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# Robust output-feedback control to eliminate stick-slip oscillations in drill-string systems

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**Abstract:** The aim of this paper is to design a robust output-feedback controller to eliminate torsional stick-slip vibrations. A multi-modal model of the torsional dynamics with a nonlinear bit-rock interaction model is used. The controller design is based on skewed-$\mu$ DK-iteration and the stability of the closed-loop nonlinear system is analyzed. The proposed controller design strategy offers significant advantages compared to existing strategies. First, it requires only surface measurements, second, it can effectively deal with multiple torsional flexibility modes, third, it provides robustness with respect to uncertainties in the bit-rock interaction and finally, control performance specifications can be taken into account. Simulation results confirm that stick-slip vibrations are indeed eliminated using the designed controller.

*Keywords:* Stick-slip oscillations, robust control, $\mu$-synthesis, output-feedback, drilling systems.

## 1. INTRODUCTION

Efficiency, reliability, and safety are important aspects in the drilling of deep wells for the exploration and production of oil, gas, mineral resources, and geo-thermal energy. Drill-strings of several kilometers in length are used to transmit the axial force and torque necessary to drill the rock formations. These drill-string systems are known to exhibit different types of self-excited vibrations, which decrease the drilling efficiency, accelerate bit wear, and may cause drill-string failure due to fatigue.

Modelling of the torsional dynamics of the drill-sting is an important step towards the control of torsional vibrations. Most controller designs presented in literature rely on one- or two degree-of-freedom (DOF) models for the torsional dynamics only, see e.g. Jansen and Van den Steen (1995); Tucker and Wang (1999); Serrarens et al. (1998); De Bruin et al. (2009). The resisting torque-on-bit (TOB) is typically modelled as a frictional contact with a velocity weakening effect. Although experiments using single cutters to identify the bit-rock interaction law (Detournay and Defourny, 1992) do not reveal such a velocity weakening effect, analysis of models that take the coupled axial and torsional dynamics into account show that such coupling effectively leads to a velocity weakening effect in the TOB (Richard et al., 2007). This motivates a modelling-for-control approach involving the torsional dynamics only and a velocity weakening bit-rock interaction law. In contrast to other studies, however, we use a multi-modal model of the torsional dynamics as field observations have revealed that multiple torsional resonance modes play a role in the onset of stick-slip oscillations.

Controllers for drilling systems aim at drill-string rotation at a constant velocity and the mitigation of stick-slip vibrations. Moreover, the following control specifications are important. First, only surface measurements can be used for feedback. Second, the controller should be able to cope with dynamics related to multiple torsional flexibility modes. Third, robustness with respect to uncertainty in the bit-rock interaction has to be guaranteed and, fourth, control performance specifications, related to e.g. measurement noise sensitivity and actuator constraints, need to be taken into account in the control design.

A well-known control method, which aims at damping the first torsional mode, is the *Soft Torque Rotary system* (Halsey et al., 1988). The same objective is set in Jansen and Van den Steen (1995), which uses a PI-controller based on the top drive velocity. Other control methods including, torsional rectification (Tucker and Wang, 1999), observer-based output-feedback (De Bruin et al., 2009; Doris, 2013), weight-on-bit control (Canudas-de Wit et al., 2008) and robust control (Serrarens et al., 1998; Karkoub et al., 2010) have been developed and are documented in literature.

Although important steps have been made, an approach that satisfies all mentioned requirements has not yet been developed. A robust control approach, as proposed in the latter two works, is particularly suitable for this problem since both robustness with respect to uncertainty of the

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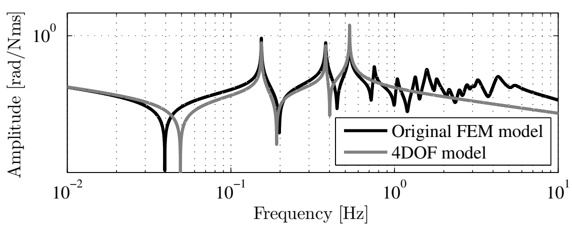


Fig. 1. Frequency response function of the FEM model and 4-DOF model from bit torque $v$ to bit velocity $\omega_1$, i.e. bit mobility.

system dynamics and control performance specifications can be taken into account in the control design. In Serrarens et al. (1998), an $\mathcal{H}_\infty$ controller synthesis method is applied to a 2-DOF drill-string model, using the twist in the drill-string is used as measurement, i.e. knowledge about the angular position of the bit is assumed to be known. Karkoub et al. (2010) uses the $\mu$-synthesis technique through DK-iteration procedure to obtain less conservative bounds on the uncertainty to obtain robustness with respect to the nonlinear bit-rock interaction. The used model is a similar 2-DOF model and also in this case down-hole measurements (to asses the twist of the drill-string) are used. Moreover, the employed 2-DOF models only take the first flexibility mode into account.

