

# Optimal Digital Control of Analog Systems

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## Introduction

In the last decades, there has been a huge development in the field of digital computers. Many analog control systems have been replaced by digital ones because of lower cost, increased flexibility and superior accuracy [1]. However, most control designs are still performed in the continuous time domain. After designing a continuous time (CT) controller, a discretization step is performed, where the analog controller is approximated by a discrete time (DT) one. Due to the increasing demands on the speed and accuracy of industrial motion systems, the discretization step is undesirable in the control design procedure, since it can result in performance degradation and instability of the closed-loop system.

## Current Design Approach

Recently, a CT model-based control design approach based on  $\mathcal{H}_\infty$  optimization has successfully been applied on a high-performance multi-input multi-output motion system [2]. The CT model, used in the optimization algorithm, is estimated from sampled observations of the system. Moreover, performance requirements are specified by means of weighting filters. The current design is based on loopshaping ideas, since the  $\mathcal{H}_\infty$  norm of a transfer function equals its peak value in the Bode magnitude plot. After subsequent controller synthesis [3], the resulting CT controller is discretized. In the next section, two approaches are discussed that avoid the explicit controller discretization step.

## Alternative Methods

Two norm-based approaches can be distinguished that directly deliver a DT controller without the need of an *a posteriori* discretization step. These are discussed below.

*Discrete Time Approach* The first approach is to estimate a DT model from the sampled observations of the system. After appropriate choice of DT weighting filters, the controller synthesis algorithm typically delivers a DT controller [4]. The major drawback of this approach is that certain performance specifications can only be guaranteed at the sample instants. This is a necessary, but not sufficient condition for

satisfactory performance in continuous time. In particular, a poorly chosen control law can achieve excellent performance at the sampling instants, whereas it results in a highly oscillating behavior between the sampling instants.

*Sampled-Data Approach* Another approach to directly design a DT controller for a CT plant is given by sampled-data control. This approach allows the control designer to specify the performance requirements in the continuous time domain. The controller synthesis algorithm in [5] computes an  $\mathcal{H}_\infty$ -optimal DT controller, given a CT plant, CT weighting filters and digital-analog/analog-digital convertors. A major complication is that for sampled-data systems, *i.e.*, systems consisting of both continuous and discrete time components, the frequency separation principle is lost. Hence, weighting filters based on loopshaping ideas cannot be applied directly [6]. Moreover, the identification of a CT model is not a straightforward task for the control designer.

## References

- [1] K. J. Åström and B. Wittenmark. *Computer-Controlled Systems: Theory and Design*. Prentice Hall, 1990.
- [2] M. M. J. van de Wal, G. van Baars, F. Sperling, and O. H. Bosgra. Multivariable  $\mathcal{H}_\infty/\mu$  feedback control design for high-precision wafer stage motion. *Contr. Eng. Pract.*, 10(7):735–755, 2002.
- [3] J. C. Doyle, K. Glover, P. P. Khargonekar, and B. A. Francis. State-space solutions to standard  $\mathcal{H}_2$  and  $\mathcal{H}_\infty$  control problems. *IEEE Trans. Automat. Contr.*, 34(8):831–847, 1989.
- [4] P. A. Iglesias and K. Glover. State-space approach to discrete-time  $\mathcal{H}_\infty$  control. *Int. J. Contr.*, 54(5):1031–1073, 1991.
- [5] B. A. Bamieh and J. B. Pearson. A general framework for linear periodic systems with applications to  $\mathcal{H}_\infty$  sampled-data control. *IEEE Trans. Automat. Contr.*, 38(5):717–732, 1992.
- [6] M. W. Cantoni and K. Glover. Frequency-Domain Analysis of Linear Periodic Operators with Application to Sampled-Data Control Design. *Proc. CDC*, 4318–4323, 1997.