Achieving Perfect Causal Feedforward Control in Presence of Nonminimum-Phase Behavior - Exploiting Additional Actuators and Squaring Down

Jurgen van Zundert, Fons Luijten, Tom Oomen - `ACC2018_v10(03/02/2018)`

***Abstract*— Control performance is limited by nonminimum-phase zeros, for example through the Poisson integral in feedback control and "unstable" poles in inverse model feedforward control. The aim of this paper is to exploit the additional freedom in overactuated systems to overcome these limitations. In particular, an approach for causal and exact inversion of nonminimum-phase systems is presented for application in inverse model feedforward control. The proposed method is based on the observation that non-square systems often have no invariant zeros. A squaring-down approach is employed to create a square system without nonminimum-phase zeros to enable direct inversion. The proposed approach is successfully demonstrated on a benchmark system. The method enables exact inversion for non-square systems without requiring preview.**

I. Introduction

System inversion is essential in tracking control applications such as inverse model feedforward, iterative learning control, and repetitive control. The system to be inverted can either be open-loop (as in inverse model feedforward [1]) or closed-loop (process sensitivity in iterative learning control [2] and complementary sensitivity in repetitive control [3]). Inversion is not always straightforward, for example when the system is nonminimum phase since direct inversion yields unbounded responses. Nonminimum-phase behavior typically results from noncollocated actuator and sensor placement due to system inherent physical restrictions.

To obtain bounded responses, inversion techniques can be used. Traditional inversion techniques, such as ZPETC [4], ZMETC and NPZ-Ignore, are restricted to singlevariable systems [5] and stable inversion [6] is restricted to square systems (same number of inputs as outputs). $\mathcal{H}_2$-preview control, $\mathcal{H}_\infty$-preview control [7], [8] and norm-optimal control [9] are directly applicable to non-square systems. The key aspect for these inversion techniques is the use of preview to compensate nonminimum-phase behavior, i.e., they require nonzero input before the start of the task. However, the use of preview can be undesired, for example, in high throughput systems where the time between tasks is limited. A recent overview and comparison of inversion techniques can be found in [8].

Non-square systems with more inputs than outputs exhibit design freedom at the input side. Examples include overactuated systems which have more actuators than sensors, but also multirate systems [10] are essentially non-square due to the difference between input and output sample rate. The main idea of this paper is to exploit this design freedom to obtain a satisfactory inverse. In particular, the interest is in avoiding the use of preview. Related work can be found in [11] in which a squaring-down approach is used to create a square system with certain properties. In this paper, the interest is in squaring-down approaches that yield minimum-phase square systems since this enables direct inversion and thereby avoids the use of preview.

Although there are many inversion techniques available for nonminimum-phase systems, they all yield noncausal solutions. Noncausality implies preview information of the trajectory is required, which is not always available, and limits the throughput. In this paper, the additional design freedom in overactuated systems is exploited to obtain exact inversion without preview. The approach is presented for linear time-invariant (LTI), discrete-time systems, with extension to continuous-time systems being straightforward.

The main contribution of this paper is a causal inverse model feedforward solution for overactuated systems with nonminimum-phase behavior. The following contributions are identified: (I) systematic representation of non-square systems; (II) squaring-down approaches with static and dynamic compensators; (III) systematic design framework for design of inverse model feedforward for overactuated systems; and (IV) application of the design framework on a benchmark system with nonminimum-phase behavior.

II. Problem formulation

One of the main challenges in tracking control is the inversion of nonminimum-phase systems as shown in Fig. 1, see also [8]. This paper aims to exploit the design freedom in overactuated systems to exactly invert nonminimum-phase systems without preview. In this section, the main idea of the proposed inversion technique is presented through several examples. A systematic design procedure is presented in subsequent sections.

Let the discrete-time system H in Fig. 1 be given by the minimal realization

$$H: \quad \begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ y(k) &= Cx(k), \end{aligned} \tag{1}$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^{\bar{m}}$, $y \in \mathbb{R}^{\bar{p}}$. Without loss of generality, it is assumed that B, C are of full rank. For notational simplicity, strictly proper systems are considered, but the presented

Van Zundert, Luijten, and Oomen are with the Eindhoven University of Technology, Department of Mechanical Engineering, Control Systems Technology group, The Netherlands. E-mail: `j.c.d.v.zundert@tue.nl`.

