Abstract—Many control applications are nonlinear and have to perform a range of different tasks. Iterative Learning Control (ILC) enables high performance for a single task, but is highly sensitive to task variations. The aim of this paper is to develop an ILC framework for Linear Parameter Varying (LPV) systems, which allows for trial-varying reference signals. This is achieved by exploiting parameter varying basis functions, such that perfect tracking is enabled for LPV systems. The proposed approach is applied to a printer sheet positioning unit, thereby validating that the tracking performance is significantly enhanced with respect to existing approaches.

I. INTRODUCTION

Iterative Learning Control (ILC) enables high tracking performance by learning a dedicated input sequence to perform a specific task [1]. This is shown to be an effective strategy for systems that perform identical tasks such as additive manufacturing systems [2] and wafer stages [3].

ILC typically exhibits poor extrapolation properties with respect to trial-varying tasks or dynamics. Indeed, a key assumption in ILC is that the both disturbances and the system dynamics remain identical during each task [4]. Consequently, the learned input signal is only optimal for a specific task and the corresponding dynamic response of the system. The performance typically deteriorates significantly when this input is applied for the completion of a different task [5], or when the dynamics are different [6], [7].

ILC has been extended to increase flexibility with respect to varying tasks in case the system dynamics are Linear Time-Invariant (LTI) and trial-invariant. A segmentation approach is suggested in [8] and is extended in [9], where the tasks are assumed to consist of standard subtasks which are learned individually and can be retrieved from a signal library as building blocks for different tasks. ILC with basis functions provides an alternative method to parametrize the ILC input in terms of the task and can be interpreted as a way to learn a parametrized feedforward compensator from the previous trials [10], [11], [12].

Various control systems exhibit trial-varying dynamics and ILC is extended in several ways to accommodate for this. In [13], the case is treated where changing system parameters result in different LTI dynamics from one trial to the next. This is accounted for by using an ILC update law that exploits all previous trials. Trial-varying Linear Time-Varying (LTV) dynamics arise when a nonlinear system is linearized along a specific trajectory, which changes when a different task is performed. In [7], a robust approach is suggested to achieve convergence to a neighborhood around the nominal solution which is bounded by bounding the variation in dynamics. Alternatively, the global nonlinear dynamics are considered in [14] to determine convergence with respect to a given single reference.

ILC methods for nonlinear systems typically impose severe requirements on system identification and controller synthesis. The Linear Parameter-Varying (LPV) systems framework provides a suitable alternative by modeling the nonlinear system as a family of LTV systems [15]. By relating each LTV system to a specific realization of a measurable scheduling signal, a linear description is obtained that facilitates black-box identification [16] and optimal controller synthesis [17]. The latter is exploited for infinite time ILC for LPV systems in [18].

Although many nonlinear control systems perform varying tasks, at present, the performance of these systems cannot be improved through iterative learning. This paper aims to fill this gap by developing an ILC framework for LPV systems that explicitly accommodates for trial-varying tasks. This is achieved through the following subcontributions.

(C1) A generalizing ILC with basis functions framework is developed for LPV systems from which the LTI case is recovered as a special case (Section IV).

(C2) LPV basis functions are derived that enable perfect tracking for both general tracking and point-to-point motion tasks (Section V).

(C3) The proposed approach is applied to a sheet positioning system, which shows that performance enhancement and increased flexibility is obtained with respect to existing approaches (Section VI).

Contribution C1 extends the approaches in [10] and [19] to LPV systems. The proposed linear parametrization in C2 enables perfect tracking for rational LPV systems, whilst enabling an explicit solution to the proposed ILC optimization problem. This parametrization provides an input-output alternative to the state-space methods presented in [18], to explicitly describe LPV compensators that enable perfect tracking by using noncausal dependence on the scheduling parameter. Contribution C3 shows that the proposed method addresses a key challenge in sheet positioning control [20], [21], as is elaborated in Section III.

