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Frequency Domain Design of Iterative Learning Control and Repetitive Control for Complex Motion Systems

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Iterative learning control (ILC) and repetitive control (RC) enable high performance for systems that execute repeating tasks. The aim of this paper is to provide an introduction to the design of ILC algorithms for motion systems and to indicate important recent developments. To enforce robust convergence, the design of ILC/RC algorithms often require detailed parametric specifications of the nominal model and its uncertainty. In this paper, a frequency-domain design procedure is outlined that enables robust design through using FRF measurements, which are often inexpensive, accurate and fast to obtain. Application to a consumer electronic printer is reported.

Keywords: Iterative Learning Control

1. Introduction

Many motion systems perform repetitive tasks. Iterative learning control (ILC) [1] and repetitive control (RC) [2] can significantly improve the performance of such systems, since they can effectively compensate the reproducible, or repeating, part of the error, using limited model knowledge. Successful applications that include repetitive tasks are for instance printers, pick-and-place machines, and wafer stages. In this paper the focus is on the design of ILC algorithms, the RC case follows similarly, [2].

Robust convergence is crucial to deal with limited model knowledge. Time-domain design approaches often require detailed parametric specifications on the model uncertainty, which can be expensive and time consuming to obtain. Instead, frequency-domain ILC/RC design allows to use inexpensive FRF measurements to cope with modelling errors.

In this paper, a frequency-domain design framework for ILC is presented and experimentally applied to a printer. Relations to frequency-domain RC design are provided, which is conceptually similar, and extensions are presented towards multivariable systems and robustness to non-repeating tasks.

2. Problem Formulation

2.1 ILC Setup The control scheme as depicted in Figure 1 is considered. This control scheme consists of an LTI plant $G(z)$ and a stabilizing linear feedback controller $C(z)$. Each repetition j of the reference signal r is referred to as a task. This reference is iteration invariant, i.e., $r_j = r$. The feedforward and output signal of iteration j are denoted by f_j and y_j, respectively. The error in iteration j is given by

$$e_j = Sr - GSf_j \text{ with } S = (I + GC)^{-1}. \quad (1)$$

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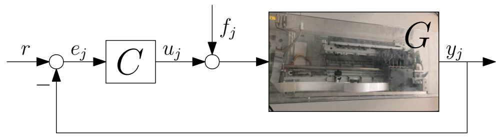

Fig. 1. Schematic representation of ILC configuration with printer setup depicted in G.

The goal of ILC is to minimize, or achieve convergence of, e_j in terms of an appropriate norm, ideally $e_\infty = \lim_{j\to\infty} e_j = 0$ subject to robust convergence of the algorithm. To this purpose, e_j is measured in task j and used to construct f_{j+1} for task $j+1$. Typically, an ILC algorithm of the form:

$$f_{j+1} = Q(f_j + Le_j) \quad (2)$$

is used, where L, Q can be non-causal, i.e., $L, Q \in \mathcal{RL}_\infty$.

2.2 Convergence Analysis The closed-loop error propagation of the ILC algorithm (2) is given by

$$e_{j+1} = Q(1 - GSL)e_j + (1 - Q)Sr, \quad (3)$$

which converges montonically in $\|e\|_2$ if

$$|Q(e^{j\omega})(1 - G(e^{j\omega})S(e^{j\omega})L(e^{j\omega}))| < 1, \forall\omega \in [0, 2\pi]. \quad (4)$$

2.3 Problem Formulation The problem addressed is the design of the filters L and Q to guarantee

- robust convergence of the error, i.e., satisfying (4), and
- high performance, i.e., e_∞ should be small.

It can be shown that these conditions are satisfied if $Q = 1$ and (4) is satisfied. This motivates to design $L \approx (GS)^{-1}$ and Q as close to 1 as possible. Furthermore, it is emphasized that robustness to modelling errors, i.e., $GSL \neq 1$, can be accounted for through appropriate design of Q.

In this paper, it is shown how to design L and Q in the frequency-domain, using only limited knowledge of GS and a FRF measurement.

Remark 2.1 The design of RC is conceptually similar to ILC, see [2]. Stability of an RC scheme is achieved if $|1 - TL||Q| < 1, \forall\omega \in [0, 2\pi]$, with $T = (1 + GC)^{-1}GC$, from which the design aims $L = T^{-1}$ and $Q = 1$ follow.

3. Frequency-domain design of L and Q in ILC

The procedure to design the L and Q-filter consists out of the following three steps

(1) obtain approximate parametric model $\widehat{GS}$,
(2) design L to approximate $(\widehat{GS})^{-1}$,
(3) design Q based on non-parametric FRF measurements.

Next, the design steps (2)-(3) are investigated, given a model $\widehat{GS}$ from step (1).

3.1 Design of L: inverting $\widehat{GS}$ In case GS is minimum phase, L can be constructed using straightforward inversion. In case GS is nonminimum phase, direct inversion leads to unstable poles. This can be effectively dealt with through non-causal filtering operations. In [3] several methods are outlined to compute or approximate a stable inverse of a nonminimal phase system.