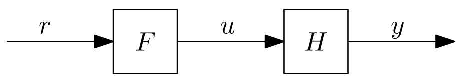


Fig. 1. Unified diagram for system inversion in tracking control applications. Given system H, the aim is to let output $y \in \mathbb{R}^{\bar{p}}$ track reference $r \in \mathbb{R}^{\bar{p}}$ through design of F such that $u \in \mathbb{R}^{\bar{m}}$ remains bounded.

results can readily be extended to nonstrictly proper systems. System H is stable if and only if $|\lambda_i(A)| < 1$, $\forall i$, where λ_i denotes the ith eigenvalue. System H is minimum phase if and only if all invariant zeros z_i (Definition 1) satisfy $|z_i| < 1, \forall i$. System H is unstable (resp. nonminimum phase) if it is not stable (resp. minimum phase).

Definition 1 (Invariant zeros [12], [13]). *Invariant zeros of H in* (1) *are those values of $z \in \mathbb{C}$ for which*

$$\operatorname{rank}\{H_{RSM}(z)\} < \text{normal rank}\{H_{RSM}(z)\},$$

with $H_{RSM}(z) = \left[\begin{array}{c|c} zI_n - A & -B \\ \hline C & 0 \end{array}\right].$

Given H in Fig. 1, the objective is to let output y track the reference trajectory r, while input u remains bounded. The following examples illustrate that the straightforward design $F = H^{-1}$ is not always a satisfactory solution.

Example 1 *Let H be square and given by*

$$H = \tfrac{(z-0.6)(z+2)}{(z-0.1)(z+0.8)(z-0.4)},$$

then

$$F = H^{-1} = \tfrac{(z-0.1)(z+0.8)(z-0.4)}{(z-0.6)(z+2)}$$

is unique, but unstable (pole at $z = -2$) as a consequence of H being nonminimum phase. Hence, input u is unbounded.

Example 2 *Let H be non-square and given by*

$$H = \tfrac{1}{(z-0.1)(z+0.8)(z-0.4)} \left[\begin{smallmatrix} (z-0.6)(z+2) & (z-5) \end{smallmatrix}\right],$$

then F such that $HF = 1$ is not unique. The solution

$$F = \left[\begin{smallmatrix} 1 \\ -0.3 \end{smallmatrix}\right] \tfrac{(z-0.1)(z+0.8)(z-0.4)}{(z+0.5)(z+0.6)},$$

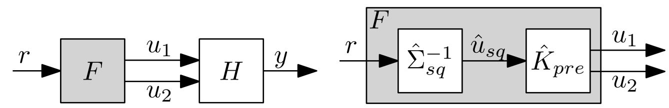
yields bounded input u since F is stable.

Example 3 *Let H be non-square and given by*

$$H = \tfrac{1}{(z-0.1)(z+0.8)(z-0.4)} \left[\begin{smallmatrix} (z-0.6)(z+2) & (z-5)(z+0.9) \end{smallmatrix}\right],$$

then F such that $HF = 1$ is not unique. The only difference with Example 2 is one additional minimum-phase zero at the second input. However, the design of a stable F such that $HF = 1$ is not as straightforward as in Example 2.

The examples show that inversion of H is nontrivial if H is nonminimum phase or non-square. Example 2 shows that additional freedom in the inputs can be exploited to create a stable system that yields exact inversion. However, such a design is not straightforward as illustrated by Example 3. In this paper, a systematic design framework for such systems is presented. The main concept is presented in the next section.



(a) Tracking control of $r \in \mathbb{R}^{\bar{p}}$ by H with more inputs $u_1 \in \mathbb{R}^{\bar{p}}$, $u_2 \in \mathbb{R}^{\bar{m}-\bar{p}}$ than outputs $y \in \mathbb{R}^{\bar{p}}$.

(b) Minimimum-phase, square $\hat{\Sigma}_{sq} = H\hat{K}_{pre}$ is created and inverted such that $HF = I$ and hence $y = r$.

Fig. 2. Inverse model feedforward approach for a system H with more inputs than outputs. Perfect tracking is obtained without preview through squaring down and direct inversion of the minimum-phase, square system.

III. Exploiting additional freedom in overactuated systems

In this paper, a systematic design approach is presented that exploits the additional freedom of overactuated systems H in Fig. 1 (i.e., with $\bar{m} > \bar{p}$) to create stable F such that $HF = I$ and hence $y = r$. A key aspect is that non-square systems only have invariant zeros in very specific situations.

