}})$. However, C^{RP9} depends in a complicated, non-analytic manner on $\mathcal{P}$, hence the use of C^{RP9} as an identification criterion is not directly possible. Thereto, as in [8], an upper bound is employed, leading to the following robust-control-relevant identification criterion.

Definition 2 (Robust-control-relevant identification): Given a stabilizing controller C^{exp}, determine

$$\mathcal{P}^{\mathrm{RCR}} = \arg\min_{\mathcal{P}} \mathcal{J}^9_{\mathrm{WC}}(\mathcal{P}, C^{\mathrm{exp}}) \quad \text{subject to (7).} \tag{8}$$

The criterion (8) provides an upper bound for the ideal criterion, i.e., the bound

$$\mathcal{J}_{\mathrm{WC}}(\mathcal{P}, C^{\mathrm{RP}}) \leq \mathcal{J}_{\mathrm{WC}}(\mathcal{P}, C^{\mathrm{exp}})$$

holds for any $\mathcal{P}$. The robust-control-relevant identification criterion in Definition 2 extends the procedure in [13] by addressing the inferential control situation. Note that a stabilizing controller C^{exp} is required, see [13] for a discussion. The controller C^{exp} used in this paper is a PID-controller, see Figure 7. In addition, the criterion (8) is at the basis of certain iterative identification and robust control approaches, including [14], [8].

C. Approach and outline

In view of the system identification for robust inferential control approach presented in Section II-B, the following procedure is followed in this paper.

1) A novel controller goal based on $\mathcal{H}_\infty$-optimization and extended controller structures are presented in Section III that are suitable for inferential servo control.

2) A procedure for the identification of a control-relevant model $\hat{P}$ is presented and applied to the experimental setup in Section IV.

3) A novel model uncertainty structure that can deal with the inferential control situation is presented in Section V and it is shown that this structure in conjunction with the nominal modeling procedure in Section IV jointly aim at identifying a robust-control-relevant model set in view of Definition 2.

4) The synthesis and implementation of robust optimal inferential controllers is presented in Section VI, as well as a comparison with prior results.

III. $\mathcal{H}_\infty$-OPTIMAL INFERENTIAL CONTROL

The inferential control problem requires several extensions to the control structure and control goal. With respect to the controller structure, it is argued in [8] that a single DOF controller is inadequate for the inferential control case. As

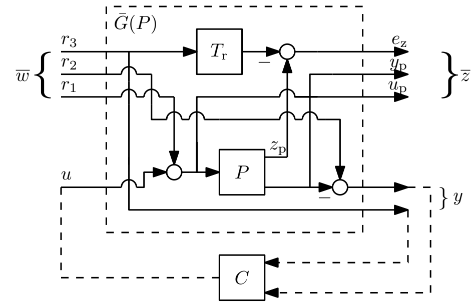

Fig. 3. Standard plant configuration for inferential control.

a supporting example, note that the involved error signal cannot be computed directly since y_{p} and z_{p} may have different dimensions in the general situation. Among the various alternatives, it is argued in [8] that the two DOF controller

$$C = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \tag{9}$$

has certain advantageous properties.

The controller (9) and the inferential servo goal discussed in Section II differ from the commonly used formulation in system identification for control, which is based on single DOF controllers and associated controller goals based on 4-block problems. To deal with the inferential servo goal and controller (9), the extended control goal

$$\mathcal{J}^9(P, C) = \|M(P, C)\|_\infty . \tag{10}$$

$$M(P, C) : w \mapsto z = W \mathcal{F}_l(\bar{G}(P), C) V \tag{11}$$

is considered, where $\bar{G}(P)$ is defined by the operator $\begin{bmatrix} \overline{z} \\ y \end{bmatrix} = \bar{G}(P) \begin{bmatrix} \overline{w} \\ u \end{bmatrix}$, and the involved signals are defined in Figure 3. Also, the bistable weighting filters $W = \mathrm{diag}\,(W_{\mathrm{e}}, W_{\mathrm{y}}, W_u)$ and $V = \mathrm{diag}\,(V_3, V_2, V_1)$ are considered and T_{r} is a stable reference model.

