Multivariable Learning Using Frequency Response Data: A Robust Iterative Inversion-Based Control Approach with Application

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Abstract—Learning control methods enable significant performance improvements for systems that operate repetitively. Typical methods rely on a parametric plant model to achieve fast and robust convergence. The aim of this paper is to develop a framework for multivariable systems that enables fast and robust learning without requiring a parametric plant model. This is achieved by connecting nonparametric frequency response function identification and robust control, which enables synthesis on a frequency-by-frequency basis. A nonconservative approach is obtained by ensuring that the identified uncertainty is directly compatible with the developed synthesis framework. Application to a multivariable benchmark motion system confirms the potential of the developed framework.

I. INTRODUCTION

Learning from past data can significantly increase the performance of systems that perform repeating tasks. Indeed, Iterative Learning Control (ILC) [1], Repetitive Control (RC) [2], and Iterative Inversion-based Control (IIC) [3], increase the tracking performance of systems that start each tasks with identical initial conditions, or that operate continuously in a periodic fashion [4]. Examples of high precision applications include nanopositioning devices [5], additive manufacturing machines [6], and printing systems [7], [8].

The convergence behavior is an essential aspect in iterative control and should typically meet certain requirements. These include: convergence in the presence of uncertainties, improved performance upon convergence, high convergence speed, and monotonicity of convergence [9, §5.1.1].

These aspects are typically addressed in ILC and RC by designing a learning and robustness filter as in the following archetypal example. First, a parametric model plant is estimated from measured data, which introduces a modeling error [10]. Second, to achieve fast convergence, a learning filter is obtained by inverting the plant model [1], which is requires dedicated techniques if the model is nonminimum phase [11]. Third, a robustness filter is designed to achieve convergence in the presence of modeling errors, which limits the achievable performance [1].

Design for robustness is well-developed for Single-Input Single-Output (SISO) systems, whereas for Multi-Input Multi-Output (MIMO) systems this remains challenging. The directionality aspect of the modeling error adds additional complexity, such that existing approaches are typically limited to diagonal designs in combination with MIMO stability criteria [7]. In this way, the full potential of MIMO iterative control is left unexploited. Formal robust control approaches [12] offer a general framework for MIMO ILC and RC [13], but typically require substantial effort to obtain parametric uncertainty models [14], and lead to synthesis problems of high computational complexity that are often intractable for systems of industrial scale.

In IIC, the systematic the modeling errors are eliminated by directly using Frequency Response Function (FRF) data [3]. This is enabled by explicitly using the Fourier transform to update the input. In this way, the learning and robust filters are replaced by complex coefficients, where the learning coefficients are simply obtained by inverting the plant FRF. Various extensions successfully improved the robustness in IIC [4], [15], but are limited to SISO systems or are restricted to diagonal controller designs [16]. Model-free IIC [17] presents a data-based extension for SISO systems, by incorporating the inverse FRF estimation step in the update law. It is shown that this can result in erratic iteration dynamics and that the performance is limited by the ability of the reference to sufficiently excite the system [8].

Although fast and robust learning control is well-developed for SISO systems, presently available techniques are not yet suitable to address complex large-scale multivariable systems. The aim of this paper is to overcome the limitations imposed by multivariable aspects in learning, which is achieved by developing a new FRF-based learning control framework. This main contribution is achieved in combination with the following sub-contributions.

(C1) An FRF-based robust MIMO iterative inversion-based control method is developed (Section III).

(C2) A synthesis method is developed to ensure robust convergence and optimal nominal performance of the iterative control method (Section IV).

(C3) An identification method is developed to estimate the relevant uncertain FRF models (Section V).

(C4) The potential of the developed approach is confirmed by application to a flexible motion system (Section VI). Contribution C1 substantially extends [16] by allowing for full MIMO learning- and robustness matrices. Contribution C2 employs well-developed concepts from $\mu$-synthesis [18] yet does not require heuristic D-scales fitting [12]. Furthermore, C2 is fundamentally different from [19], [20], where robustness is optimized for time domain ILC. Contribution C3 establishes a connection between stochastic FRF identification and $\mu$-synthesis [18], which closely connects to [21].