The tracking control application of Fig. 1 with more inputs than outputs ($\bar{m} > \bar{p}$) is shown in Fig. 2(a), with input $u \in \mathbb{R}^{\bar{m}}$ divided into $u_1 \in \mathbb{R}^{\bar{p}}$, $u_2 \in \mathbb{R}^{\bar{m}-\bar{p}}$. It is assumed that H is right invertible as defined by Definition 2.

Definition 2 (Invertibility). *System H in* (1) *is*

- *left invertible if and only if $r_n = \bar{m}$,*
- *right invertible if and only if $r_n = \bar{p}$,*
- *invertible if and only if $r_n = \bar{m} = \bar{p}$,*

with $r_n = \text{normal rank}\{C(zI_n - A)^{-1}B\}$.

The design approach consists of two parts as, see Fig. 2(b). First, a precompensator $\hat{K}_{pre}$ is designed such that

$$\hat{\Sigma}_{sq} = H\hat{K}_{pre} \tag{2}$$

is square with dimensions $\bar{p} \times \bar{p}$. Second, F is selected as

$$F = \hat{K}_{pre}\hat{\Sigma}_{sq}^{-1} \tag{3}$$

such that perfect tracking is obtained since $y = HFr = H\hat{K}_{pre}\hat{\Sigma}_{sq}^{-1}r = H\hat{K}_{pre}(H\hat{K}_{pre})^{-1}r = r$. Note that the second step is straightforward once $\hat{K}_{pre}$ is determined.

The proposed approach is also used in Example 2 where squaring down is performed by the static precompensator $\hat{K}_{pre} = \left[\begin{smallmatrix} 1 \\ -0.3 \end{smallmatrix}\right]$. The precompensator yields a minimum-phase, square system as desired. It is shown later on that there does not exist a static precompensator for the system in Example 3 such that the square system is minimum phase. However, there does always exist dynamic compensators such that the square system is minimum phase. The dynamic compensator design for Example 3 is presented later on.

The precompensator design $\hat{K}_{pre}$ introduces additional invariant zeros in the square system $\hat{\Sigma}_{sq}$ in (2). The interest is in stable precompensator designs $\hat{K}_{pre}$ that yield minimum-phase, square systems $\hat{\Sigma}_{sq}$ as they result in stable F and thus bounded u. If the compensator is stable, but the square system is nonminimum phase, then inversion techniques are required to compute bounded outputs of $\hat{\Sigma}_{sq}^{-1}$, since the invariant nonminimum-phase zeros become unstable poles in $\hat{\Sigma}_{sq}^{-1}$. Inversion techniques, such as stable inversion, are not preferred since they require preview. The critical step in the proposed approach is the design of $\hat{K}_{pre}$ which is presented in the following sections.

IV. Systematic representation for dynamic squaring down

In this section, the system is reformulated such that important properties, such as invariant zeros, are displayed to facilitate the squaring-down approach in the next section. This section constitutes contribution (I).

A. Dual problem

Most inversion problems in literature consider left-invertible systems. In the related work [11, III.B] it is claimed that stability of a dynamic compensator cannot be guaranteed using a direct squaring-down approach for right-invertible systems. For these reasons, the right-invertible system H of Fig. 2(a) is transformed to a left-invertible system by considering its dual as given by Lemma 3.

Lemma 3 (Dual system)**.** *The dual of the state-space system defined by the triplet (A, B, C) is given by $(A^\top, C^\top, B^\top)$.*

By Lemma 3, the dual $\hat{\Sigma} = H^d$ is given by

$$\hat{\Sigma}: \quad \begin{aligned} \hat{x}(k+1) &= \hat{A}\hat{x}(k) + \hat{B}\hat{u}(k), \\ \hat{y}(k) &= \hat{C}\hat{x}(k), \end{aligned} \tag{4}$$

with $(\hat{A}, \hat{B}, \hat{C}) = (A^\top, C^\top, B^\top)$ and $\hat{x} \in \mathbb{R}^n$, $\hat{y} \in \mathbb{R}^p$, $\hat{u} \in \mathbb{R}^m$, where $p = \bar{m}$, $m = \bar{p}$. Note that since H is right invertible, $\hat{\Sigma} = H^d$ is left invertible, see also Definition 2.

B. Special coordinate basis

The compensator design approach exploits properties of the special coordinate basis (s.c.b.). The transformation of a left-invertible system to an s.c.b. is given by Theorem 4. For right-invertible systems see, for example, [14]. For non-strictly proper systems see, for example, [15].