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II. Preliminaries

A matrix $A \in \mathbb{R}^{n \times n}$ is positive definite, i.e., $A > 0$, if $\| x \|_A^2 > 0 \forall x \in \mathbb{R}^n$, where $\| x \|_A^2 = x' A x$. Throughout, Discrete Time (DT) signals $s : \mathbb{Z} \rightarrow \mathbb{R}$ are considered. A Single-Input Single-Output (SISO) DT-LPV system $G(\rho) : u \rightarrow y$ refers to the following recurrence relation [16] between the input sequence $u$ and the output $y$,

$$G(\rho[k], q) = \frac{B(\rho[k], q)}{A(\rho[k], q)} = \frac{\sum_{i=0}^{b_q} b_i(\rho[k]) q^{-i}}{\sum_{i=0}^{a_q} a_i(\rho[k]) q^{-i}}, \quad (1)$$

where $q$ is the shift operator, i.e., $q^{-\tau} s[k] = s[k - \tau]$, $\rho$ is a scheduling signal which is said to be admissible if $\rho[k] \in \mathcal{D} \subset \mathbb{R}^n$, and $\kappa \in \mathbb{N}$ is the constant relative degree if $b_k(\rho) \neq 0 \forall \rho \in \mathcal{D}$ [18]. Alternatively, $G(\rho)$ is represented as,

$$G(\rho[k], q) = \sum_{i=0}^{\infty} g_i(\rho[k]) q^{-i},$$

and is asymptotically stable if $\lim_{k \rightarrow \infty} g_i(\rho[k]) = 0$ for all admissible $\rho$, [22]. Moreover, $G(\rho)$ is at rest at $k = k_0$ if $y[k] = 0, k \in [k_0 - n_a - 1, k_0]$. Additionally, a signal is lifted and related to trial $j$, with length $N$, as follows,

$$s^{(j)} = [s^{(j)}[0] \ldots s^{(j)}[N - 1]]^\top \in \mathbb{R}^N,$$

and $y = G(\rho) u$ denotes the lifted signal of $y = G(\rho) u$.

III. Problem Formulation

In this section, the general tracking and point-to-point tracking problem are introduced, along with a case study that is a typical example from this class of problems.

A. The Control Problem

The aim in general tracking is to achieve a desired output $r$ at all time, which is formulated as follows.

**Problem 1** (General tracking). Given an LPV system $G(\rho)$ given by (1) that is at rest for $k = 0$, find $u$ such that $y = r \forall k \in [0, N - 1]$, for all admissible $\rho$.

In contrast, the aim of point-to-point tracking is to steer the output of a system $G(\rho)$ from rest to a desired end value $r_f$ within finite time and to keep it at this constant value [19].

**Problem 2** (Point-to-point tracking). Given an LPV system $G(\rho)$ given by (1) that is at rest for $k = 0$, find $u$ such that $y[k] = r_f \in \mathbb{R} \forall k \in [k_p, N - 1]$, for all admissible $\rho$.

Compared to general tracking, the requirement that $y$ should equal an arbitrary signal is relaxed to requiring that $y$ should be equal to a constant value $r_f$ after a given number of samples $k_p$. This allows for more design freedom since the output can follow less demanding trajectory prior to $k = k_p$.

B. Motivating Case Study

In this paper, solutions to Problem 1 and 2 are developed which are applicable to a wide range of applications, including the sheet positioning unit considered in this paper, as is shown in Figure 1. The objective of this control system is the following. At time $k = 0$, a sheet enters the correction area with a constant velocity $v_x$ and is clamped between two actuated pinch wheels. At this point, the sheet has an angular offset $d_{\phi}$, which is assumed to be known. The aim is to correct the angle with the actuated pinches, while the sheet is moving in the longitudinal $x$-direction. This can be achieved with general tracking or point-to-point tracking, where $r$ is a predetermined correcting reference trajectory based on $d_{\phi}$.