The main contribution of this paper is the design of a robust output-feedback controller methodology to eliminate stick-slip vibrations, with the following advantages over existing controllers: 1) usage of surface measurements only, 2) application of the controller to multi-modal drill-string models while guaranteeing local stability of the desired set-point, 3) optimization of the robustness with respect to uncertainty in the bit-rock interaction and, 4) integration of control performance specifications in the design approach.

## 2. DRILL-STRING MODEL

A lumped-parameter model that represents a drilling system is proposed as a basis for controller design. The proposed model is based on a finite element method (FEM) model representation of a realistic drilling system (representing a discretization of a distributed parameter (PDE) model of the drill-string dynamics), see Vromen et al. (2014) for more details on the FEM model. The bit mobility (see Fig. 1) gives an indication of the important resonance modes in the onset of stick-slip vibrations, it is clearly visible that the first three resonance modes are dominant. Therefore, a 4-DOF model is developed, which incorporates 4 rotating inertias, connected to each other with springs and dampers to model the torsional flexibility and damping characteristics of a drill-string (see Fig. 2). The lower disc represents the drill bit in practice, the upper disc the top drive inertia, and the other degrees of freedom characterize additional flexibility modes. The inclusion of these extra modes in the model is a key improvement with respect to existing models used for controller design.

The driving input of the system is the motor torque $T_m$. The available measurements are the top drive velocity $\omega_{td}$ and the pipe torque $T_{pipe}$, which is defined as the torque in the drill-string right below the top drive. The drill-string-borehole interaction torques $\phi_i(\omega_i)$, $i = 2, 3, 4$, are modelled as set-valued Coulomb friction laws ($\phi(q_2) := [\phi_2(\omega_2)\ \phi_3(\omega_3)\ \phi_4(\omega_4)]^\top$, with $q_2 := [\omega_2\ \omega_3\ \omega_4]^\top$) and the

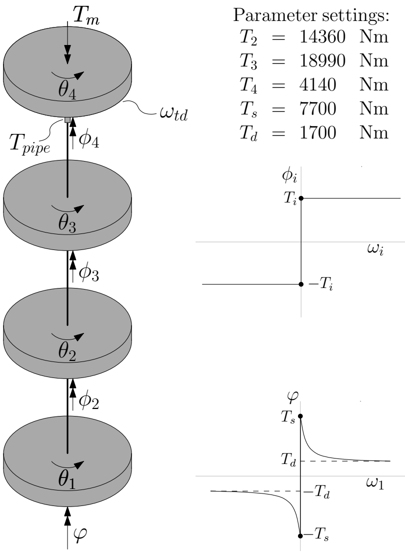


Fig. 2. 4-DOF model of the drill-string.

interaction torque $\varphi(\omega_1)$ at the bit-rock interface is defined by a set-valued Coulomb friction law with Stribeck effect, see Fig. 2. The resulting equations of motion are written in first-order state-space form:

$$\begin{aligned} \dot{x} &= Ax + Gv + G_2 v_2 + B u_t \\ q &= Hx \\ q_2 &= H_2 x \\ y &= Cx \\ v &\in -\varphi(q) \\ v_2 &\in -\phi(q_2). \end{aligned} \tag{1}$$

Herein, $x = [\theta_1 - \theta_2,\ \omega_1,\ \omega_2,\ \theta_2 - \theta_3,\ \omega_3,\ \theta_3 - \theta_4,\ \omega_4]^\top \in \mathbb{R}^7$ is the state, where $\theta_i$, $i = 1, 2, 3, 4$, describes the rotational displacement of the inertias, $\omega_i := \dot{\theta}_i$, and the bit velocity is defined as $q := \omega_1$. Moreover, the bit-rock interaction torque is given by $v \in \mathbb{R}$ and the drill-string-borehole interaction torques are given by $v_2 \in \mathbb{R}^3$. In addition, $u_t := T_m \in \mathbb{R}$ is the control input and, $y := [\omega_{td}\ T_{pipe}]^\top \in \mathbb{R}^2$ is the measured output.