Theorem 4 (Special coordinate basis (s.c.b.))**.** *For left-invertible system $\hat{\Sigma}$ in* (4)*, there exist transformations*

$$\hat{x} = \Gamma_1 \begin{bmatrix} x_a \\ x_b \\ x_f \end{bmatrix}, \quad \hat{y} = \Gamma_2 \begin{bmatrix} y_f \\ y_s \end{bmatrix}, \quad \hat{u} = \Gamma_3 u,$$

with non-singular $\Gamma_1, \Gamma_2, \Gamma_3$ and $y_f \in \mathbb{R}^m, y_s \in \mathbb{R}^{p-m}$, such that the transformed system Σ satisfies an s.c.b., i.e.,

$$\begin{aligned} x_a(k{+}1) &= A_{aa}x_a(k) + A_{af}y_f(k) + A_{as}y_s(k), \\ x_b(k{+}1) &= A_{bb}x_b(k) + A_{bf}y_f(k), \\ x_{if}(k{+}1) &= A_{if}x_{if}(k) + L_{if}y_f(k) + B_{if}u_i(k) \\ &\quad + B_{if}\big(E_{ia}x_a(k) + E_{ib}x_b(k) + \sum_{j=1}^{K} E_{ij}x_{jf}(k)\big), \\ y_{if}(k) &= C_{if}x_{if}(k) = x_{1,i-1}(k), \quad y_f(k) = C_f x_f(k), \\ y_{is}(k) &= C_{is}x_{bis}(k) = x_{b1,i-1}(k), \quad y_s(k) = C_s x_b(k). \end{aligned} \tag{5}$$

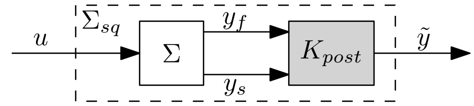


Fig. 3. Postcompensator K_{post} combines outputs y_f, y_s of the left-invertible system Σ in s.c.b. form into a new output $\tilde{y}$ such that the combined system Σ_{sq} is square. The freedom in design of K_{post} is exploited to make Σ_{sq} minimum phase.

where

$$\begin{aligned} A_{if} &= \begin{bmatrix} 0 & I_{(i-1)q_i} \\ 0 & 0 \end{bmatrix}, \quad B_{if} = \begin{bmatrix} 0 \\ I_{q_i} \end{bmatrix}, \\ C_f &= \operatorname{diag}\{C_{1f}, C_{2f}, \dots, C_{Kf}\}, \quad C_{if} = \begin{bmatrix} I_{q_i} & 0 \end{bmatrix}, \\ C_s &= \operatorname{diag}\{C_{1s}, C_{2s}, \dots, C_{Ks}\}, \quad C_{is} = \begin{bmatrix} I_{r_i} & 0 \end{bmatrix}, \end{aligned}$$

with $q_i, r_i \in \mathbb{N}$, $i = 1, \dots, K$, $K \leq n$.

Proof. The proof follows along similar lines as for continuous-time systems in [14, Theorem 2.1]. □

The invariant zeros are easily determined from the special coordinate basis, see Property 5.

Property 5 (Invariant zeros s.c.b. [14, Property 2.5])**.** *The invariant zeros (Definition 1) of $\hat{\Sigma}$ in* (5) *are the eigenvalues $\lambda(A_{aa})$ with A_{aa} in* (5)*.*

The s.c.b. structure of Theorem 4 is exploited in the squaring-down approach presented in the next section.

V. Squaring down

In this section, the squaring-down approach is presented for the left-invertible system Σ satisfying the s.c.b. in Theorem 4 and constitutes contribution (II). The concept of squaring down is outlined in section V-A. Two types of compensators are presented: static compensators in section V-B and dynamic compensators in section V-C. An overview of the complete feedforward design approach for the right-invertible system H in Fig. 2(a) is presented in section VI.

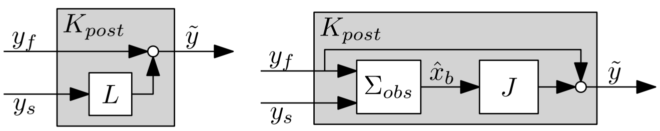
A. Squaring down a left-invertible system

The concept of squaring down a left-invertible system Σ in (5) is illustrated in Fig. 3. The outputs $y_s \in \mathbb{R}^m$, $y_f \in \mathbb{R}^{p-m}$ are combined into a new output $\tilde{y} \in \mathbb{R}^m$ through a postcompensator: $\tilde{y} = K_{post}\begin{bmatrix} y_f \\ y_s \end{bmatrix}$, such that $\Sigma_{sq} = K_{post}\Sigma$ is square with dimensions $m \times m$. The main idea is to design K_{post} such that Σ_{sq} is invertible and has no nonminimum-phase zeros to enable direct inversion.