Since the offsets are different for each sheet, the control system has to perform a similar, but trial-varying task [21], as is shown in Figure 2. Moreover, the inertia of the sheet depends on the the distance between the mass-center of the paper and mid-point between the pinches, which is denoted by $\rho$. Since $\rho$ changes rapidly due to longitudinal velocity $v_x$, the system dynamics are nonlinear. Throughout, $v_x$ is considered an exogenous variable, hence, the system can be modeled as an LPV system with scheduling parameter $\rho$ [15]. In addition, since sheets of several dimensions should be handled at various velocities, the scheduling trajectories $\rho$ are similarly trial-varying, as is shown in Figure 2.

In the next section, an ILC framework is proposed with required extrapolation flexibility to solve the class of problems presented in this section.
IV. ILC with Basis Functions for LPV Systems

In this section, an ILC framework for LPV systems is developed that accommodates for trial-varying tasks, which constitutes contribution C1. This is achieved by combining parametrized LPV feedforward and input shaping controllers whose parameters are iteratively updated by using ILC.

A. Combined feedforward and input-shaping control

To solve Problem 1 and 2, the control architecture as shown in Figure 3 is proposed, where \( G(\rho) \) is the plant, \( C_{ff}(\rho) \) is a feedback controller, \( C_{ff}(\rho, \theta) \) is the feedforward controller, and \( C_y(\rho, \theta) \) is the input-shaper. Moreover, \( r \) is a known reference, which for Problem 2 ends in the desired point \( r_f \), as is shown in Figure 4. Input-shaping is included to exploit the freedom in Problem 2, where \( y \) may differ from \( r \). It is shown in Section V that this configuration enables perfect tracking when \( C_{ff}(\rho) \) and \( C_y(\rho) \) are parametrized as Finite Impulse Response (FIR) filters. Such a parametrization ensures stability and allows for an explicit solution in norm-optimal ILC, as is shown next.

B. The ILC optimization problem

Optimizing the parameters \( \theta^{(j)} \) of \( C_{ff}(\theta^{(j)}, \rho) \) and \( C_y(\theta^{(j)}, \rho) \) can be achieved by minimizing the norm of the predicted error during the next trial \( e^{(j+1)} \), based on the reference \( r^{(j)} \), the scheduling trajectory \( \rho^{(j)} \), and the measured tracking error \( e^{(j)} \) of the previous task. To this end, consider \( e^{(j)} \) when the system starts from \( r^{(j)} \),

\[
e^{(j)} = S(\rho^{(j)})(C_y(\rho^{(j)}, \theta^{(j)})) - G(\rho^{(j)}))C_{ff}(\rho^{(j)}, \theta^{(j)})r^{(j)},
\]

where \( S(\rho) = (1 + G(\rho)C_{ff}(\rho))^{-1} \).

The lifted error during the next trial is then predicted as,

\[
\hat{e}^{(j+1)} = e^{(j)} - \hat{\mathcal{E}}(\theta^{(j)}, \rho^{(j)}, r^{(j)}) + \hat{\mathcal{E}}(\theta^{(j+1)}, \rho^{(j)}, r^{(j)}),
\]

where it is assumed that during the next task the same reference is to be tracked under the same scheduling. Here, \( \hat{\mathcal{E}} \) is an estimate of \( \mathcal{E} \) that is obtained by replacing \( S(\rho) \) and \( G(\rho) \) with parametric models \( \hat{S}(\rho) \) and \( \hat{G}(\rho) \). The flexible ILC problem for LPV systems is formulated as follows.