## 3. CONTROL PROBLEM FORMULATION

The desired operation of the drill-string system is a constant angular velocity $\omega_{eq}$ for all four inertias. So, the objective is to regulate this set-point of the nonlinear drill-string system by means of an output-feedback controller. The available measurements for the controller are the top drive angular velocity $\omega_{td}$ and the pipe torque $T_{pipe}$. The system can be controlled by the top drive torque $T_m$. As briefly mentioned in the introduction, the controller should

(1) locally stabilize the desired velocity of the drill-string, therewith eliminating torsional (stick-slip) vibrations;
(2) ensure robustness with respect to uncertainty in the nonlinear bit-rock interaction $\varphi$;
(3) guarantee the satisfaction of closed-loop performance specifications, in particular on measurement noise sensitivity, i.e., limitation of the amplification of measurement noise, and limitation of the control action such that top drive limitations can be satisfied;
(4) guarantee robust stability and performance in the presence of multiple flexibility modes dominating the torsional dynamics.

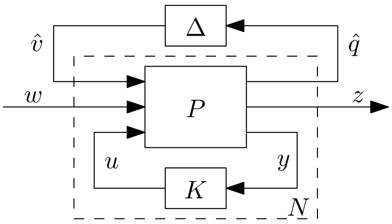


Fig. 3. Control configuration with uncertainty block $\Delta$.

To facilitate controller synthesis, the drill-string dynamics (1) are reformulated. The desired constant angular velocity $\omega_{eq}$ for all discs can be associated with a desired equilibrium $x_{eq}$ for the state of the system. To ensure that $x_{eq}$ is an equilibrium of the closed-loop system, the control input $u_t = u_c + \tilde{u}$ is decomposed in a constant feedforward torque $u_c$ (inducing $x_{eq}$) and the feedback torque $\tilde{u}$. For the feedforward design, we assume that $\omega_i > 0$, for $i = 2, 3, 4$, hence $\phi$ is constant (see Fig. 2) and can be compensated for by $u_c$ and we determine $x_{eq}$ and $u_c$ using the equilibrium equation of system (1), i.e. $Ax_{eq} - G\varphi(Hx_{eq}) - G_2\phi(H_2x_{eq}) + Bu_c \ni 0$. Next, we define $\xi := x - x_{eq}$ and apply a linear loop transformation such that the slope of a transformed nonlinearity $\tilde{\varphi}(q)$ (associated to $\varphi(q)$ through the loop transformation) is equal to zero at the equilibrium velocity, i.e. $\partial\tilde{\varphi}/\partial q|_{q=\omega_{eq}} = 0$. This results in a state-space representation of the transformed drill-string dynamics in perturbation coordinates:

$$\dot{\xi} = A_t\xi + B\tilde{u} + G\tilde{v} \quad (2a)$$
$$\tilde{y} = C\xi \quad (2b)$$
$$\tilde{q} = H\xi \quad (2c)$$
$$\tilde{v} \in -\tilde{\varphi}(\tilde{q}) \quad (2d)$$

with $A_t := A + \delta GH$, $\delta = -\partial\varphi/\partial q|_{q=\omega_{eq}} > 0$, $\tilde{y} := y - Cx_{eq}$, $\tilde{q} := q - Hx_{eq}$, $\tilde{\varphi}(\tilde{q}) := \varphi(\tilde{q} + Hx_{eq}) - \varphi(Hx_{eq}) + \delta\tilde{q}$ and $\tilde{v} := v - v_{eq} - \delta\tilde{q}$. The dynamics in (2) represents a Lur'e-type system, with the linear dynamics $G_{ol}$ ((2a) - (2c)), having inputs $\tilde{u}$ and $\tilde{v}$ and outputs $\tilde{y}$ and $\tilde{q}$, and the nonlinearity $\tilde{\varphi}$ in the feedback loop. The open-loop transfer function $G_{ol}(s)$ is defined as

$$\begin{bmatrix}\tilde{q}(s)\\ \tilde{y}(s)\end{bmatrix} := G_{ol}(s)\begin{bmatrix}\tilde{v}(s)\\ \tilde{u}(s)\end{bmatrix} = \begin{bmatrix}g_{11}(s) & g_{12}(s)\\ g_{21}(s) & g_{22}(s)\end{bmatrix}\begin{bmatrix}\tilde{v}(s)\\ \tilde{u}(s)\end{bmatrix}. \quad (3)$$

As a next step, we model the nonlinearity $\tilde{\varphi}$ by an uncertainty $\Delta$. This model formulation is used in the controller design procedure, presented in Section 4.

## 4. DESIGN OF A ROBUST OUTPUT-FEEDBACK CONTROLLER

In this section, we present a robust control design approach based on skewed-$\mu$ DK-iteration. This technique combines several concepts from robust control theory to design a controller that achieves robust stability and performance of a system with model uncertainties (Skogestad and Postlethwaite, 2005).