The precompensator K_{pre} for the right-invertible dual system system Σ^d is obtained as the dual of the postcompensator of Σ, i.e., $K_{pre} = K_{post}^d$. The precompensator $\hat{K}_{pre}$ for system H in Fig. 2(b) is given by

$$\hat{K}_{pre} = \Gamma_2^{-\top} K_{pre} = \Gamma_2^{-\top} K_{post}^d. \tag{6}$$

Next, two types of postcompensator design are presented: static postcompensators in section V-B and dynamic postcompensators in section V-C.



(a) Static postcompensator with static gain L.

(b) Dynamic postcompensator with observer Σ_{obs} to obtain $\hat{x}_b$ for use in state feedback.

Fig. 4. Two proposed postcompensator designs for K_{post} in Fig. 3.

B. Static compensator

The static compensator design of K_{post} in Fig. 3 is given by Theorem 6 and illustrated in Fig. 4(a).

Theorem 6 (Static compensator)**.** *Given Fig. 3 with Σ in* (5)*, the static compensator is given by*

$$K_{post} = \begin{bmatrix} I_m & L \end{bmatrix},$$

i.e., $\tilde{y} = y_f + Ly_s$, with $L \in \mathbb{R}^{m\times(p-m)}$.

Proof. The proof follows along similar lines as for continuous-time systems in [11, III.A]. □

Properties of the square system Σ_{sq} for the static compensator of Theorem 6 are provided by Theorem 7, where n_a, n_b are the dimensions of x_a, x_b in (5), respectively.

Theorem 7 (Properties Σ_{sq} static compensator)**.** *Given H in* (1) *and Σ in* (5)*, the square system Σ_{sq} in Fig. 3 with K_{post} the static compensator of Theorem 6 has the properties:*

- *invertible,*
- *$n_a + n_b$ invariant zeros: $\lambda(A_{aa}) + \lambda(A_{bb} - A_{bf}LC_s)$,*
- *n poles: $\lambda(\hat{A})$.*

Proof. The proof follows along similar lines as for continuous-time systems in [11, Theorem 3.1]. □

Theorem 7 shows that the static compensator introduces new invariant zeros in addition to the invariant zeros $\lambda(A_{aa})$ of Σ. When inverting the square system, these zeros become poles. Hence, these zeros are preferred to be minimum phase to avoid the use of inversion techniques which require preview. The zeros $\lambda(A_{aa})$ are fixed, but since Σ is non-square, they are often non-existing, and they may all be minimum phase. Theorem 8 shows how the additional zeros can possibly be placed through static output feedback.

Theorem 8 (Invariant zero placement static compensator)**.** *The invariant zeros introduced by the static compensator in Theorem 6 can possibly be placed by solving the static output feedback problem for the triplet (A_{bb}, A_{bf}, C_s).*

Proof. By Theorem 7, the additional invariant zeros are given by $\lambda(A_{bb} - A_{bf}LC_s)$ and thus affected by L. These zeros are also the poles of the state-space system $(A, B, C) = (A_{bb}, A_{bf}, C_s)$ with static output feedback gain $-L$. The placement is equivalent to a static output feedback problem which is not always solvable [16]. □

The static compensator design of Theorem 6 is applied to Example 2 and Example 3.

Example 2 (continued) *The dual left-invertible system, see also Lemma 3, is given by*

$$H^d = \frac{1}{(z-0.1)(z+0.8)(z-0.4)} \begin{bmatrix} (z-0.6)(z+2) \\ (z-5) \end{bmatrix},$$

with $m = 1$, $p = 2$. An s.c.b, see Theorem 4, is given by

$$\begin{aligned} x_b(k+1) &= \begin{bmatrix} -1.4 & 1 \\ 1.2 & 0 \end{bmatrix} x_b(k) + \begin{bmatrix} 1 \\ -5 \end{bmatrix} y_f(k), \\ x_f(k+1) &= \begin{bmatrix} -0.657 & 0.345 \end{bmatrix} x_b(k) + 1.1x_f(k) + u(k), \\ y_f(k) &= x_f(k), \\ y_s(k) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_b(k), \end{aligned}$$