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Fig. 1. The control configuration with exogenous output disturbances.

A. Preliminaries

The set real numbers, complex numbers, and integers are denoted by \( \mathbb{R}, \mathbb{C}, \) and \( \mathbb{Z} \). For a matrix \( A \in \mathbb{C}^{n \times m} \), \( |A|_{ij} \) denotes the \( ij \)-th element, \( A^H \) is the conjugate transpose, and \( \rho(A) \) and \( \sigma(A) \) represent the largest eigenvalue and largest singular value. For \( a \in \mathbb{C}^n, |a|_2 \) denotes the 2-norm, whereas for \( A \in \mathbb{C}^{n \times m}, |A|_{2,2} \) denotes the induced 2,2-norm. \( S(k) \) denotes the Discrete Fourier Transform (DFT) of a signal \( s(t) \) and is defined on the frequency grid \( \Omega \triangleq \{ \omega_k \in \mathbb{C} | \omega_k = \frac{2\pi k}{N}, k = 0, ..., N-1 \} \). In this article, a square MIMO LTI DT system \( G \) : \( u \mapsto y \) is considered, which is represented by its transfer function \( G(z) \in \mathbb{C}^{n \times n}, z \in \mathbb{C}, \) and \( G(e^{j\omega k}) \) is the FRF of \( G \) on the DFT grid \( \Omega \).

II. Problem Formulation

The problem considered in this paper is to achieve disturbance rejection and accurate tracking for stable MIMO LTI processes that perform identical tasks, where during a task the system is approximately in steady state. This is formalized by the following assumptions.

Assumption 1. The system \( G \) is square, stable and LTI. The output is disturbed by \( v(t) \) and \( d(t) \), such that \( y(t) = G(z)u(t) + d(t) + v(t) \), as shown in Figure 1.

The results in this paper are readily extended to the case that \( G \) is nonsquare, as will be indicated where relevant.

Assumption 2. During the \( i \)-th task, that consists of \( N \) samples, the output \( y(t) \) is measured and its DFT satisfies

\[
Y_i(k) = G(e^{j\omega k})U_i(k) + T_{U_i}(k) + V_i(k) + D(k),
\]

where \( T_{U_i} \) is the transient induced by \( U_i \) \cite[§2.6.3]{10}, \( D \) describes the disturbances that are identical for each trial, and \( V_i \) describes the trial-varying disturbances.

To improve the performance during subsequent tasks, the approach taken in this paper is to iteratively update \( U_i(k) \).

Problem 1. Define the tracking error during the \( i \)-th task as

\[
E_i(k) = R(k) - Y_i(k),
\]

where \( Y_i(k) \) is as given by (1). Determine \( U_{i+1}(k) \) such that \( \lim_{t \to \infty} |E_i(k)|_2 \) is small in an appropriate sense.

Since \( |E|_2 = |e|_2 \) by Parsevals theorem, the time domain signal energy is minimized as in norm optimal ILC \cite{1}.

The stochastic disturbance \( V_i \) presents a limitation to the achievable performance and limits the accuracy up to which the plant FRF, \( G(e^{j\omega k}) \), can be estimated. This uncertainty may cause the iterative procedure to become instable. In contrast to pre-existing approaches, this paper focuses on achieving robust iteration stability by directly exploiting the statistical properties of \( V \), through a combination of system identification and learning.

III. Robust Learning from Data

In this section, a robust FRF-based MIMO learning algorithm is developed as a solution to Problem 1, which constitutes contribution C1. First, the iterative control algorithm is presented. This is followed by introducing an uncertain plant description, which facilitates a robust analysis of performance and convergence. These results are then reformulated into a constrained robust synthesis problem.

The following algorithm fits several existing approaches \cite{3, 4, 8, 15}, whereas the contribution of the present paper is a nonconservative design framework for the learning and robustness matrices \( Q(k), L(k) \in \mathbb{C}^{n \times n} \).