Problem 3 (ILC with basis functions for LPV systems). Let \( e^{(j)} \) be the lifted tracking error during trial \( j \) and define \( \hat{e}^{(j+1)} \) as (3). Then, determine the new parameters \( \theta^{(j+1)} \) as,

\[
\theta^{(j+1)} = \arg \min_{\theta} \mathcal{J}(\theta^{(j+1)}),
\]

where \( \mathcal{J}(\theta^{(j+1)}) = \|e^{(j+1)}\|_{W_e}^2 + \|\theta^{(j+1)}\|^2_{W_\theta} + \|\theta^{(j+1)} - \theta^{(j)}\|^2_{W_{\Delta \theta}}, \)

and \( W_e, W_\theta \) and \( W_{\Delta \theta} \) are predefined weighting matrices.

The optimization problem (4) depends on the way \( e^{(j+1)} \) depends on \( \theta^{(j+1)} \), which is determined by the parametrization of \( C_{ff}(\theta) \) and \( C_y(\theta) \). Next, a linear parametrization is adopted and an explicit solution to Problem 3 is derived.

C. Explicit ILC for linear parametrizations

Consider the following parametrization of \( C_{ff}(\theta) \) and \( C_y(\theta) \),

\[
C_{ff}(\rho, \theta) = \sum_{i=0}^{n} \phi_i(\rho) \psi_i(\theta) \theta_i^{ff}, \tag{5a}
\]

\[
C_y(\rho, \theta) = \left( \varphi_0(\rho) + \sum_{i=1}^{m} \varphi_i(\rho) \psi_i(\theta) \theta_i^{ff} \right) \eta^\top, \tag{5b}
\]

where \( \varphi_i(\rho) \) and \( \phi_i(\rho) \) are basis functions of \( \rho \), \( \eta, \theta \) are the converged error is given by,

\[
\hat{e}(\theta, \rho, r) = \hat{S}(\rho)\varphi(\rho)r - \hat{\Phi}(\rho, \theta), \quad \theta = \left[ \begin{array}{c} \theta^y \\ \theta^{ff} \end{array} \right],
\]

\[
\hat{\Phi}(\rho, \theta) = \left[ \begin{array}{c} \hat{\Phi}_f(\theta) \\ \hat{\Phi}_i(\theta) \\ \hat{\Phi}_i(\theta) \\ \hat{\Phi}_i(\theta) \end{array} \right],
\]

\[
\hat{\Phi}_f(\theta) = \left[ \begin{array}{c} \theta_i^{ff} \\ \theta_n^{ff} \end{array} \right], \quad \theta = \left[ \begin{array}{c} \theta^y \\ \theta_i^{ff} \end{array} \right],
\]

where \( \hat{\Phi}(\rho, \theta) = \mathbb{R}^{n_{\theta} \times n_{\theta}} \) with \( n_{\theta} = n + m + 1 \). The linear-in-the-parameters property enables the following result [12].

Lemma 1. Considering Problem 3 with parametrization (5), then the optimal parameter update is given by,

\[
\theta^{(j+1)} = \left( I - Q^{(j)} \right) e^{(j)} + Q^{(j)} r^{(j)} \theta^{(j)},
\]

where \( L \in \mathbb{R}^{n_{\theta} \times n} \) and \( Q \in \mathbb{R}^{n_{\theta} \times n_{\theta}} \) are given by,

\[
L(\rho, r) = (\hat{\Phi}^\top W_e \hat{\Phi} + W_\theta + W_{\Delta \theta})^{-1} \hat{\Phi}^\top W_e,
\]

\[
Q(\rho, r) = (\hat{\Phi}^\top W_e \hat{\Phi} + W_\theta + W_{\Delta \theta})^{-1} \hat{\Phi}^\top W_e \hat{\Phi} + W_{\Delta \theta}.
\]

