Inverting Nonminimum-Phase Systems from the Perspectives of Feedforward and ILC ⋆

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Abstract: System inversion is at the basis of many feedforward and learning control algorithms. This paper aims to analyze several of these approaches in view of their subsequent use, showing inappropriate use that is previously overlooked. This leads to new insights and approaches for both feedforward and learning. The methods are compared in various aspects, including finite vs. infinite preview, exact vs. approximate, and quality of inversion in various norms which directly relates to their use. The results are validated on a nonminimum-phase benchmark system.

Keywords: Model inversion, Nonminimum phase, Feedforward control, ILC

1. INTRODUCTION

The quality of inversion depends on its goal. The aim of this paper is to investigate, compare, and develop inversion techniques for the purpose of both feedforward and learning control. The key difference is in which norm the approximation is most relevant. The model to be inverted includes the closed-loop process sensitivity in iterative learning control (ILC) (Steinbuch and van de Molengraft, 2000), the closed-loop complementary sensitivity in repetitive control (Bristow et al., 2006), or the open-loop system in inverse model feedforward (Boeren et al., 2015). For nonminimum-phase or strictly proper systems, several approaches can be pursued for inversion.

System inversion has received significant attention, also from a theoretical perspective (Silverman, 1969). Successful approximate solutions include ZPETC (Tomizuka, 1987), ZMETC, NPZ-Ignore (Gross et al., 1994), and EBZPETC (Torfs et al., 1992), see also Butterworth et al. (2012) for an overview. Additionally, standard $\mathcal{H}_\infty$ without preview has been used (Wang et al., 2016; De Roover and Bosgra, 2000) to design ILC filters, as well as $\mathcal{H}_\infty$ preview in feedforward (Hazell and Limebeer, 2008; Mirkin, 2003). Furthermore, optimization-based approaches include techniques based on LQ tracking control (Athans and Falb, 1966), also known as norm-optimal ILC (Gunnarsson and Norrlöf, 2001), where in addition to inversion a weight on the input signal is imposed. In Wen and Potsaid (2004), ZPETC, ZMETC, and a model matching approach are compared. The model matching approach is similar to the $\mathcal{H}_\infty$ preview control presented in the present work, yet without preview.

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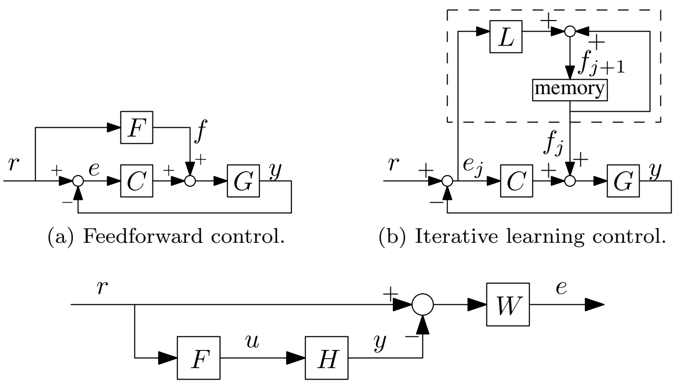
Although many algorithms and approaches are available for model inversion, there is a striking misuse of inappropriate techniques that hampers performance. For example, ZPETC is often used for the design of ILC filters, but requires an additional robustness filter at the cost of performance (Steinbuch and van de Molengraft, 2000), while better $\mathcal{H}_\infty$ approaches are available (Wang et al., 2016; De Roover and Bosgra, 2000). This paper provides guidelines on suitable use of inversion techniques for both inverse model feedforward and learning control by addressing the application specific objective. The aim of this paper is to compare existing approaches and provide several new approaches with clear benefits. In this respect it extends Butterworth et al. (2012); Teng and Tsao (2015) with additional approaches and by explicitly addressing the control goal.

The outline of the paper is as follows. In section 2, the inverse model feedforward and ILC optimization problems are cast in a single general framework, and the associated challenges, optimization criteria, and properties are presented. In section 3, the benchmark system used for assessing the inversion techniques is introduced. In section 4, the inversion techniques are presented. First, the well-known approximate inverse techniques NPZ-Ignore, ZPETC, and ZMETC are recapitulated, followed by stable inversion. Second, inversion techniques based on norm-optimal feedforward/ILC are presented. Third, $\mathcal{H}_\infty$ and $\mathcal{H}_2$ preview control are presented. In section 5, an overview of the techniques is presented and the techniques are evaluated on the benchmark system of section 3 in both a feedforward and an ILC setting. Section 6 contains conclusions and guidelines.