Algorithm Robust finite-time ILC

Given a possibly zero initial input \( u_0(t) \).

1) Apply \( u_0(t) \) and record \( e_i(t) = r(t) - y_i(t) \), when \( y_i \) is approximately in steady state.

2) a) Obtain \( U_i(k) \) and \( E_i(k) \) by applying the DFT.

b) Update \( U_i(k) \) \( \forall k \) as

\[
U_{i+1}(k) = Q(k)(U_i(k) + L(k)E_i(k)).
\]

3) Obtain \( u_{i+1}(t) \) by applying the IDFT.

c) Set \( i \leftarrow i+1 \) and repeat from step 1) until \( e_i \) converges.

By awaiting steady state at 1), \( T_{U_i} \to 0 \) \cite[§2.6.3]{10}. For \( T_{U_i} = 0 \), the relation between \( Y(k) \) and \( U(k) \) is decoupled in the frequency domain. Consequently, the design of \( Q(k) \) and \( L(k) \) can be considered on a frequency-by-frequency basis. Therefore, the argument \( k \) is occasionally omitted from the notation. First, robust convergence is considered, which is facilitated by formulating a set of possible plant realizations.

A. Uncertain FRF model

In practice, the true plant FRF is not perfectly known. This uncertainty can be described by using following model.

Definition 1. Let \( \mathcal{G}(\Delta) \in \mathbb{C}^{n \times n} \) be an FRF model set

\[
\mathcal{G}(\Delta) = G_n + W_y \Delta W_u, \quad \Delta \in \Delta,
\]

where \( W_y, W_u \in \mathbb{C}^{n \times n} \) are scaling matrices.

Here, \( G_n \) is referred to as the nominal model and \( \Delta \) is a set of unstructured perturbations. In Section V, a method is developed to estimate (3) from data that results from dedicated experiments. Robust analysis of (2) is facilitated by assuming that the true system \( G_\circ \) is captured by \( \mathcal{G}(\Delta) \).

Assumption 3. Let \( \mathcal{G}(\Delta) \) be given by Definition 1. There exist a \( \Delta_\circ \in \Delta \) such that \( \mathcal{G}(\Delta_\circ) = G_\circ \).

In the next section, robust convergence and asymptotic performance of (2) are treated.

B. Robust convergence & asymptotic performance

The following assumption enables a deterministic analysis of convergence.

2216
Assumption 4. The transient response and the trial-varying disturbances are zero during learning, i.e., $T_{U_i} = 0$, $V_i = 0$.

To ensure that (2) converges without displaying learning transients [1], the concept of monotonic convergence is considered, which ensures that after each iteration the distance towards the limit value decreases.

Definition 2. A sequence $\{X_i\}, X \in \mathbb{C}^n$, is said to converge monotonically in $\|\cdot\|_2$ to a fixed point $X_\infty \in \mathbb{C}^n$ if $\exists \kappa \in [0, 1)$ such that $\|X_{i+1} - X_\infty\|_2 \leq \kappa \|X_i - X_\infty\|_2 \forall i$.

Note that $\kappa^{-1}$ reflects the speed of convergence. If (2) results in monotonically converging sequences $\{U_i\}$ for all possible realizations of the plant FRF, i.e., $\forall \Delta \in \Delta$, then (2) is said to be robustly convergent. It is emphasized that $\Delta$ is independent of the trials $i$.

Lemma 1. Consider (2) under Assumption 1 to 4 and let $G(\Delta)$ be given by Definition 1. If

$$
\|Q(I - LG(\Delta))\|_{2,2} < 1 \quad \forall \Delta \in \Delta,
$$

(4)

then, sequences $\{U_i\}$ converge monotonically to $U_\infty$, and sequences $\{E_i\}$ converge to $E_\infty$, where

$$
U_\infty = (I - Q(I - LG_o))^{-1}QL(R - D),
$$

(5)

$$
E_\infty = (I - G_o(I - Q(I - LG_o))^{-1}QL)(R - D).
$$

(6)

A proof follows by substituting $U_\infty = U_i = U_{i+1}$ in (2), determining $U_\infty$, and substituting in $E_\infty = R - GU_\infty - D$. Evaluating the condition in Definition 2 for $X = U$ yields that $\kappa$ equals the left hand side of (4).

