Bi-Orthonormal Basis Functions for Improved Frequency Domain Identification

By Tom Oomen and Robbert van Herpen

Frequency domain identification

Typical criterion: \( V(\theta) = \sum_{k=1}^{m} |W_k(G_o(\xi_k) - \hat{G}(\theta, \xi_k))|^2 \)

Enhanced convergence in iterative algorithms:

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Computation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Levy [1]</td>
<td>( \theta b = b )</td>
</tr>
<tr>
<td>SK [2]</td>
<td>Iterate: ( \theta b = b )</td>
</tr>
<tr>
<td>IV [3]</td>
<td>Iterate: ( C^T \theta b = C^T b )</td>
</tr>
</tbody>
</table>

Numerical aspects

Conditioning:
- **SK**: \( \kappa(A) \)
- **IV**: \( \kappa(C^T A) \approx \kappa(A)^2 \)
- \( \kappa(A) \) is very high (beyond \( 10^{16} \)) \( \Rightarrow \) inaccurate solution
- partial solutions
  - frequency & amplitude scaling, . . .
  - use rational bases: OBF, FLBF, . . .

Enhanced convergence at expense of numerical conditioning? Optimal conditioning for SK

- **SK**: \( \theta b = b \), \( A = W\Phi \)

\[
\langle \phi_i, \phi_j \rangle = \sum_{k=1}^{m} \phi_j(\xi_k)^H W_k^H W_{(k,k)} \phi_i(\xi_k) \quad (1)
\]

- model
  - \( \hat{G} = \frac{b(\xi, \theta)}{a(\xi, \theta)} \)
  - \( \left[ \begin{array}{c} b(\xi, \theta) \\ a(\xi, \theta) \end{array} \right] = \sum_{i=1}^{n} \phi_i \theta_i \)
- if \( \phi \) orthonormal w.r.t. (1), then \( \kappa(A) = 1 \)

Orthogonal polynomials not suitable for IV case
- \( C^T A \) indefinite ([5]) & non-symmetric

New bi-orthogonal polynomials [6]

- **IV**: \( C^T \theta b = C^T b \), \( C^T A = \Psi^H W_2^H W_1 \Phi \)
- data-dependent bi-linear form

\[
\langle \phi_i, \psi_j \rangle = \sum_{k=1}^{m} \psi_j(\xi_k)^H W_{2,(k,k)}^H W_{1,(k,k)} \phi_i(\xi_k) \quad (2)
\]

**Result**: if \( \phi \) and \( \psi \) bi-orthonormal w.r.t. (2), then \( \kappa(C^T A) = 1 \)

Bilinear form (2) extends inner product (1)

Illustration: IV vs. SK

Optimal conditioning (\( \kappa = 1 \)) for IV-type identification
- both continuous and discrete time
- efficient (\( C(\bar{m}) \)) and reliable computation of bases
- extended to multivariable case

Enhanced convergence at expense of numerical conditioning?

Bi-orthogonal polynomials in identification [6]

| Alg. | basis | \( \kappa \) | \( V(\theta^*) \) | \( ||\frac{\partial V(\theta^*)}{\partial \theta^*}|| \) |
|------|-------|-----------|----------------|------------------|
| SK   | monomial | 1.010 · 10^9 | 27.915 | 0.229 |
| SK   | ort. w.r.t. (1) | 1.000 | 27.915 | 0.229 |
| IV   | monomial | 5.669 · 10^15 | 6.003 | 5.588 · 10^{-13} |
| IV   | bi-ort. w.r.t. (2) | 1.000 | 6.003 | 5.588 · 10^{-13} |

Acknowledgement

The authors gratefully acknowledge important contributions by Okko Bosgra in an early stage of this research.

References


This work is supported by ARN Research and by the International Research Insitituion Scheme under the VENI grant "Precision Motion Beyond the Nanometer" (no. 13073) awarded by NWO (The Netherlands Organisation for Scientific Research) and STW (Dutch Science Foundation).