The following remarks are appropriate with respect to the weighting filter design.

1) The criterion $\mathcal{J}^9$ in (10) enables the specification of versatile control goals through the use of weighting filters and guarantees internal stability of the closed-loop system, see [8, Proposition 4].

2) The weighting filter selection can be based on an identified frequency response function of the system P, which is discussed in Section IV.

3) The weighting filters $W_{\mathrm{y}}, W_u, V_2, V_1$ are based on an $\mathcal{H}_\infty$-loopshaping control design, see, e.g., [15]. As a result, a standard controller that aims at a good response in terms of the measured variables can be computed for comparison. Specifically, the controller

$$C^{\mathrm{RP4}} = \arg\min_{C} \sup_{P \in \mathcal{P}} \left\| \begin{bmatrix} W_{\mathrm{y}} P_{\mathrm{y}} \\ W_u \end{bmatrix} (I + C P_{\mathrm{y}})^{-1} \begin{bmatrix} C V_2 & V_1 \end{bmatrix} \right\|_\infty \tag{12}$$

is compared with C^{RP9} in Section VI. Further details of the weighting filter selection can be found in [16, Chapter 7].

IV. NOMINAL MODEL IDENTIFICATION FOR CONTROL

As is discussed in Section II, a robust-control-relevant model set $\mathcal{P}^{\mathrm{RCR}}$ is to be identified, where the control criterion is given by (10). The first step in the construction of a robust-control-relevant model set is the identification of a nominal model $\hat{P}$, followed by the quantification of model uncertainty Δ in Section V, where it is also shown that these separate steps jointly aim at identifying a robust-control-relevant model set.

The rationale behind identifying a control-relevant model stems from the triangle inequality

$$\|M(P_o, C)\|_\infty \leq \|M(\hat{P}, C)\|_\infty + \|M(P_o, C) - M(\hat{P}, C)\|_\infty.$$

Specifically, by substituting C^{exp} in the latter expression, the following control-relevant identification criterion is obtained.

Definition 3: The control-relevant identification criterion is defined as

$$\hat{P} = \arg\min_P \|M(P_o, C^{\text{exp}}) - M(P, C^{\text{exp}})\|_\infty. \quad (13)$$

Clearly, knowledge regarding $M(P_o, C^{\text{exp}})$ is required to solve (13). This knowledge is obtained from data in a frequency response-based procedure, as is discussed below.

The control-relevant identification criterion in Definition 3 differs from the results in [13] by considering a more general control goal and control structure. The distinction is even more pronounced when the control-relevant identification criterion is recast as a coprime factor optimization problem as is considered next.

To anticipate on the results in Section V, it turns out that a coprime factorization of the nominal model is essential to construct a model uncertainty that is guaranteed to satisfy (7) for a certain $\mathcal{H}_\infty$-norm-bounded perturbation Δ. A main results in [8] is that the 9-block control-relevant identification criterion in Definition 3 can be recast as the 3-block problem

$$\left\| W \left(\begin{bmatrix} N_{z,o} \\ N_o \\ D_o \end{bmatrix} - \begin{bmatrix} \hat{N}_z \\ \hat{N} \\ \hat{D} \end{bmatrix} \right) \right\|_\infty, \quad \begin{bmatrix} N_z \\ N \\ D \end{bmatrix} = \begin{bmatrix} P_z \\ P_y \\ I \end{bmatrix} (\tilde{D}_e + \tilde{N}_{e,2} V_2^{-1} P_y)^{-1}. \quad (14)$$

In (14), $\tilde{D}_e$ and $\tilde{N}_{e,2}$ can be computed from C^{exp} and V, see [8]. The advantage of the formulation (14) is twofold. Firstly, it reduces the complexity of the problem by a factor three. Secondly, it is shown in [8] that the pair $\{N, D\}$ is a right coprime factorization of P_y and the pair $\{N_z, D\}$ is a stable factorization of P_z, which is essential for the subsequent developments in Section V.