Clearly, (4) presents a constraint to ensure robust convergence. In combination with (6), a performance optimization problem can be formulated in terms of $Q$ and $L$.

C. Optimizing $L$ and $Q$ for robustness and performance

In this section, the design of $Q$ and $L$ is formulated as a constrained optimization problem.

Since $G_o$ is unknown, (6) cannot directly be minimized. Instead, nominal performance is considered by using $G_n$. This is motivated by the identification method in Section V, which is such that $G_n = G_o$ is the realization with the highest probability. Combining (4) and (6) results in the following optimization problem for each frequency $k$.

Problem 2. Let $G(\Delta)$ be given by Definition 1, and consider

$$
\begin{align*}
\min_{L, Q \in \mathbb{C}^{n \times n}} & \quad \|Q(I - LG(\Delta))\|_{2,2}, \\
\text{subject to} & \quad \|Q(LG(\Delta))\|_{2,2} < 1 \quad \forall \Delta \in \Delta.
\end{align*}
$$

(7a)

Before tackling this problem in the next section, a sequential strategy is proposed to reduce the complexity of (7).

D. Inverse FRF-based learning

Problem 2 depends nonlinearly on the matrix variables $Q$ and $L$. This can be alleviated by optimizing $L$ and $Q$ sequentially. Here, it is chosen to take $L = G_n^{-1}$, where the pseudo inverse is used when $G_n$ is not invertible. The motivation for this is twofold. First, perfect tracking is obtained for $Q = I$, since (7a) equals zero in this case. Second, for $L = G_n^{-1}$, the worst-case convergence speed is maximized. This is illustrated for the SISO case in Figure 2, which shows (7b), i.e., $\|Q(1 - L(G_n + \Delta))\| < |QG_n^{-1}\Delta| < 1$.

IV. Q-SYNTHESIS

In this section, a $Q$-synthesis method is developed by reformulating Problem 2 as a Semi-Definite Program (SDP), which constitutes contribution C2.

This key result is presented by the following Theorem.

Theorem 1. If $L = G_n^{-1}$, then (7) is equivalent to

$$
\begin{align*}
\min_{Q \in \mathbb{C}^{n \times n}, x_2, \beta \in \mathbb{R}} & \quad \beta, \quad \text{subject to:} \quad \beta > 0, \quad x_2 > 0, \\
\begin{bmatrix}
-G_n(I - Q)G_n^{-1}W_R -\beta I \\
x_2W_n^HW_n - I -\gamma QG_n^{-1}W_g -\delta R
\end{bmatrix} & \begin{bmatrix}
-G_n(I - Q)G_n^{-1}W_R \\
-\delta R
\end{bmatrix} \leq 0,
\end{align*}
$$

(11)

A proof follows by reformulating (7b) using the main loop theorem [18, Thm 4.3] and using that the resulting the $\Delta$ condition can be determined exactly by using optimal D-scaling, since the inflated perturbation consist of only 2 full complex blocks. The Linear Matrix Inequalities (LMIs) result by using the Schur complement. Due to space limitations, a detailed proof will be published elsewhere.

The main consequence of this theorem is that the $Q$-matrix can be synthesized in a nonconservative manner by solving (8), which can be done using various algorithms that are known to converge globally in polynomial time [22]. Recall that each frequency $k$ is optimized independently, such that the optimization of the entire $Q(k)$ is readily parallelized.

In this section, a $Q$-synthesis method is developed to maximize the performance while ensuring robust convergence. In the next section, these results are connected to an FRF identification approach to comprise a complete framework.
V. IDENTIFICATION OF THE MODEL SET

In this section, a method is developed to identify the model set \( \mathcal{G}(\Delta) \), as given by (3), which constitutes contribution C3.