\( \theta^{(j+1)} \) globally minimizes \( \mathcal{J}(\theta) \) if \( \hat{\Phi}^\top W_e \hat{\Phi} + W_\theta + W_{\Delta \theta} > 0 \). Moreover, \( \|e^{(j+1)} - \theta^{(j)}\|^2 \) converges monotonically in the trial domain if \( \hat{\sigma}(Q - L\hat{\Phi}) < 1 \), where \( \sigma(\Phi) \) represents (6) with the true system transfers \( S(\rho) \) and \( G(\rho) \) and where the converged error is given by, \( e^{\infty} = e_0 - \Phi e^{\infty}, \) with

\[
e^{(j)} = \left( I - Q^{(j)} \right) e_0.
\]
The proof of this lemma follows along similar lines as in [12]. This lemma provides the explicit expression of the LPV learning and robustness matrices $L$ and $Q$ that are used to update $C_{ff}(\rho)$ and $C_{y}(\rho)$. It also provides guidelines to tune the weighting filters such that monotonic convergence of $\theta$, and a satisfactory $e_{\infty}$ is achieved.

In this section, an ILC method is proposed for LPV systems, which enables improved tracking performance for systems with variable dynamics that perform non-repeating tasks. The next section treats the selection of basis functions.

V. SELECTING LPV BASIS FUNCTIONS

In this section, scheduling basis functions $\phi_i(\rho)$ and $\varphi_i(\rho)$ are derived that solve Problem 1 and 2, which constitutes contribution C2. It is shown that these functions are distinctly different from what would result by ad hoc extension of LTI systems, which enables improved tracking performance for different from what would result by ad hoc extension of LTI contribution C2. It is shown that these functions are distinctly derived that solve Problem 1 and 2, which constitutes

$$C_{ff}(\rho) = \sum_{i=0}^{n_k} \alpha_i(\rho) q^{-i}, \quad C_y(\rho) = \sum_{i=\kappa}^{n_k} \beta_i(\rho) q^{-i}, \quad (7)$$

from which $\phi_i(\rho)$ and $\varphi_i(\rho)$ are obtained as follows.

**Lemma 2.** Parametrization (5) and (7) are equivalent for $\tau = \kappa$, $\theta_f = 1$, $\theta_s = 1 \forall i$ and,

$$\left[ \begin{array}{c} \psi_0(\rho) \ldots \psi_n(\rho) \end{array} \right] = \left[ \begin{array}{c} \alpha_0(\rho) \ldots \alpha_n(\rho) \end{array} \right] T_{n+1}^{-1},$$

$$\left[ \begin{array}{c} \varphi_0(\rho) \ldots \varphi_\kappa(\rho) \end{array} \right] = \left[ \begin{array}{c} \beta_0(\rho) \ldots \beta_\kappa(\rho) \end{array} \right] T_{\kappa+1}^{-1},$$

$$T_n = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 0 & T_x & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & T_x^n \end{bmatrix}, \quad D_n = \begin{bmatrix} 0 & 0 & \ldots & 0 \\ 1 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 2 & \ldots & 0 \end{bmatrix}. \quad (8)$$

Proof. The proof follows directly by recognizing that $\left[ \psi_0 \ldots \psi_n \right]^T = T_n \left[ 1 \ldots q^{-1} \right]^T$. \hfill $\square$

A. General tracking

From $e^{(i)}$ as given by (2) it is clear that Problem 1 is solved for $C_{y}(\rho) = 1$, and $C_{ff}(\rho) = G^{-1}(\rho)$. The following theorem presents the explicit expression of $G^{-1}(\rho)$.

**Theorem 1.** For any $G(\rho)$ given by (1) with constant relative degree $\kappa$, the right-inverse $G^{-1}(\rho)$ of $G$, which is such that $r = G(\rho)G^{-1}(\rho)r$ for any $r$ and $\rho$, is given by,

$$G^{-1}(\rho[k + \kappa], q) = \frac{\sum_{i=0}^{n_k} a_i(\rho[k + \kappa]) q^{-i}}{\sum_{i=\kappa}^{n_k} b_i(\rho[k + \kappa]) q^{-i}}. \quad (8)$$

Proof. The proof follows by computing $r = G(\rho)G^{-1}(\rho)r$ on signal level, which shows that $r = r$. \hfill $\square$

This theorem shows that in case the relative degree of $G(\rho)$ is non-zero, i.e. $\kappa > 0$, $G^{-1}(\rho)$ is noncausal in both $r$ and $\rho$. Equation (8) naturally leads to a linear parametrization for the following class of systems.