The general robust control configuration for a (LTI) plant $P$ with an uncertainty $\Delta$ and (LTI) controller $K$ is shown in Fig. 3, where $y$ is the measured output, $u$ the control output and the exogenous inputs $w$ and outputs $z$. The system $P$, in Fig. 3, can be described by

$$\begin{bmatrix}\hat{q}\\ z\\ y\end{bmatrix} = \begin{bmatrix}P_{11} & P_{12} & P_{13}\\ P_{21} & P_{22} & P_{23}\\ P_{31} & P_{32} & P_{33}\end{bmatrix}\begin{bmatrix}\hat{v}\\ w\\ u\end{bmatrix}. \quad (4)$$

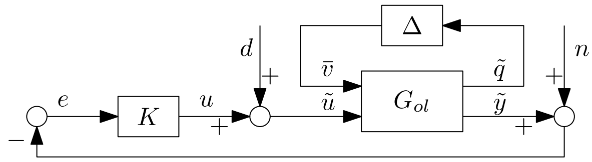


Fig. 4. Linear drill-string dynamics $G_{ol}$ in closed loop with the controller $K$ and including model uncertainty $\Delta$.

The system $N$ is defined as the lower linear fractional transformation (LFT) of $P$ with the controller $K$, that is

$$N := F_l(P, K) = \begin{bmatrix}P_{11} & P_{12}\\ P_{21} & P_{22}\end{bmatrix} + \begin{bmatrix}P_{13}\\ P_{23}\end{bmatrix}K(I - P_{33}K)^{-1}[P_{31}\ P_{32}].$$

### *4.1 Nominal stability and performance*

As mentioned in Section 3, the controller design aims at stability, performance, and robustness for the uncertainty $\Delta$. In this section, the focus is on the first two aspects. Robustness is considered in the next section. Hereto, consider the system without uncertainty given by

$$\begin{bmatrix}z\\ y\end{bmatrix} := \underline{P}\begin{bmatrix}w\\ u\end{bmatrix} = \begin{bmatrix}P_{22} & P_{23}\\ P_{32} & P_{33}\end{bmatrix}\begin{bmatrix}w\\ u\end{bmatrix} \quad (5)$$

and the lower LFT of $\underline{P}$ and $K$, that is, $N_{22} := F_l(\underline{P}, K)$.

Based on the system representation in Fig. 3, the closed-loop system of the linear drill-string dynamics $G_{ol}$ in feedback with the linear, dynamic controller $K$ to be designed is shown in Fig. 4. In this representation, additional inputs $n$ and $d$ are introduced, representing measurement noise and actuator noise, respectively. Define the unweighted inputs $\bar{w} := [n\ d]^\top$ and unweighted outputs $\bar{z} := [e\ u]^\top$ such that the closed-loop transfer functions between $\bar{w}$ and $\bar{z}$ equal the relevant sensitivity functions, as follows:

$$\begin{bmatrix}e\\ u\end{bmatrix} = -\begin{bmatrix}(I + g_{22}K)^{-1} & (I + g_{22}K)^{-1}g_{22}\\ K(I + g_{22}K)^{-1} & K(I + g_{22}K)^{-1}g_{22}\end{bmatrix}\begin{bmatrix}n\\ d\end{bmatrix}, \quad (6)$$

where $g_{22}$ is the open-loop transfer function from input $\tilde{u}$ to output $\tilde{y}$ as defined in (3). Performance specifications can now be introduced by the design of weighting functions for these sensitivity functions. Performing so for multiple sensitivity functions is often referred to as *mixed sensitivity* (Kwakernaak, 1993). For example, consider the sensitivity function $S = (I + GK)^{-1}$ (for a SISO plant $G$ and controller $K$) and the upper bound $1/|w_P(s)|$, where $w_P(s)$ is the weighting filter to be specified. Then

$$|S(j\omega)| < 1/|w_P(j\omega)|,\ \forall\omega \ \Leftrightarrow\ |w_P(j\omega)S(j\omega)| < 1,\ \forall\omega.$$

The latter fact implies that the bound on the sensitivity can be written as a norm-bound on the product of the weighting filter and sensitivity function, i.e. $\|w_PS\|_\infty < 1$, where we used the definition of the $\mathcal{H}_\infty$-norm $\|H(s)\|_\infty := \sup_{\omega\in\mathbb{R}}\bar{\sigma}(H(j\omega))$. This is a key element in the design of a controller that guarantees nominal performance.

The concept nominal performance is defined as follows: for a system without uncertainty $\Delta$ the closed-loop system $N_{22} = F_l(\underline{P}, K)$ is internally stable and the $\mathcal{H}_\infty$-norm of this system (from $w$ to $z$) is smaller than 1, that is

$$\|N_{22}\|_\infty = \sup_\omega \bar{\sigma}(F_l(\underline{P}, K)) < 1.$$

This means that nominal performance can be achieved by solving the "standard" $\mathcal{H}_\infty$ optimal control problem, which aims to find the internally stabilizing controller $K$ to minimize $\|F_l(\underline{P}, K)\|_\infty$. As proven in (Zhou et al., 1996, Sec. 5.3), internal stability of the closed-loop is guaranteed by choosing $w$ and $z$ as the weighted version of $\bar{w}$ and $\bar{z}$.