To select weighting filters and to understand the behavior of the open-loop system P_o, it is useful to compute the frequency response function of the open-loop system. Indeed, $P_o(\omega) = \begin{bmatrix} N_{z,o}(\omega) & N_o(\omega) \end{bmatrix}^T (D_o(\omega))^{-1}$, see Figure 5 for the results. Based on a thorough analysis, the following interpretation is given.

1) The digital computer implementation introduces delay.

2) The resonance phenomena at 4 Hz and 10 Hz correspond to rigid-body modes suspended by the leaf springs.

3) Up to 300 Hz, $P_{y,o}$ has a -2 slope due to collocation of y_p and u_p as explained in Section II-A.

4) At ≈ 32 Hz, P shows a different behavior than $P_{y,o}$, since the slope changes from -2 to -4. This corresponds to the first bending mode of the beam and non-collocation of u_p and z_p, which is schematically illustrated in Figure 2.

Next, weighting filters W and V are designed using the frequency response function in Figure 5, hence all ingredients are present to identify a control-relevant nominal model. Coprime factors of the true system, i.e., $N_{z,o}, N_o, D_o$ are identified and a parametric model $\hat{N}_z, \hat{N}, \hat{D}$ is estimated using the frequency domain system identification algorithm described in [8], see Figure 4. The advantage of the coprime factor domain in Figure 4 is that by virtue of the criterion (14), the parametric model should be accurate in the region where the gain of the coprime factors is large. Clearly, the coprime factor model is control-relevant from visual inspection of Figure 4.

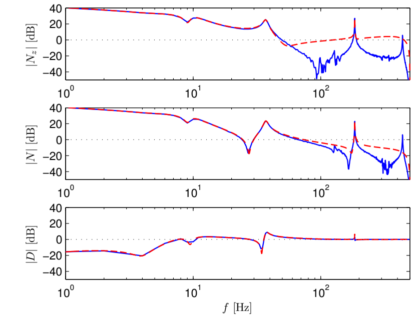

Fig. 4. Bode magnitude diagram of identified frequency response function of $N_{z,o}, N_o, D_o$ (solid) and identified parametric model $\hat{N}_z, \hat{N}, \hat{D}$ (dashed).

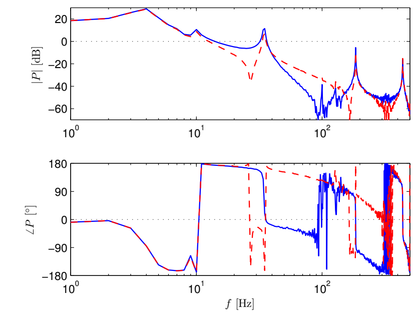

Fig. 5. Frequency response function estimate $P_{z,o}(\omega)$ (solid), $P_{y,o}(\omega)$ (dashed).

V. Towards Robust-Control-Relevant Model Sets

As is discussed in Section II, the model $\hat{P}$ inevitably is an approximation of P_o. Hence, it remains to quantify the model quality. In view of the control goal of the model, this amounts to constructing a model set in view of Definition 2. Two aspects are important: the model uncertainty i) structure, and ii) size.

A. Model uncertainty structure

Since the LFT of an LFT is again an LFT, $M(P, C)$ in (11) for any $P \in \mathcal{P}$ can be written as $\mathcal{F}_u(\hat{M}, \Delta)$, where $\Delta \in \boldsymbol{\Delta} \subset \mathcal{H}_\infty$. Next, (10) yields

$$\mathcal{J}^9_{\text{WC}}(\mathcal{P}, C^{\text{exp}}) = \sup_{\Delta \in \boldsymbol{\Delta}} \|\hat{M}_{22} + \hat{M}_{21}\Delta(I - \hat{M}_{11}\Delta)^{-1}\hat{M}_{12}\|_\infty, \quad (15)$$

The use of general uncertainty structures, including additive uncertainty, may have two disadvantages: 1) (7) may not be satisfied for an admissible $\Delta_u \in \mathcal{H}_\infty$, and 2) the connection between (15) and (8) is not clear and consequently the resulting model set may not be robust-control-relevant.