Consider the open-loop configuration in Figure 1. An approach [10, §2.7.2] to estimate the MIMO FRF \( G_c(k) \) is to perform \( n \) experiments and construct the matrices

\[
\mathbb{U}(k) = \begin{bmatrix} U_1(k) & \ldots & U_n(k) \end{bmatrix} \in \mathbb{C}^{n \times n}, \tag{9}
\]

\[
\Psi(k) = \begin{bmatrix} Y_1(k) & \ldots & Y_n(k) \end{bmatrix} \in \mathbb{C}^{n \times n}, \tag{10}
\]

\[
\mathcal{G}(k) = \Psi(k)\mathbb{U}^{-1}(k), \tag{11}
\]

where it is assumed that \( \mathbb{U}(k) \) is regular \( \forall k \), which is readily achieved. To quantify the uncertainty of (11), the nature of the disturbances during the experiments is specified.

**Assumption 5.** The output \( Y_i(k) \) satisfies Assumption 2, where \( D(k) = 0 \), and \( V_i(k) \) is circularly symmetric complex normally distributed with regular covariance matrix \( C_V(k) \), and different realizations \( V_i(k) \) are mutually independent.

This is commonly assumed in FRF identification methods, [10, §2.5, §7, §16.16]. Consequently, (11) can be written as

\[
\mathcal{G}(k) = \Psi(k)\mathbb{U}^{-1}(k) = G_c(e^{j\omega_k}) + \Psi(k)\mathbb{U}^{-1}(k), \tag{12}
\]

\[
\Psi(k) = \begin{bmatrix} V_1(k) & \ldots & V_n(k) \end{bmatrix} \in \mathbb{C}^{n \times n}, \tag{13}
\]

where \( \Psi(k) \) is a random matrix, which enables a probabilistic uncertainty description as is shown next.

A. Probabilistic unstructured uncertainty

The following lemma makes a direct correspondence between the FRF estimate (12) and model set \( \mathcal{G}(\Delta) \) as given by Definition 1. This is achieved by considering the cumulative probability of the largest singular value of \( \Psi \).

**Lemma 2.** If \( \Psi \), as given by (13), satisfies Assumption 5, then the cumulative density function of \( \sigma(C_{\Psi}^{-1/2}\Psi) \) satisfies

\[
\Pr\left(\sigma(C_{\Psi}^{-1/2}\Psi) \leq \gamma\right) = \frac{\det(\Psi_c(\gamma^2))}{\prod_{j=1}^{n}\Gamma(n-j+1)^2}, \tag{14}
\]

where \( \Gamma(x) \) is the gamma function [23, eqn. 6.1.1] and \( \Psi_c(x) \in \mathbb{R}^{n \times n} \) is a Hankel matrix function of \( x \in (0,\infty) \), with \( [\Psi_c(x)]_{ij} = \Gamma(i+j-1)\psi(i+j-1,x) \), where \( \psi(a,x) \) is the incomplete gamma function [23, eqn. 6.5.1].

A proof follows from [24, eqn. 8] since \( C_{\Psi}^{-1/2}\Psi \) is a matrix of circularly symmetric complex normally distributed entries. This lemma enables the following key result, which establishes the connection between FRF identification and the developed robust synthesis approach.

**Theorem 2.** Consider (11) under Assumption 1, 2 and 5. Let \( \mathcal{G}(\Delta) \) be given by Definition 1 with \( G_n = \mathcal{G} \), \( W_y = C_{\Psi}^{-1/2} \) and \( W_u = \mathbb{U}^{-1} \). If \( \gamma \) is such that (14) \( \geq \alpha \), then \( \mathcal{G}(\Delta) \) contains \( G_o(k) \) with a probability greater than or equal to \( \alpha \).