with $\Gamma_2 = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$. The s.c.b. confirms H^d has no invariant zeros since x_a is non-existent. Selecting $L = -0.3$ in Theorem 6 yields $K_{post} = \begin{bmatrix} 1 & -0.3 \end{bmatrix}$ and $\lambda(A_{bb} - A_{bf}LC_s) = \{-0.5, -0.6\}$. Hence, by (6)*, $\hat{K}_{pre} = \left[\begin{smallmatrix} 1 \\ -0.3 \end{smallmatrix}\right]$ which corresponds to the compensator design presented earlier.*

Example 3 (continued) *An s.c.b, see Theorem 4, of the dual left-invertible system (see Lemma 3), is given by*

$$\begin{aligned} x_b(k+1) &= \begin{bmatrix} -1.4 & 1 \\ 1.2 & 0 \end{bmatrix} x_b(k) + \begin{bmatrix} -5.5 \\ -3.3 \end{bmatrix} y_f(k), \\ x_f(k+1) &= \begin{bmatrix} 0.609 & -0.294 \end{bmatrix} x_b(k) + 1.1x_f(k) + u(k), \\ y_f(k) &= x_f(k), \\ y_s(k) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_b(k), \end{aligned} \tag{7}$$

with $\Gamma_2 = \left[\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}\right]$. It can be verified that there does not exist an $L \in \mathbb{R}$ such that $|\lambda(A_{bb} - A_{bf}LC_s)| < 1$. Hence, for this system, there does not exist a static compensator that yields a square system which is minimum phase.

Theorem 8 shows that the problem of designing a post-compensator reduces to solving a static output feedback problem. However, the static output feedback problem does not always have a solution, as also illustrated by Example 3. Therefore, in the next section a dynamic compensator is constructed for which the additional invariant zeros can always be placed arbitrarily.

C. Dynamic compensator

In this section, an observer is used to reconstruct full state x_b, followed by state feedback on this observed state $\hat{x}_b$. The resulting dynamic compensator is presented in Theorem 9 and illustrated in Fig. 4(b). Note that the observer poles $\lambda(N)$ can be placed arbitrarily since (A_{bb}, A_{bf}) is controllable as Σ is a minimal realization.

Theorem 9 (Dynamic compensator)**.** *Given Fig. 3 with Σ in* (5)*, the dynamic compensator K_{post} is given by*

$$K_{post} = \begin{bmatrix} I_m & 0 \end{bmatrix} + J\Sigma_{obs},$$

i.e., $\tilde{y}(k) = y_f(k) + J\hat{x}_b(k)$, $\hat{x}_b(k) = \Sigma_{obs} \begin{bmatrix} y_f(k) \\ y_s(k) \end{bmatrix}$, where Σ_{obs} denotes a minimal order observer, see e.g. [17, section 2.3], for the matrix triplet (A_{bb}, A_{bf}, C_s) with poles $\lambda(N)$.

Proof. Due to space restrictions the proof is omitted. □

Properties of the square system Σ_{sq} for the dynamic compensator of Theorem 9 are provided by Theorem 10.

Theorem 10 (Properties Σ_{sq} dynamic compensator)**.** *Given H in* (1) *and Σ in* (5)*, the minimal realization of the square system Σ_{sq} in Fig. 3 with K_{post} the dynamic compensator of Theorem 9 has the properties:*

- *invertible,*
- *$n_a + n_b$ invariant zeros: $\lambda(A_{aa}) + \lambda(A_{bb} - A_{bf}J)$,*
- *n poles: $\lambda(\hat{A})$.*

Proof. The proof follows along similar lines as for continuous-time systems in [11, Theorem 3.2]. □

The additional invariant zeros $\lambda(A_{bb} - A_{bf}J)$ are affected by J and can be arbitrarily placed through static state feedback of the pair (A_{bb}, A_{bf}). This requires that (A_{bb}, A_{bf}) is controllable which is satisfied since Σ is a minimal realization. Hence, Σ_{sq} can always be made minimum phase which enables the use of direct inversion without preview.

Next, the dynamic compensator is applied to Example 3.