To resolve these two disadvantages, the specific coprime factorization of (14), in conjunction with a (W_u, W_y)-normalized coprime factorization of C_2^{exp} given by $\{N_{c,2}, D_{c,22}\}$ is used to construct the model set

$$\left\{ P \middle| P = \begin{bmatrix} \hat{N}_z + W_e^{-1}\Delta_z \\ \hat{N} + D_{c,22}\Delta_u \end{bmatrix} (D - N_{c,2}\Delta_u)^{-1}, \begin{bmatrix} \Delta_z \\ \Delta_u \end{bmatrix} \in \mathcal{RH}_\infty \right\}. \quad (16)$$

TABLE I

ROBUST-CONTROL-RELEVANT IDENTIFICATION AND ROBUST CONTROLLER SYNTHESIS RESULTS.

	C^{exp}	C^{RP4}	C^{RP9}
$\mathcal{J}^4(\hat{P}_y, C)$	123.01	4.09	–
$\mathcal{J}^9(\hat{P}, C)$	196.82	41.18	9.34
$\mathcal{J}^4_{\text{WC}}(\mathcal{P}^{\text{RCR}}, C)$	125.50	4.10	–
$\mathcal{J}^9_{\text{WC}}(\mathcal{P}^{\text{RCR}}, C)$	201.38	49.45	10.88

Compared to (15), the model set (16) leads to the stronger result

$$\mathcal{J}^9_{\text{WC}}(\mathcal{P}, C^{\text{exp}}) \leq \|\hat{M}_{22}\|_\infty + \sup_{\Delta \in \boldsymbol{\Delta}} \|\Delta\|_\infty, \qquad (17)$$

see [8], where $\Delta = \begin{bmatrix} \Delta_z^T & \Delta_u^T \end{bmatrix}^T$. The result (16) is an significant extension of the dual-Youla-Kučera-based uncertainty structure, e.g., as is considered in [17], [18], in two aspects: i) it extends existing results to the two DOF controller (9) and associated control goal (10), and ii) besides rendering $\hat{M}_{11}$ to zero, it removes $\hat{M}_{21}$ and $\hat{M}_{12}$ from the expression (15) by exploiting the freedom in coprime factorizations. The importance of the result (17) is at least twofold: 1) it connects the size of Δ and the control criterion, hence in conjunction with the result (14) it provides a solution to the robust-control-relevant identification problem 2, and 2) it enables the use of unstructured model uncertainty, hence the controller synthesis problem (6) and (12) can be solved efficiently.

B. Model uncertainty size

To evaluate the model quality of $\hat{P}$, a large number of validation experiments are performed under relevant operating conditions. Subsequently, the validation-based uncertainty modeling procedure of [19] is employed, which is especially useful in view of the relevant error sources in Section II-B.1. The size of model uncertainty in the model uncertainty structure (16) is given by $\sup_{\Delta \in \boldsymbol{\Delta}} \|\Delta\|_\infty \leq 5.6$.

The resulting model set is, similar to the results in Figure 4, formulated in an abstract coprime factor domain. Thereto, the procedure in [20], which is based on generalizations of the structured singular value, is adopted to visualize the candidate model in terms of the open-loop system behavior. The results are depicted in Figure 6. It appears that the model is most accurate around the desired cross-over region at approximately 50 Hz, see [20] for a further interpretation.