**Corollary 1.** For any $G(\rho)$ given by (1) with constant relative degree $\kappa$ and with $B(\rho) = b_\kappa(\rho[k]) q^{-\kappa}$, $C_y(\rho)$ and $C_{ff}(\rho)$ as given by (7) result in perfect tracking for,

$$a_i(\rho[k]) = b_i(\rho[k + \kappa]), \quad \beta_\kappa = 1, \quad \beta_i = 0. \quad \beta_i(\rho[k]) = 1, \quad \forall k \in [k_p, N - 1], \quad (12)$$

and holds for $\rho[k] = \rho_f \in \mathbb{R}^n$, $k \in [k_p, N - 1]$, if

$$\sum_{i=\kappa}^{n_k} \beta_i(\rho[k + i]) = 1, \quad \forall k \in [k_p, N - 1] \quad (13)$$
Similarly, the proof of this lemma will be presented elsewhere. This lemma shows that when $\rho$ is constant when $r$ is constant, (13) can be combined with (11) to result in a square system of equations that is independent of $k$. The solution of which results in $\alpha_i(\rho)$ and $\beta_i(\rho)$ that solve Problem 2. In contrast, when $\rho$ is not constant in the performance interval, (12) must be satisfied in combination (11) which results in a set of equations that depends explicitly on $k$. Consequently, R1 and R2 cannot be simultaneously satisfied for arbitrary $\rho$ by using $C_{tf}(\rho)$ and $C_{v}(\rho)$ that depend explicitly on $\rho$ and are given by (7), as is illustrated in Section VI.

C. Causal scheduling functions

Corollary 1 and Lemma 3 show that perfect tracking for LPV systems requires knowledge about future values of $\rho$. In some situations this information may not be available and a causal parametrization is desired. Such a parametrization is obtained by ignoring the time-varying behavior and assuming commutativity, which results in,

$$\begin{align*}
[\alpha_0(\rho) \ldots \alpha_{n_i}(\rho)] &= \gamma^{-1}(\rho) [a_0(\rho) \ldots a_{n_i}(\rho)] , \\
[\beta_0(\rho) \ldots \beta_{n_i}(\rho)] &= \gamma^{-1}(\rho) [b_0(\rho) \ldots b_{n_i}(\rho)] , \\
\gamma(\rho) &= \sum_{i=K}^{n_i} b_i(\rho) \quad \text{if} \quad \gamma(\rho) \neq 0 \forall \rho \in D. \quad (14)
\end{align*}$$

Although these basis functions do not enable perfect tracking, significant performance improvement is enabled with respect to LTI basis functions. Moreover, note that this parametrization satisfies R2 since (12) holds, which results in superior point-to-point performance for arbitrary scheduling.

In this section, guidelines are provided for the selection of scheduling functions that: (1) enable perfect tracking, and (2) depend on $\rho$ in a causal fashion. These variations are compared by application to a sheet positioning system next.

VI. APPLICATION TO A PRINTER SHEET POSITIONER

In this section, the ILC method for LPV systems, as presented in Section IV, is applied to a simulated printer sheet positioning system to achieve point-to-point tracking as discussed in Section V-B. This shows that a significant improvement is enabled by using the LPV basis functions as discussed in Section V, which constitutes contribution C3.