3.2 Design of Q based on FRF measurements As mentioned $Q = 1$ will lead to high performance. However, due to L being not equivalent to $(GS)^{-1}$, due to the model uncertainties, Q should be designed to guarantee robustness. FRF measurement data can be used to compute $|1-GSL|$ as given for a specific application in Figure 2. From this figure it is concluded only by designing the Q-filter properly, Condition (4) can be satisfied. In order to mitigate the phase shift of the Q-filter, another filtering with the the adjoint Q^* (which is enabled by off-line filtering) is used, such that $Q^*(e^{j\omega})Q(e^{j\omega}) = Q(e^{-j\omega})Q(e^{j\omega}) = |Q(e^{j\omega})|^2$. By applying this specific filter Condition (4) translates into

$$|Q(e^{j\omega})|^2|(1-G(e^{j\omega})S(e^{j\omega})L(e^{j\omega}))| < 1, \forall\omega \in [0, 2\pi]. \quad (5)$$

FRF measurement data of GS can now be used to the tune the magnitude of $Q(e^{j\omega})$ such that (5) is satisfied.

It is important to note that using this approach it is not necessary to have a model of the uncertainty of $\widehat{GS}$. By using the FRF measurement data to design Q all the model uncertainties are handled. This is a distinctive advantage compared to alternative robust ILC frameworks.

4. ILC applied to a printer setup

The proposed design approach is applied to the printer setup depicted in Figure 1. A parametric model of the plant is obtained by approximating the data from FRF measurements. The L-filter is designed by determining the inverse of $\widehat{GS}$ using a zero phase-error tracking control approach [3]. A Q-filter is designed as a first-order low-pass filter with a cut-off frequency of 50Hz. From Figure 2 it is observed that Condition (5) is satisfied when this Q-filter is applied.

In Figure 3 experimental results are presented of a 4th order reference signal that is repetitively applied to the printer. As can be observed the norm of the error decreases as the number of trials increases.

5. Extensions

5.1 Multivariable ILC In this paper solely single-input and single-output systems (SISO) are discussed. In [2][4] an approach to use similar frequency domain tools to design the filters L and Q for multi-input multi-output (MIMO) systems for both ILC and RC algorithms are proposed. The proposed approach in [4] is to ignore the interaction when determining L, i.e., L is designed as for a SISO system. Next, the Q-filter is designed taking the ignored iteraction into account in order to guarantee robustness using only a FRF of the MIMO system.

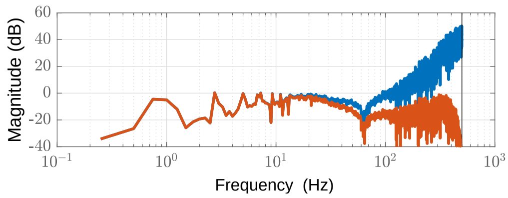

Fig. 2. Bode magnitude of $|1-GSL|$ (—) and $|Q|^2|1-GSL|$ (—) with Q a first order low-pass filter.

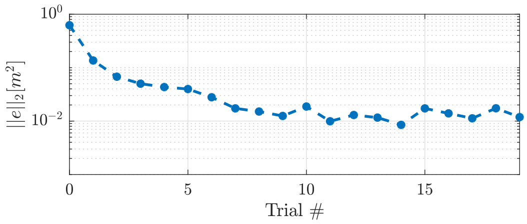

Fig. 3. Norm of the error signal for each trial.

5.2 Flexible ILC In this paper, the focus has been on system with iteration-invariant disturbances, i.e., $r_j = r$. In [5][6] an ILC configuration is developed that is able to cope with disturbance on the system that vary over time. The approach used by [5][6] is to parameterize the feedforward signal using basis functions. It is shown that a finite number of basis functions can be adopted to parameterize the inverse of the plant. Using this in an ILC scheme at each task the system uses the obtained data to adjust the basis functions. This increases the accuracy of the estimation of the inverse of the plant which leads to a better feedforward signal. This approach can significantly improve the performance while maintaining flexibility in the reference signal.

6. Conclusions

In this paper, a brief overview of frequency-domain ILC design is presented. It is shown that robust convergence and robust performance can be obtained using an intuitive manual loop-shaping design, by making use of FRF measurements. The proposed design approach is applied to a printer setup, and experimental results are presented. Moreover, extensions of this approach to multivariable ILC systems and flexible ILC systems are mentioned.

References

(1) D. A. Bristow, M. Tharayil, and A. G. Alleyne. A survey of iterative learning control. *IEEE Control Systems*, 26(3):96–114, June 2006.
(2) L. Blanken, T. Hazelaar, S. Koekebakker, and T. Oomen. Multivariable repetitive control design framework applied to flatbed printing with continuous media flow. In *IEEE Conf. on Decision and Control (CDC)*, 2017.
(3) J. van Zundert and T. Oomen. On inversion-based approaches for feedforward and ILC. *Mechatronics*, 2017.
(4) L. Blanken, S. Koekebakker, and T. Oomen. Design and modeling aspects in multivariable iterative learning control. In *IEEE Conf. on Decision and Control (CDC)*, pages 5502–5507. IEEE, 2016.
(5) F. Boeren, A. Bareja, T. Kok, and T. Oomen. Frequency-domain ILC approach for repeating and varying tasks: With application to semiconductor bonding equipment. *IEEE/ASME Trans. on Mech.*, 21(6):2716–2727, 2016.
(6) L. Blanken, G. Isil, S. Koekebakker, and T. Oomen. Flexible ILC: Towards a convex approach for non-causal rational basis functions. In *IFAC Triennial World Congress*, 2017.