The performance of the model (set) can also be quantified in terms of the control criterion (10). Indeed, from Table I, it is observed that the nominal model $\hat{P}$ achieves a performance of $\mathcal{J}^9(\hat{P}, C) = 196.82$. The uncertain models satisfy a bound equal to $\mathcal{J}^9_{\text{WC}}(\mathcal{P}^{\text{RCR}}, C) = 201.38$. Since the norm bound on Δ_u equals 5.6, the bound in (17) indeed is satisfied.

VI. ROBUST CONTROL DESIGN

In this section, robust controllers are synthesized for the model set $\mathcal{P}$ as described in Section IV and Section V using the inferential control design framework based on $\mathcal{H}_\infty$-optimization as described in Section III. Hereto, two controllers are synthesized

1) a robust inferential controller C^{RP9} as in (6) and

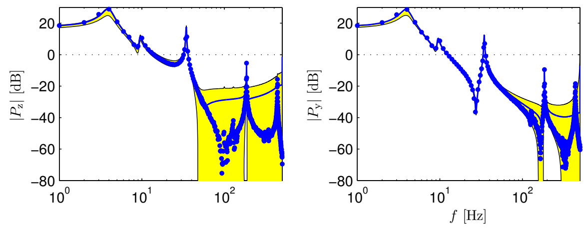

Fig. 6. Bode magnitude diagram: frequency response function corresponding to Figure 5 (dot), nominal model $\hat{P}$ (solid), model set $\mathcal{P}$ (shaded).

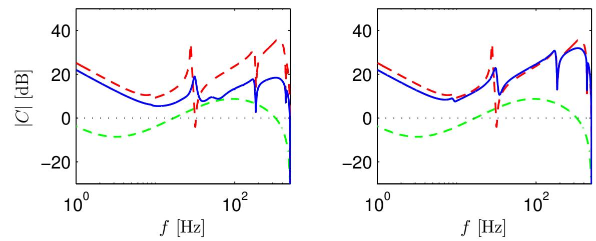

Fig. 7. Bode diagram of controller: robust inferential controller C^{RP9} (solid), robust 4-block controller C^{RP4} (dashed), initial controller C^{exp} (dash-dotted). Left: C_1, right: C_2.

2) a standard 4-block controller C^{RP4} as in (12) using standard robust control design approaches, i.e., using a single DOF controller and 4-block criterion.

Identical weighting filters are used for the synthesis of C^{RP4} and C^{RP9}, thereby enabling a fair comparison of the results.

The resulting controllers are depicted in Figure 7. Clearly, C^{RP9} and C^{RP4} have a significantly higher gain compared to the initial controller C^{exp}. Also, it can clearly be observed that C^{RP4} is a single DOF controller, while C^{RP9} indeed exploits the additional controller freedom.

The controller performance in the criterion (10) is given in Table I. Observe that C^{RP9} results in a value of $\mathcal{J}^9_{\text{WC}}(\mathcal{P}, C^{\text{RP9}}) = 10.88$. The controller C^{RP4}, which aims at minimizing $\mathcal{J}^4_{\text{WC}}$, yet analyzed in terms of the inferential performance criterion, leads to a performance of $\mathcal{J}^9_{\text{WC}}(\mathcal{P}, C^{\text{RP4}}) = 49.45$.

Finally, the controllers are implemented on the experimental system in Figure 1. The resulting step responses of the z_{p} and y_{p} variables are depicted in Figure 8. It is emphasized that none of the controllers has access to the performance variable z_{p}. Hence, the measurement of z_{p} is only used for an *a posteriori* analysis of the performance variables. The following observations are made

1) Controller C^{RP4} significantly improves the response in terms of y_{p} compared to the initial controller C^{exp}, which is in correspondence with the results regarding $\mathcal{J}^4_{\text{WC}}$ in Table I.

2) However, controller C^{RP4} also results in a poor performance, i.e., the response in terms of the z_{p} variable, when compared to the response in terms of y_{p}. Note that this leads to a potentially dangerous situation, since the poor performance in terms of z_{p} cannot be observed from signals inside the single DOF feedback loop.

