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Bi-Orthonormal Basis Functions for Improved Frequency Domain Identification

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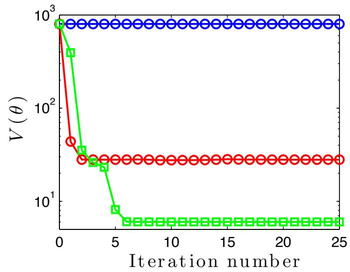
Frequency domain identification

Typical criterion: $V(\theta) = \sum_{k=1}^{m} |W_k(G_o(\xi_k) - \hat{G}(\theta, \xi_k))|^2$

Enhanced convergence in iterative algorithms:

Algorithm	Computation
Levy [1]	$A\theta = b$
SK [2]	iterate: $A\theta = b$
IV [3]	iterate: $C^T A\theta = C^T b$



Numerical aspects

Conditioning:

- SK: $\kappa(A)$
- IV: $\kappa(C^T A) \approx \kappa(A)^2$
- $\kappa(A)$ is very high (beyond 10^{16}) $\Rightarrow$ inaccurate solution
- partial solutions
  - frequency & amplitude scaling, ...
  - use rational bases: OBF, FLBF, ...

Enhanced convergence at expense of numerical conditioning?

Optimal conditioning for SK

- SK: $A\theta = b, \quad A = W\Phi$
- data-dependent inner product [4]

$$\langle \phi_i, \phi_j \rangle = \sum_{k=1}^{m} \phi_j(\xi_k)^H W_{(k,k)}^H W_{(k,k)} \phi_i(\xi_k) \qquad (1)$$

- model
  - $\hat{G} = \frac{b(\theta,\xi)}{a(\theta,\xi)}$
  - $\begin{bmatrix} b(\xi,\theta) \\ a(\xi,\theta) \end{bmatrix} = \sum_{i=1}^{n} \phi_i \theta_i$
- if ϕ orthonormal w.r.t. (1), then $\kappa(A) = 1$

Orthonormal polynomials not suitable for IV case

- $C^T A$ indefinite ([5]) & non-symmetric

New bi-orthonormal polynomials [6]

- IV: $C^T A\theta = C^T b, \quad C^T A = \Psi^H W_2{}^H W_1 \Phi$
- data-dependent bi-linear form

$$\langle \phi_i, \psi_j \rangle = \sum_{k=1}^{m} \psi_j(\xi_k)^H W_{2,(k,k)}{}^H W_{1,(k,k)} \phi_i(\xi_k) \qquad (2)$$

Result: if ϕ and ψ bi-orthonormal w.r.t. (2), then $\kappa(C^T A) = 1$

Bilinear form (2) extends inner product (1)

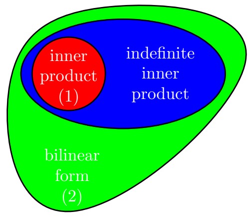
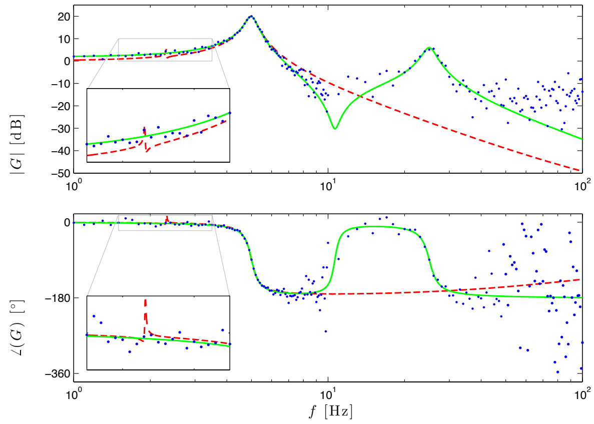


Illustration: IV vs. SK



Alg.	basis	κ	$V(\theta^\star)$	$\left\Vert \frac{\partial V(\theta)}{\partial \theta^T} \Big\vert_{\theta=\theta^\star} \right\Vert$
SK	monomial	$1.010 \cdot 10^6$	27.915	0.229
SK	ort. w.r.t. (1)	1.000	27.915	0.229
IV	monomial	$5.669 \cdot 10^{15}$	6.003	$5.588 \cdot 10^{-13}$
IV	bi-ort. w.r.t. (2)	1.000	6.003	$5.588 \cdot 10^{-13}$

Bi-orthonormal polynomials in identification [6]

- optimal conditioning ($\kappa = 1$) for IV-type identification
- both continuous and discrete time
- efficient ($\mathcal{O}(m)$) and reliable computation of bases
- extended to multivariable case

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