### *4.2 Alternative robust performance*

Robust performance means that the performance objective in Section 4.1 is achieved for all possible models in the uncertainty set (Skogestad and Postlethwaite, 2005).

*Remark 1.* Standard robust performance techniques aim at optimizing the performance for all possible plants in the uncertainty set. In contrast, we aim to optimize the robustness with respect to the uncertainty while still guaranteeing internal stability and satisfaction of the performance objectives. This is called alternative robust performance.

Consider the system $P$ in Fig. 3, including the uncertainty block $\Delta$. The input-output pair $\hat{v}$, $\hat{q}$ is related to this uncertainty block and the (weighted) closed-loop transfer function $N(s) = F_l(P, K)$ is given by

$$\begin{bmatrix} \hat{q} \\ w \end{bmatrix} = N \begin{bmatrix} \hat{v} \\ z \end{bmatrix} = F_l(P,K) \begin{bmatrix} \hat{v} \\ z \end{bmatrix}. \tag{7}$$

Robust stability is obtained by designing a controller $K$ such that the system $N$ is internally stable and the upper LFT, $F := F_u(N, \Delta)$, is stable for all $\Delta \in \mathbf{\Delta}$. Herein, the uncertainty set is a norm-bounded subset of $\mathcal{H}_\infty$, i.e., $\mathbf{\Delta} = \{\Delta \in \mathcal{RH}_\infty \,|\, \|\Delta\|_\infty < 1\}$. However, the aim is to find a stabilizing controller that also meets certain performance specifications. Therefore, we use a similar approach as in (Skogestad and Postlethwaite, 2005, Sec. 8.10) and consider the fictitious 'uncertainty' $\Delta_P$. The uncertainty $\Delta_P$ is a complex unstructured uncertainty block which represents the $\mathcal{H}_\infty$ performance specifications. Moreover, note that $\Delta_P \in \mathbf{\Delta}_P$, with $\mathbf{\Delta}_P = \{\Delta_P \in \mathcal{RH}_\infty \,|\, \|\Delta_P\|_\infty < 1\}$. The result given in (Zhou et al., 1996, Thm. 11.8) states that a robust performance problem is equivalent to a robust stability problem with the augmented uncertainty

$$\hat{\Delta} = \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_P \end{bmatrix} \tag{8}$$

with $\hat{\Delta}$ a block-diagonal matrix. In other words, both the performance specifications and uncertainty are taken into account in a similar fashion. The robust performance condition can now be formulated as follows:

$$\mu_{\hat{\mathbf{\Delta}}}(N(j\omega)) < 1, \quad \forall\omega, \tag{9}$$

where $\mu_{\hat{\mathbf{\Delta}}}$ is the structured singular value with respect to the uncertainty set $\hat{\mathbf{\Delta}}$ with the structure as given in (8) and any $\Delta \in \mathbf{\Delta}$ and $\Delta_P \in \mathbf{\Delta}_P$. The structured singular value is defined as the real non-negative function

$$\mu_{\hat{\mathbf{\Delta}}}(N) = \frac{1}{\bar{k}_m}, \; \bar{k}_m = \min \left\{ k_m \,\middle|\, \det\left(I - k_m N \hat{\Delta}\right) = 0 \right\} \tag{10}$$

with complex matrix $N$ and block-diagonal uncertainty $\hat{\Delta}$.

To optimize the robustness with respect to the uncertainty $\Delta$ (i.e. part of $\hat{\Delta}$ in (8)), the skewed structured singular value $\mu^s$ can be used. The skewed structured singular value is used if some uncertainty blocks in $\hat{\Delta}$ are kept fixed ($\Delta_P$ in this case) to investigate how large another source of uncertainty ($\Delta$ in this case) can be before robust stability/performance cannot be guaranteed anymore. In this case, we aim to obtain the largest possible uncertainty set $\Delta$ (i.e. uncertainty w.r.t. the bit-rock interaction), given a fixed $\Delta_P$ (i.e. fixed performance specifications). Hereto, we introduce the matrix $K_m^s := \operatorname{diag}(k_m^s,\ I)$ and the skewed structured singular value $\mu^s_{\hat{\Delta}}(N)$ can then be defined as $\mu^s_{\hat{\mathbf{\Delta}}}(N) = \frac{1}{\bar{k}_m^s}$, $\bar{k}_m^s = \min\left\{k_m^s \,\middle|\, \det\left(I - K_m^s N \hat{\Delta}\right) = 0\right\}$. Thus, the robust performance condition (9), with additional scaling (through $K_m^s$) in terms of the skewed structured singular value, can be written as