Hence, \( \gamma \) can be used to increase the probability that \( G_o(k) \) is contained in \( \mathcal{G}(\Delta) \). This is visualized in Figure 3, which displays (14) for various dimensions \( n \). Note that Theorem 2 considers a single frequency and that \( G_o(k) \) is contained in \( \mathcal{G}(\Delta) \) for all frequencies with probability \( \alpha^{1/2} \).

In summary, combining the developed identification and robust synthesis methods, results in a framework that enables iterative control that converges with a known probability. The complete design approach is summarized in Section VI-A, after a discussion on reducing the uncertainty.

**Remark 1.** The presented results are readily extended to the real-valued DC and Nyquist frequency coefficients, i.e., \( k \in \{0, \frac{m}{2}\} \), by considering real Gaussian matrices [25].

**Remark 2.** In contrast to traditional identification approaches, a bound on \( \sigma(\Delta) = \|\Delta\|_2 \) is formulated here, as opposed to a bound on \( \|\vec{V}(\Delta)\|_2 = \|\Delta\|_F \) [10, §2.7.2], which is not directly compatible with the \( \mu \)-framework.

B. Reducing the uncertainty

The developed results require \( C_V \), which is practically unknown, and can be estimated by collecting \( M \) independent observations of the same experiment \( i \). This allows the uncertainty to be decreased by averaging [21]. This is achieved by replacing the columns of \( \Psi \) by the sample mean \( \hat{Y}_i \), and replacing \( C_V \) by the mean sample covariance of the sample mean \( \hat{C}_V \), where \( \hat{C}_V = \frac{1}{M} \sum_{i=1}^{M} (Y_i^T Y_i) \). Since \( C_V = \frac{1}{M} C_V \), and \( W_y = C_{\Psi}^{-1/2} \), this reduces the uncertainty by a factor \( \frac{1}{\sqrt{M}} \). The independent observations can be readily obtained by using periodic excitations [10, §10.1]. In the next section, it is validated that for a moderate number periods \( M \), Lemma 2 holds with practically relevant accuracy. In this section, an FRF identification method is developed, which, in conjunction with the \( Q \)-synthesis method comprises a complete learning control framework that is applied to a motion system in the next section.

VI. APPLICATION TO A FLEXIBLE MOTION SYSTEM

In this section, the developed framework is applied to increase the performance of a benchmark flexible beam system, which confirms its potential and constitutes contribution C4.

A. Design framework

The complete \( L \) and \( Q \) design framework, as developed in the previous sections, is summarized as follows.

1) Obtain input-output data by performing \( n \) identification experiments. Construct \( \mathbb{U}(k) \) and \( \Psi(k) \), given by (9) and (10). Ensure that the experimental conditions satisfy Assumption 5 and are such that \( \mathbb{U}(k) \) is regular.
Optionally, estimate $C_V(k)$ and reduce the uncertainty as described in Section V-B.

2) Compute $G(k)$ given by (11), and set $G_n(k) = \hat{G}(k)$.
3) Compute $L(k) = G_n^{-1}(k) \forall k$.
4) Compute the $Q$-matrix through the following steps.
   a) Choose a probability $\alpha \in (0, 1)$, and use (14) or Figure 3 to determine $\gamma$, such that (14) $\geq \alpha$.
   b) Set $W_p(k) = C_V^{1/2}(k)$, $W_a(k) = U^{-1}(k)$ and solve the LMI problem given by (8) $\forall k$.

In the next section this approach is illustrated by application.

B. The plant

Application of the developed approach to the benchmark beam system, as presented in [26], is simulated using a 16-th order model with dimension $n = 2$. This model was estimated using the method developed in [27]. The resulting FRF of the discretized parametric model is shown in Figure 4.