3) When analyzing the response of the robust inferential controller C^{RP9}, it appears that it results in a peculiar response with respect to the y_{p} variable at 0.07 s. When analyzing the performance variable z_{p}, the reason for this oscillation becomes clear, since the peculiar response seems required to ensure that z_{p} has a desired response. Indeed,

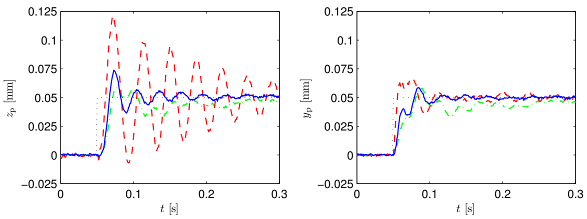

Fig. 8. Experimental step responses using robust inferential controller C^{RP9} (solid), robust 4-block controller C^{RP4} (dashed), initial controller C^{exp}.

the trajectories of z_p and y_p are inherently coupled, hence a single input u_p cannot result in an arbitrary response in terms of both z_p and y_p. Similar responses have been observed in other systems where performance variables and measured variables can be distinguished, including optimal control of overhead cranes, see [21].

4) Comparing the robust inferential controller with the standard 4-block controller, it is clear from Figure 8 that C^{RP9} results in a significantly better performance compared to C^{RP4}. This is also in agreement with the results in Table I, since $\mathcal{J}^{9}_{\mathrm{WC}}(\mathcal{P}, C^{\mathrm{RP9}}) < \mathcal{J}^{9}_{\mathrm{WC}}(\mathcal{P}, C^{\mathrm{RP4}})$.

The conclusions are twofold. In the situation where a dynamic relation is present between the measured variables and performance variables

1) standard feedback control designs that aim at a good response in terms of the measured variables may lead to poor performance. It is emphasized that this can lead to potentially dangerous situations, since the poor performance cannot be detected from the single DOF feedback loop.

2) the proposed robust inferential control situation appropriately deals with the unmeasured performance variables by (implicitly) inferring these from the measured variables through a model.

VII. Conclusions

In this paper, a novel system identification and robust control design framework with experimental verification on a prototype next-generation motion system is presented that appropriately deals with unmeasured performance variables during normal operation. The main conclusions are twofold.

Firstly, it is shown that although common control design techniques, such as the robust $\mathcal{H}_\infty$-loopshaping controller C^{RP4}, can significantly improve the response in terms of the measured variables, these techniques may lead to a deteriorated performance in terms of the performance variables. This is a potentially dangerous situation, since the poor performance cannot be detected from signals within the feedback control loop. Of course, the poor performance will be detected when using the system for a specific task, e.g., it may lead to defective products, but in this case it may be unclear how the feedback control design should be adjusted.

Secondly, it is shown that the proposed system identification and robust control design methodology can result in high inferential performance. The key step is to use model knowledge to infer the performance variables from the measured variables. This model is identified using a temporary sensor during the identification step that is not available during normal operation of the system. This assumption is not considered restrictive, since the sensor is not required during normal operation. Furthermore, the model quality is tuned towards the control goal to enable high performance control. Also, since the model is used for inferring the performance variables, the performance hinges on the model quality. This motivates the need for an uncertainty model and a robust control design. Both the control-relevant identification and model uncertainty are aimed at the identification of a robust-control-relevant model set, enabling high performance robust inferential control. Experimental results confirm that the proposed procedure indeed outperforms prior control design techniques and enables the high performance control design for next-generation positioning systems exhibiting flexible dynamical behavior.

References

[1] G. J. Balas and J. C. Doyle, "Control of lightly damped, flexible modes in the controller crossover region," *J. Guid., Contr., Dyn.*, vol. 17, no. 2, pp. 370–377, 1994.