$$\mu^s_{\hat{\mathbf{\Delta}}}(N(j\omega)) < 1, \quad \forall\omega. \tag{11}$$

To support controller design satisfying particular performance specifications, weighting filters and scaling matrices are introduced in the loop in Fig. 4, as shown in Fig. 5. Those frequency-domain weighting filters allow us to specify the (inverse) maximum allowed magnitudes of the closed-loop transfer functions (6). Moreover, the scaling matrices are introduced to improve the numerical conditioning of the problem and to tune the desired bandwidth. The (weighted) generalized plant $P$ with input weighting filters $V_i(s)$ and output weighting filters $W_i(s)$, with $i \in \{1, 2, 3\}$, and scaling matrices $W_{sc}$ and $V_{sc}$, is specified by

$$\begin{bmatrix} \hat{q} \\ \hat{e} \\ \hat{u} \\ e \end{bmatrix} = \underbrace{\begin{bmatrix} W_1 & 0 & 0 & 0 \\ 0 & W_2 W_{sc} & 0 & 0 \\ 0 & 0 & W_3 V_{sc}^{-1} & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \bar{P} \begin{bmatrix} V_1 & 0 & 0 & 0 \\ 0 & {W_{sc}}^{-1} V_2 & 0 & 0 \\ 0 & 0 & V_{sc} V_3 & 0 \\ 0 & 0 & 0 & I \end{bmatrix}}_{P} \begin{bmatrix} \hat{v} \\ \hat{n} \\ \hat{d} \\ u \end{bmatrix}$$

Herein, $\bar{P}(s)$ is the MIMO transfer function of the unweighted system $\bar{P}$ with inputs $[\tilde{v}\ n\ d\ u]^\top$ and outputs $[\tilde{q}\ e\ u\ e]^\top$ with its state-space realization given by

$$\bar{P} \overset{s}{=} \left[\begin{array}{c|cccc} A_t & G & 0 & B & B \\ \hline H & 0 & 0 & 0 & 0 \\ -C & 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 & I \\ -C & 0 & -I & 0 & 0 \end{array}\right]. \tag{12}$$

In this section, an alternative robust performance framework is introduced. An efficient procedure to synthesize a controller that minimizes the skewed structured singular value $\mu^s_{\hat{\mathbf{\Delta}}}$, for the purpose of obtaining robust performance, is known as the DK-iteration procedure. In DK-iterations, a $\mu$-analysis ($D$-step) and $\mathcal{H}_\infty$-optimization ($K$-step) are solved alternately (see Oomen et al. (2014) and Skogestad and Postlethwaite (2005) for more details).

### *4.3 Closed-loop stability analysis*

The main purpose of the controller is to stabilize the equilibrium $\xi = 0$ of the nonlinear system (2). Let us assume a controller $K$ has been designed that meets the performance specifications and is robust with respect to the uncertainty $\Delta$. Hence, the designed controller guarantees stability for the *linear* closed-loop system $N(s)$ and achieves robustness with respect to the specified uncertainty $\Delta$.

Stability of the equilibrium $\xi = 0$ of the closed-loop nonlinear system can be investigated using the circle criterion (Khalil, 2002, Thm. 7.1). Consider the closed-loop transfer function $G_{cl}$, of system (2) with controller $K$, that is $G_{cl} := g_{11} - g_{12}K(I + g_{22}K)^{-1}g_{21}$. Moreover, consider a symmetric sector condition on the nonlinearity, i.e. $\tilde{\varphi} \in [-\gamma, \gamma]$ for $\gamma > 0$. It can be shown that for the system to be absolutely stable the condition $\|G_{cl}(j\omega)\|_\infty < 1/\gamma$ should be satisfied. Hence, the $\mathcal{H}_\infty$-norm of the closed-loop bit mobility $G_{cl}$ gives an upper bound on the sector

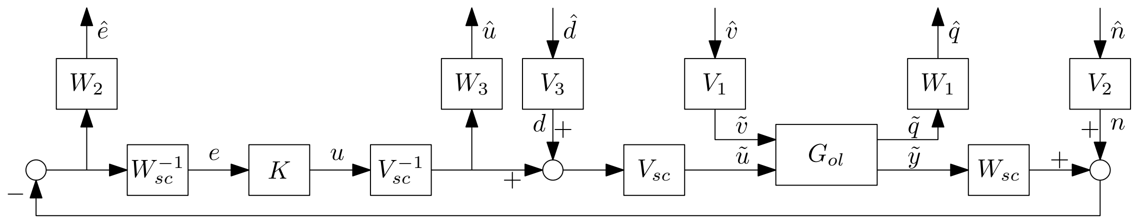


Fig. 5. Closed-loop system with weighting filters and scaling matrices.

