# Numerically reliable identification of complex systems

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## 1 Introduction

Parametric identification of complex systems in the frequency domain, typically requires a model to be fitted over a wide range of frequencies with large variations in amplitude. This imposes challenges with respect to numerical reliability of the identification methods. These issues are evidenced by the multitude of methods found in literature that aim to improve this numerical stability[1][2]. The aim of this work is to compare these different methods and to establish new connections. This leads to two new identification methods. These new methods and the considered methods from literature are compared using simulated and experimental data.

## 2 Methods

In this work two aspects are considered. First, the convergence properties of the underlying identification algorithms are compared. Second, the numerical properties of the different implementations are studied. The considered identification algorithms are the Sanathanan-Koerner algorithm (SK) [3] and the recently developed Instrumental Variable (IV) algorithm [4]. For both these algorithms various implementations are considered, which utilise different basis functions to describe the parametric model. The considered methods are listed below; new methods are marked with $^*$.

- Frequency localising basis functions (FLBF's) using pole relocation (similar to Vector Fitting)$^*$ (SK FLBF);
- Vector Fitting as proposed by Gustavsen [5] (SK VF);
- Data dependent orthonormal polynomials [1] (SK OP);
- Monomial polynomial basis (SK Mon);

- IV FLBF as proposed by Gilson [2] (IV FLBF);
- Vector Fitting extended to IV$^*$ (IV VF);
- Data dependent bi-orthonormal polynomials [1] (IV BP).

## 3 Results

A set of sixteenth order models, generated randomly as the sum of eight second order resonators, is used to analyse the numerical conditioning of the considered methods.

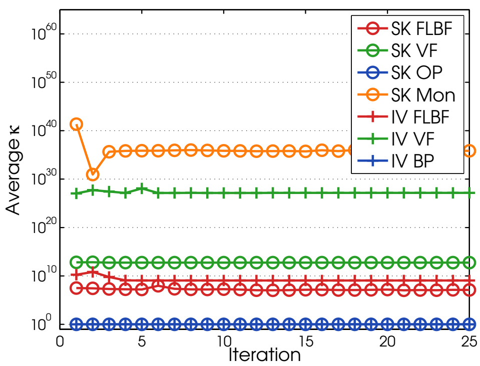


**Figure 1:** Average numerical conditioning properties for the considered methods on simulated data.

Figure 1 shows the numerical conditioning issues that are present when the choice of basis functions is not appropriate for the given problem. The monomial polynomial basis leads to a very ill conditioned problem ($\kappa > 10^{30}$) for the SK algorithm, which in general has better conditioning than the equivalent IV problem. The data dependent polynomial bases lead to optimal conditioning for both identification algorithms, further validating the theoretical results in [1].

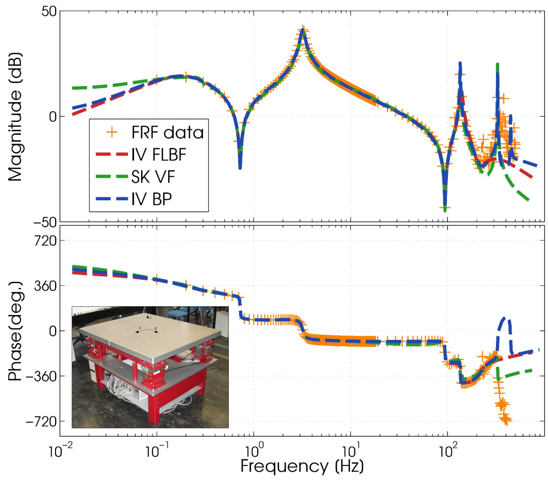


**Figure 2:** Bode diagram of AVIS (Active Vibration Isolation System) FRF measurements and resulting parametric fits.

In Figure 2 it can be seen that the resulting fits are accurate up to 200Hz for the AVIS data. The IV methods lead to a slightly lower value of the cost function (weighted 2-norm of the fitting error) than the SK methods. This result is consistent with theory.

## 4 Ongoing research

A topic of ongoing research is the efficient implementation of the data dependent (bi-)orthonormal bases for multivariable identification. Also the viability of an IV or SK implementation which utilises an orthonormal rational basis in combination with the pole relocation technique is being investigated further.

## References

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