Example 3 (continued) *The subsystem (A_{bb}, A_{bf}, C_s) in* (7) *has two states and one output. Let the desired invariant zeros be $\lambda(N) = 0.7$ and $\lambda(A_{bb} - A_{bf}J) = \{-0.5, -0.6\}$, then pole placement on the pair (A_{bb}, A_{bf}) yields $J = \begin{bmatrix}0.164 & -0.182\end{bmatrix}$. The dynamic postcompensator, see Theorem 9, is given by*

$$K_{post} = \frac{1}{(z-0.7)} \begin{bmatrix}0.6 & 0.291(z-0.531)\end{bmatrix}.$$

The system F in (3) *is given by*

$$F = \frac{(z-0.4)(z+0.8)(z-0.1)}{(z+0.6)(z+0.5)(z-0.7)} \begin{bmatrix}0.709(z+1.064) \\ 0.291(z-0.531)\end{bmatrix}$$

and is stable. It can be verified that $HF = 1$ and hence perfect tracking is obtained.

In summary, the dynamic postcompensator design in Theorem 9 can always create a stable, minimum-phase, square system, if H in (1) is stable and has no nonminimum-phase invariant zeros, which holds for most non-square systems.

VI. Application in tracking control

In this section, the design framework is summarized and applied to a benchmark system, which constitutes contribution (III) and (IV), respectively.

A. Design framework

The systematic design procedure for design of F given a right-invertible system H is shown in Fig. 5 and combines the results of previous sections. There are two main design types: static and dynamic squaring down of which the properties are given by Lemma 11 and Lemma 12, respectively.

Lemma 11 (Properties controller with static compensator)**.** *Given H in* (1) *and Σ in* (5)*, the minimal realization of F in Fig. 5 based on a static compensator has:*

- *n invariant zeros: $\lambda(\hat{A})$,*
- *$n_a + n_b$ poles: $\lambda(A_{aa}) + \lambda(A_{bb} - A_{bf}LC_s)$.*

Proof. The results follow from Theorem 7 and Fig. 5. □

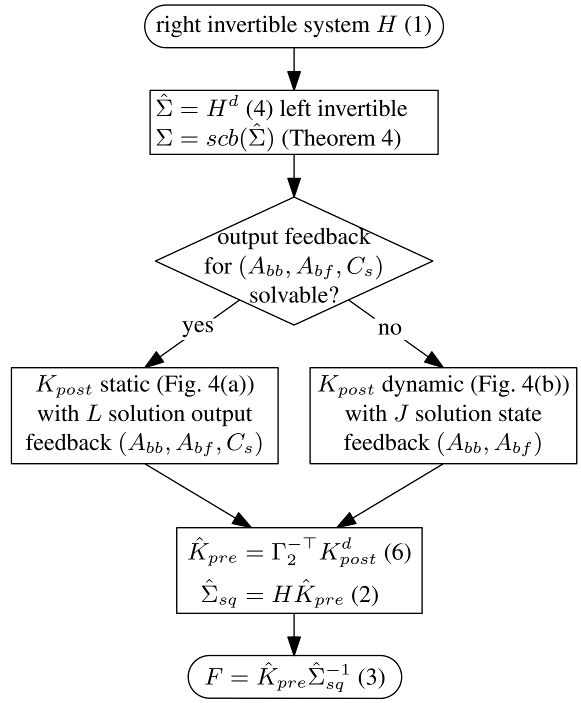
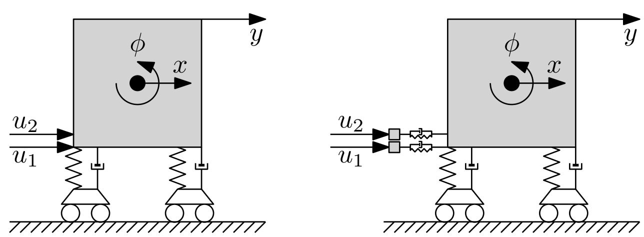


Fig. 5. Proposed inversion approach for right-invertible systems in tracking control as shown in Fig. 2. The precompensator $\hat{K}_{pre}$ is designed such that the square system $\hat{\Sigma}_{sq}$ has desired properties, for example, minimum phase. Perfect tracking is obtained through squaring down and inversion of the square system.



(a) System of [8, section 6]. (b) System (a) with actuator dynamics.

Fig. 6. Two-input, one-output systems of which both individual transfers are nonminimum phase.