Table 1. Parameter settings for the weighting filters and scaling matrices.

| | |
|---|---|
| $\alpha = 1625$ | $\beta = 0.006$ |
| $\omega_3 = 3.342$ | |
| $b_1 = 0.1$ | $b_2 = 1$ |
| $\omega_b = 0.2\pi$ | $M = 10$ |
| $\omega_{ro} = 2\pi$ | $\zeta = 1$ |

that the nonlinearity $\tilde{\varphi}$ should comply with. Using the proposed controller design strategy a controller $K$ can be designed such that $\|G_{cl}\|_\infty$ is minimized. In other words, the robustness with respect to uncertainty in the bit-rock interaction is optimized.

## 5. CONTROLLER SYNTHESIS

Weighting filter design is key in satisfying the performance specifications. Moreover, achieving specific design targets such as the inclusion of integral action and high-frequency roll-off can be achieved by absorbing these filters in the loop, see Meinsma (1995).

The scalings matrices $V_{sc}$ and $W_{sc}$ are used to tune the open-loop crossover frequency of $G_{ol}$. The input scaling $V_{sc} = \alpha$ is chosen such that the crossover frequency is between the first anti-resonance and resonance of the system (see Fig. 1). The output scaling $W_{sc} = \text{diag}(1, \beta)$ with $\beta$ such that the crossover frequency is near the third resonance mode. The weighting filters are mostly chosen to be static gains for the nominal controller design, only the filters $V_1(s)$ and $V_{21}(s)$ (where $V_2(s) = \text{diag}(V_{21}, V_{22})$) are chosen to be frequency dependent. The filter $V_1(s)$ can be used to specify bounds on the closed-loop bit mobility function ($G_{cl}$). Ideally, the bit mobility should be damped as much as possible (as follows from Section 4.3). However, this typically results in high control action. To deal with this trade-off, the weighting filter $V_1(s)$ has a notch filter near the third mode and is defined as $V_1(s) = \frac{(1/\omega_3^2)s^2+(2b_1/\omega_3)s+1}{(1/\omega_3^2)s^2+(2b_2/\omega_3)s+1}$ with $\omega_3$ the resonance frequency of the third mode. The filter $V_{21}(s)$ is given by $V_{21}(s) = \frac{s+\omega_b}{Ms}$ and enables us to tune the sensitivity function. The parameter values used in the scaling matrices and the weighting filters are given in Table 1. The remaining (static) weighting filters are defined as $V_{22} = 0.001$, $V_3 = 0.01$, $W_1 = W_{21} = W_{22} = W_3 = 1$. The filters in the loop to obtain a controller that includes integral action and high-frequency roll-off are, respectively, given by $f_{ro} = \frac{\omega_{ro}^2}{s^2+2\zeta\omega_{ro}s+\omega_{ro}^2}$ and $f_{int} = \frac{1}{s}$.

Performing the DK-iteration procedure for the drill-string system with the weighting filters as specified above, results in the controller $K_t(s) = [K_{\omega_{td}}(s), K_{T_{pipe}}]$, as shown in Fig. 6. From this figure, the integral action in the controller, $K_{\omega_{td}}(s)$, that uses the top drive angular velocity

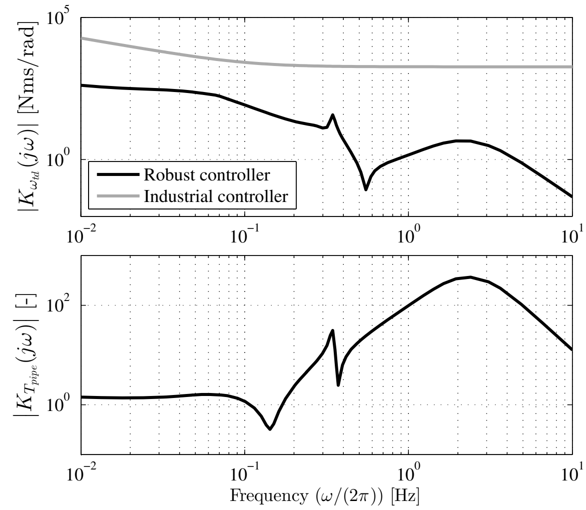


Fig. 6. Designed linear dynamic controller for the drill-string system, upper plot is the controller which uses the top drive angular velocity, the controller in the lower plot is based on the pipe torque measurement.

can be clearly recognized. Moreover, the second-order roll-off is present in both controllers. It can also be seen that the designed controller is active in the frequency range of the torsional resonance modes of the drill-string system (see Fig. 1). In the same figure, an industrial controller, which only uses top drive velocity measurements is shown. This controller is a properly tuned active damping system (i.e. PI-control of the angular velocity) which aims at damping the first torsional mode of the drill-string dynamics. A comparison with this controller by means of a simulation is presented in Section 6.