Notation Discrete, single-input, single-output (SISO), linear, time-invariant (LTI) systems are considered. Let $S = (1 + GC)^{-1}$ denote the sensitivity and $\lambda_i(\cdot)$ the i-th eigenvalue. A causal LTI system is referred to as stable (minimum phase) iff all poles (zeros) are inside the unit



(a) Feedforward control.

(b) Iterative learning control.

(c) General block diagram.

Fig. 1. The feedforward (a) and ILC (b) design problem are equivalent to finding u in (c) that minimizes e.

disk, otherwise the system is referred to as unstable (nonminimum phase). For ease of presentation, it is assumed that the inverted system is hyperbolic, i.e., contains no eigenvalues on the unit circle. Note that techniques as in Devasia (1997) can be used to relax this condition.

2. PROBLEM DEFINITION

In this section, the inverse model feedforward and ILC optimization problem are detailed, the common inversion problem is formulated, and the application specific criteria are defined.

2.1 Role of inversion for feedforward and ILC

Feedback and feedforward control are typically combined to achieve high performance. Feedback control can deal with uncertainty, but its performance is limited due to Bode's sensitivity integral. For known signals, feedforward control can be used to achieve excellent performance. In the feedforward scheme of Fig. 1(a), the goal is to design feedforward f such that tracking error $e = r - y$ is minimized, where r is the desired trajectory for output y. If the system performs repetitive trajectories r, information of the previous task j can be used to enhance the performance of the next task $j+1$ through iterative learning control (ILC). For the ILC scheme of Fig. 1(b), f_{j+1} is designed based on data e_j, f_j such that $e_{j+1} = e_j - SG(f_{j+1} - f_j)$ is minimized.

Both the feedforward and the ILC design problem can be cast into the diagram of Fig. 1(c). Both problems are equivalent to finding an input u such that error e is minimized. System H can be an open-loop system as in feedforward (G) or a closed-loop system as in ILC (SG) and repetitive control (SGC).

2.2 On inversion

Consider the general block diagram in Fig. 1(c). Throughout, it is assumed that system H is proper with relative degree $d \in \mathbb{Z}$, has $p \in \mathbb{Z}$ nonminimum-phase zeros, and has state-space realization (A, B, C, D). An immediate solution to minimize e is to select $F = H^{-1}$, where

$$H^{-1} \stackrel{s}{=} \left[\begin{array}{c|c} A - BD^{-1}C & BD^{-1} \\ \hline D^{-1}C & D^{-1} \end{array}\right]. \qquad (1)$$

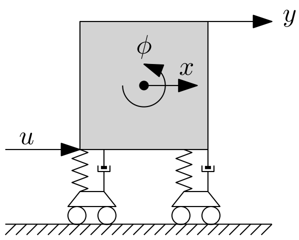


Fig. 2. The benchmark system is a mass that can translate in x direction and rotate in ϕ direction. The system has input force u and output position y.

At least three challenges are associated with the direct use of (1):

I) delay: for $d > 0$, H^{-1} does not exist since D is not invertible.
II) non-square: systems with a different number of inputs then outputs cannot be directly inverted as in (1) since D is non-square.
III) nonminimum-phase zeros: for $p > 0$, H^{-1} is unstable which, when solved forward in time, yields unbounded u.

Remark 1. The first issue can be overcome by inverting the bi-proper system $\bar{H} = z^d H$, where the a-causal z^d is implemented as a time-shift on the time domain signal. Note that for an infinite time horizon, filtering the time-shifted signal with $\bar{H}$ is equivalent to filtering the original signal with H, whereas for a finite time horizon this introduces boundary errors. The presented inversion techniques aim at the second and third issue.

2.3 Criteria

Depending on H, it might not be possible to achieve zero error $e = 0$. Therefore, the inversion techniques construct u given a certain criterion aimed at minimizing e. The criterion depends on the particular application.