[2] B. Ninness and G. C. Goodwin, "Estimation of model quality," *Automatica*, vol. 31, no. 12, pp. 1771–1797, 1995.

[3] P. M. J. Van den Hof and R. J. P. Schrama, "Identification and control - closed-loop issues," *Automatica*, vol. 31, no. 12, pp. 1751–1770, 1995.

[4] M. Gevers, "Identification for control: From the early achievements to the revival of experiment design," *Eur. J. Contr.*, vol. 11, no. 4-5, pp. 1–18, 2005.

[5] H. Hjalmarsson, "From experiment design to closed-loop control," *Automatica*, vol. 41, pp. 393–438, 2005.

[6] P. Albertos and A. Sala, Eds., *Iterative Identification and Control*. London, UK: Springer, 2002.

[7] R. Amirthalingam and J. H. Lee, "Subspace identification based inferential control applied to a continuous pulp digester," *J. Proc. Cont.*, vol. 9, no. 5, pp. 397–406, 1999.

[8] T. Oomen, O. Bosgra, and M. van de Wal, "Identification for robust inferential control," in *Proc. 48th Conf. Dec. Contr.*, Shanghai, China, 2009, pp. 2581–2586.

[9] J. R. Parrish and C. B. Brosilow, "Inferential control applications," *Automatica*, vol. 21, no. 5, pp. 527–538, 1985.

[10] F. J. Doyle, III, "Nonlinear inferential control for process applications," *J. Proc. Cont.*, vol. 8, no. 5-6, pp. 339–353, 1998.

[11] P. C. Hughes, "Space structure vibration modes: How many exist? Which ones are important?" *IEEE Contr. Syst. Mag.*, vol. 7, no. 1, pp. 22–28, 1987.

[12] R. S. Smith, "Closed-loop identification of flexible structures: An experimental example," *J. Guid., Contr., Dyn.*, vol. 21, no. 3, pp. 435–440, 1998.

[13] T. Oomen, R. van Herpen, and O. Bosgra, "Robust-control-relevant coprime factor identification with application to model validation of a wafer stage," in *15th IFAC Symp. Sys. Id.*, Saint-Malo, France, 2009, pp. 1044–1049.

[14] R. A. de Callafon and P. M. J. Van den Hof, "Suboptimal feedback control by a scheme of iterative identification and control design," *Math. Mod. Syst.*, vol. 3, no. 1, pp. 77–101, 1997.

[15] G. Vinnicombe, *Uncertainty and Feedback: $\mathcal{H}_\infty$ loop-shaping and the ν-gap metric*. London, UK: Imperial College Press, 2001.

[16] T. Oomen, "System identification for robust and inferential control with applications to ILC and precision motion systems," Ph.D. dissertation, Eindhoven University of Technology, Eindhoven, The Netherlands, 2010.

[17] B. D. O. Anderson, "From Youla-Kucera to identification, adaptive and nonlinear control," *Automatica*, vol. 34, no. 12, pp. 1485–1506, 1998.

[18] S. G. Douma and P. M. J. Van den Hof, "Relations between uncertainty structures in identification for robust control," *Automatica*, vol. 41, pp. 439–457, 2005.

[19] T. Oomen and O. Bosgra, "Well-posed model uncertainty estimation by design of validation experiments," in *15th IFAC Symp. Sys. Id.*, Saint-Malo, France, 2009, pp. 1199–1204.

[20] T. Oomen, S. Quist, R. van Herpen, and O. Bosgra, "Identification and visualization of robust-control-relevant model sets with application to an industrial wafer stage," in *Proc. 49th Conf. Dec. Contr.*, Atlanta, GA, USA, 2010, pp. 5530–5535.

[21] T. Singh and W. Singhose, "Tutorial on input shaping/time delay control of maneuvering flexible structures," in *Proc. 2002 Americ. Contr. Conf.*, Anchorage, AK, USA, 2002, pp. 1717–1731.