C. Identification of the model set

The FRF model set was identified by following step 1). To evaluate of the potential of the developed framework, the disturbance $v(t)$ was set to be a white noise sequence with covariance matrix $C_V(k) = \sigma_V^2 \begin{bmatrix} 0.16 & 0.4 \\ 0.4 & 0.01 \end{bmatrix} \forall k$, with $\sigma_V = 5 \cdot 10^{-6}$. The input-output data was obtained by performing $n = 2$ different experiments where during each experiment either one of the inputs was a random-phase multi-sine with a unity spectrum such that $U(k) = I, \forall k$. Each experiment was performed by repeating the input for $M + M_t$ periods, where the first $M_t = 4$ periods were used to achieve steady state, and where the remaining $M = 50$ periods were used to estimate the sample means $\bar{Y}_e(k)$ and their corresponding sample covariance $C_V(k)$, as is discussed in Section V-B.

D. $Q$-synthesis

After $L(k) = G_n^{-1}(k)$ was computed, $Q(k)$ was synthesized by performing step 4). By using the visual aid provided by Figure 3, $\gamma = 4$ was selected at step b), which for $n = 2$ results in a probability of $\alpha \approx 0.99996$, as is determined using (14). This results in a probability of $\alpha^{N/2} \approx 0.98$ that convergence is achieved simultaneously for all frequencies.

In the remainder, two cases are considered: i) a full complex matrix, i.e., $Q_f \in \mathbb{C}^{2 \times 2}$, and ii), a complex diagonal matrix, i.e., $[Q_d]_{ii} \in \mathbb{C}$, $[Q_d]_{ij} = 0$ for $i \neq j$. This was readily implemented at step c) by using YALMIP [28] in combination with an appropriate Semi-Definite Program (SDP) solver. The resulting $Q$-matrices are shown in Figure 5, which displays the magnitude and phase. Three key observations are made. First, it is clear that up to 60 Hz, both $Q$ matrices are equal to the identity matrix, i.e., $Q = I$, which should lead to perfect asymptotic tracking at those frequencies, as is validated in the next section. Second, beyond 60 Hz, the freedom to assume any matrix in $\mathbb{C}^{2 \times 2}$ was fully exploited by $Q_f$, i.e., the off-diagonals are non-zero, $Q$ is clearly not hermitian, and all entries are complex. In contrast, $Q_d$ turned out to be a real matrix. This is in close agreement with the notion of using phase-less robustness filters [2]. Third, and probably most remarkably, beyond 60 Hz, $Q_f$ displays relatively large peaks. This goes against the SISO intuition which requires that $|Q| < 1$ to increase the robustness [4]. The latter indeed turned out to be the optimal solution for $Q_d$. Next, the designed $L$ and $Q$ matrices are employed to iteratively increase the tracking performance.
Fig. 6. The initial tracking error $e(t)$ (---) and the error after a single iteration of applying the algorithm described in Section III for $Q_f$ (---) and $Q_d$ (——). This shows that the performance is significantly improved for both $Q_f$ and $Q_d$, yet $Q_f$ outperforms $Q_d$ in the second channel.

E. Iterative control results

The control goal is to track a high frequency triangle wave with both outputs. To achieve this, the iterative control algorithm was applied with the designed $L$ and $Q$ matrices, as is presented in Section III. The error was measured after 2 seconds to achieve steady state, and $v(t)$ was as during the identification experiments. In this case, convergence was attained after a single iteration, which resulted in a significant performance increase, as is shown in Figure 6, which shows the original tracking error, and the tracking error obtained by using $Q_d$ and $Q_f$. Clearly, $Q_f$ provides superior performance for output 2. More specifically, for $Q_f$, $\|e(t)\|_2 = 6.0 \cdot 10^{-4}$, whereas for $Q_d$, $\|e(t)\|_2 = 7.8 \cdot 10^{-4}$, where $\|e(t)\|_2 = \sqrt{\sum_{t=0}^{N} e(t)^2}$. This confirms that there is a benefit to using full robustness matrices over diagonal matrices in iterative inversion-based control.

VII. CONCLUSION

A learning control method is developed that improves the performance of multivariable systems that perform repetitive tasks. Fast and robust convergence is achieved by developing a nonconservative inverse-FRF-based approach that allows frequency-by-frequency optimization of the robustness. Simulated application of the method demonstrated its efficacy.

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