A. The Sheet Positioner Model

This example considers the simplified dynamics between the relative velocity of the pinch wheels and the rotation $d_\phi$ of sheet with respect to the fixed world, which are modeled by (1), where the coefficients are given by,

$$\begin{align*}
[a_0 \ a_1 \ a_2 \ a_3] &= [m \ d \ m \ k - 2d + 3m \ d - k - m] , \\
[b_0 \ b_1 \ b_2] &= [1 \ (\omega_0^2 - 2) \ (1 + \omega_0^2 - \omega_r^2)] \frac{k}{\omega_r^2 T_s} , \\
m(\rho) &= (1 + \rho^2) T_s^{-2} , \\
d(\rho) &= (150 + 25\rho^2) T_s^{-1} , \\
\omega_r(\rho) &= (\frac{3}{2}\rho + 1)200m T_s , \\
k &= (120m)^2 , \quad D = [0,2].
\end{align*}$$

Figure 5 illustrates the variation of the LTI dynamics for constant $\bar{\rho} \in D$ in the frequency range of interest, which shows that a large variation in the resonance frequency occurs due to the changing inertia at the load side.

B. Point-to-Point Tracking

The control objective is to solve the point-to-point tracking problem as described in Problem 2, for the various tasks as shown in Figure 2. This is achieved by using the ILC method as presented in Section IV, where the open-loop case is considered, i.e. $C_{f}(\rho) = 0$, $S(\rho) = 1$. To show the potential of using LPV basis functions, parametrization (5) is used with the following basis functions,

LTI $\varphi_1 = \phi_i = 1$, $\tau = 3$.

LPV (Causal) $\alpha_i(\rho)$, $\beta_i(\rho)$ are given by (14).

LPV (Non-causal) $\alpha_i(\rho)$, $\beta_i(\rho)$ follow from Lemma 3 by simultaneously solving (11) and (13).

For all cases $n = 3$ and $m = 2$, and for the LPV cases, $\varphi_i(\rho)$ and $\phi_i(\rho)$ follow from $\alpha_i(\rho)$, $\beta_i(\rho)$ by using Lemma 2.

ILC is applied by updating the parameter $\theta$ as given by Lemma 1, with $W_c = I$, $W_0 = W_{\Delta \theta} = 0$, and where $\hat{G}(\rho) = \frac{n}{m} G(\rho)$ to illustrate that ILC improves tracking performance when a nonperfect model of the system is used. Moreover, the output of the system is perturbed by Gaussian white noise with variance $\sigma^2 = 10^{-8}$ to mimic experimental conditions.

C. Results

The trial-domain results are shown in Figure 6, which shows that the noncausal LPV basis functions achieve tracking performance up to the noise floor after 6 iterations, regardless of the trial-varying $r$ and $\rho$, and the non-perfect model $\hat{G}(\rho)$. Moreover, the causal LPV basis achieves a significant performance improvement, whereas the LTI basis performs considerably worse. Figure 7 shows the input $u(10)$ and tracking error $e(10)$ during the final task. The top plot shows that the LPV basis’ introduce the flexibility to decrease the input amplitude according to the decreasing
effective inertia. The LTI basis is not able to account for this, which results in the large overshoot as shown in the bottom plot. Figure 8 shows the error with respect to the unshaped reference in the performance interval, i.e. $r[k] - y_{\text{ref}}^{[10]}[k] \forall k \in [k_p, N - 1]$. The top plot shows that the causal LPV basis achieves superior performance. The reason for this is that although it does not achieve $e = 0$, i.e. requirement R1 of Theorem 2, it does guarantee that the reference is unaltered the performance interval, i.e. R2, while $\|e\|_2^2$ is minimized. The opposite holds for the noncausal basis which does not satisfy R1, but does satisfy R2. However, when $\rho$ is kept constant during the performance interval, R2 is indeed satisfied by the noncausal LPV basis, which results in near perfect tracking, as is shown in Figure 8.

VII. CONCLUSION

An ILC framework for LPV systems is proposed that enables iterative performance improvement for nonlinear systems that perform varying tasks. This is achieved by using basis functions in terms of the reference and scheduling signals. Application shows that significant performance improvement is obtained with respect to existing approaches.

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