Lemma 12 (Properties controller with dynamic compensator)**.** *Given H in* (1) *and Σ in* (5)*, the minimal realization of F in Fig. 5 based on a dynamic compensator has:*

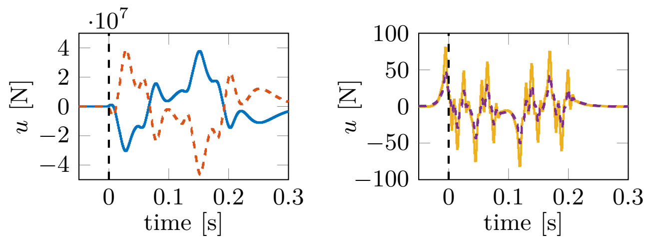
- *n invariant zeros: $\lambda(\hat{A})$,*
- *$n_a + 2n_b - p + m$ poles: $\lambda(A_{aa}) + \lambda(N) + \lambda(A_{bb} - A_{bf}J)$.*

Proof. The results follow from Theorem 10 and Fig. 5. □

The location of the invariant zeros of $\hat{\Sigma}_{sq}$ directly influence the dynamics and the resulting input signals u_1, u_2 in Fig. 2. To avoid the use of preview, these invariant zeros should be minimum phase.

B. Application to a flexible cart system

In this section, the squaring-down approach is applied to the non-square benchmark systems in Fig. 6.



(a) Dynamic squaring down: u_1 (——) and u_2 (- - -).

(b) Norm-optimal feedforward: u_1 (——) and u_2 (- - -).

Fig. 7. Input signals for the system in Fig. 6(b). The dynamic squaring-down approach avoids the use of preview, but results in large input signals.

1) Static compensator design: In this part, a static compensator is designed for the system in Fig. 6(a). The system is the same as the benchmark system in [8] with the addition of one extra input. In [8, section 6] a static precompensator is derived for this system based on physical insights. Next, the systematic design procedure illustrated in Fig. 5 is used.

The dual left-invertible system in s.c.b. form shows that the output feedback problem for (A_{bb}, A_{bf}, C_s) is solvable, as is to be expected based on the results in [8, section 6]. In particular, for $6.99 \cdot 10^7 < L < 9.77 \cdot 10^7$ the square system is minimum phase. For $L = 7.99 \cdot 10^7$ the static precompensator is given by $\hat{K}_{pre} = 5.77 \cdot 10^7 \begin{bmatrix} -1 \\ 1.1 \end{bmatrix}$. The compensator design matches that of [8, section 6] apart from a scaling factor. The scaling factor is canceled in F (see (3)) and hence the same signals u_1, u_2 as in [8, section 6] are obtained.

2) Dynamic compensator design: In this part, a dynamic compensator is designed for the system in Fig. 6(b) which includes actuator dynamics. The actuator dynamics for both inputs are modeled as identical mass-damper-spring systems with mass $m_a = 0.001$ kg, damping $d_a = 0.5$ Ns/m, and spring $k_a = 100$ N/m. Since there does not exist a satisfactory static compensator (Theorem 6) for the system in Fig. 6(b), a dynamic compensator (Theorem 9) is designed with $\lambda(N) = \{-0.8, 0.85, 0.9, 0.95\}$ and $\lambda(A_{bb} - A_{bf}J) = \{0.75, -0.8, 0.85, 0.9, 0.95\}$ such that the square system is minimum phase.

Fig. 7 shows the input signals for the dynamic squaring-down approach and the norm-optimal control solution [8]. Both solutions yield exact tracking by design, with the key difference that the squaring-down approach avoids the use of preview. However, the corresponding input signals are relatively large in magnitude. This is a consequence of the selected invariant zeros which are design variables and can be used to 'shape' the input signals.

VII. Conclusions

System inversion is essential in tracking control applications such as inverse model feedforward and iterative learning control. Direct inversion of nonminimum-phase systems leads to unbounded responses. For such systems, bounded responses can be obtained through inversion approaches. However, most inversion approaches are restricted to square or even singlevariable systems. Furthermore, the use of preview plays a key role in most inversion techniques, but is often undesired.

In this paper, the design freedom in overactuated systems, i.e., systems with more inputs than outputs, is exploited to avoid the use of preview. A systematic design approach is presented that yields exact tracking, while avoiding the use of preview. The proposed approach is applied on a benchmark system demonstrating the mentioned properties.

The generated input signals are influenced by the location of the invariant zeros of the square system. Design guidelines for the placement of these zeros is part of future research. Future work also aims at experimental validation of the presented results.

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