## 6. SIMULATION RESULTS

In this section, the controller designed in Section 5 is applied to the drill-string model presented in Section 2. First, we present a simulation result of the drill-string system in closed-loop with an existing industrial controller (based on Jansen and Van den Steen (1995)). For the simulations, we introduce a so-called startup scenario, which is based on practical startup procedures for drilling rigs. Herein, the drill-string is first accelerated to a low constant rotational velocity with the bit above the formation (off bottom) and, subsequently, the angular velocity and weight-on-bit (WOB) are gradually increased to the desired operating conditions. The increase in WOB is modelled as a scaling of the bit-rock interaction torque.

A simulation result of the drill-string model (1) in feedback with the industrial controller, shown in Fig. 6, is shown in Fig. 7. In the upper plot the top drive velocity ($\omega_{td}$) is shown along with the reference velocity $\omega_{ref}$. From the bit response, in the bottom plot, we can clearly recognize

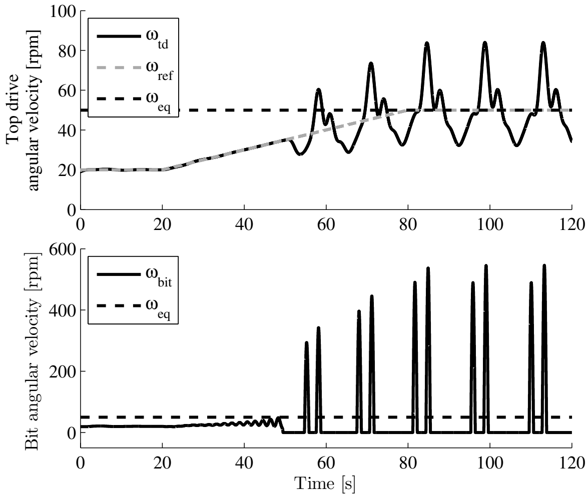


Fig. 7. Simulation result of the drill-string model with an existing industrial controller.

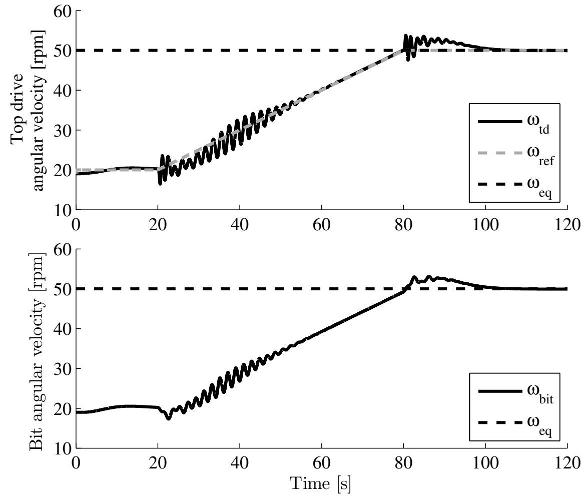


Fig. 8. Simulation result of the drill-string model with the designed output-feedback controller.

stick-slip oscillations. The increasing amplitude of the oscillations in the top drive velocity demonstrates that these vibrations arise when the WOB is increased ($20 \leq t < 80$s), i.e., when due to scaling of the TOB the velocity weakening effect affects the dynamics. A simulation result of the designed controller in Fig. 6 is shown in Fig. 8. The same startup scenario and initial conditions, as in Fig. 7, are used for this simulation. These simulation results show that the top drive and bit angular velocity converge to their set-point and stick-slip vibrations are avoided.

## 7. CONCLUSIONS

In this paper, a synthesis strategy for controllers aiming at the mitigation of torsional stick-slip oscillations in drilling systems is proposed. The controller design is based on skewed-$\mu$ DK-iteration, and offers several benefits over existing controllers. First, the designed controller is applicable to a multi-model drill-string model while guaranteeing (local) stability of the desired operating point. Second, the controller is optimized to have robustness with respect to uncertainty in the bit-rock interaction. Third, performance specifications are integrated in the controller design. Fourth, the controller uses only surface measurements. Simulation results of the proposed controller applied to the 4-DOF drill-string model show that the stick-slip oscillations are eliminated, while a simulation of an existing industrial controller, under the same conditions, shows stick-slip vibrations.

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