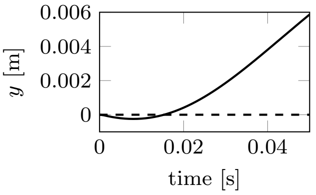
In feedforward, generally high performance in terms of the error e is pursued. Typically, this is enforced by minimizing the energy in the error signal through minimizing $\|e\|_2$ (Van der Meulen et al., 2008; Boeren et al., 2015).

In ILC, the main concern is to guarantee convergence in the error to ensure stability over trails. Superior performance is obtained by executing several trials. For update $f_{j+1} = f_j + Le_j$, see also Fig. 1(b), the error has trial dynamics $e_{j+1} = (1 - SGL)\, e_j$. Hence, to ensure the monotonic convergence of $\|e_j\|_2$ over trials it should hold $\|1 - SGL\|_\infty < 1$ (Bristow et al., 2006). Assuming this is feasible, the fastest convergence for arbitrary e_j is found by minimizing $\|1 - SGL\|_\infty$. In the general block diagram of Fig. 1(c), this is equivalent to minimizing $\|W(1-HF)\|_\infty$.

3. BENCHMARK SYSTEM

To validate the inversion techniques of section 4, the benchmark system shown in Fig. 2 is used. The same system is used in Van Zundert et al. (2016) for different purposes.

The open-loop system G from force u [N] to position y [m] in Fig. 2 is given by



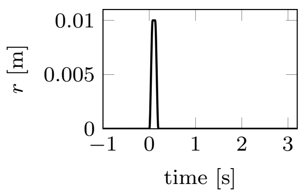


Fig. 3. Impulse response SG. Fig. 4. Trajectory r.

$$G = \frac{-3\times10^{-8}(z+0.963)(z-0.945)(z-1.141)}{(z-1)^2(z^2-1.960z+0.963)},$$

with sample time $h = 0.001$ s. The process sensitivity SG with feedback controller $C = \frac{925(z-0.998)}{z-0.981}$ is

$$SG = \frac{-3\times10^{-8}(z+0.963)(z-0.945)(z-1.141)(z-0.981)}{(z-0.990)(z^2-1.990z+0.990)(z^2-1.960z+0.964)}.$$

For both $H = G$ in feedforward and $H = SG$ in ILC, the following observations can be made:

- H is stable ($|\lambda_i(H)| < 1, \forall i$);
- H has one nonminimum-phase zero $z = 1.141$ ($p = 1$);
- H is strictly proper ($D = 0$) with relative degree $d = 1$ (see also Remark 1).

The impulse response for SG is shown in Fig. 3. Initially the system moves in opposite direction due to the nonminimum-phase zero.

The reference trajectory r is a fourth-order forward-backward motion that is non-zero for $t \in [0, 0.2]$ and of total length $N = 4201$ samples as depicted in Fig. 4. Time $t = 0$ is defined as the start of the movement. The zero values at the start and end of the trajectory allow for pre-actuation and post-actuation, respectively.

4. OVERVIEW OF TECHNIQUES

In this section, the inversion techniques are presented, developed, and implemented on the benchmark system of section 3.

4.1 Approximate inverse (NPZ-Ignore, ZPETC, ZMETC)

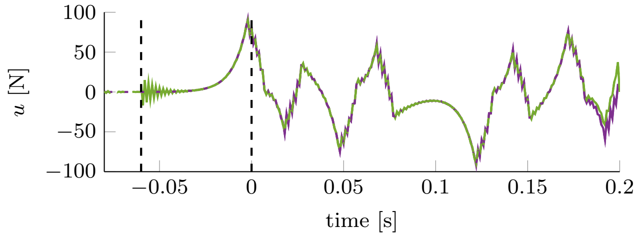
Approach As mentioned in section 2.2, nonminimum-phase zeros and delays are key challenges for system inversion. A key issue is that the inverse system is unstable. Several techniques have been proposed to approximate the inverse, including NPZ-Ignore (Gross et al., 1994), zero-phase-error tracking control (ZPETC) (Tomizuka, 1987), and zero-magnitude-error tracking control (ZMETC). If H is nonminimum phase then the three approaches are identical and exact: $HF = 1$.

Application to benchmark system Due to space restrictions, this part is omitted. See, for example, Butterworth et al. (2012) for an overview and comparison of the methods.

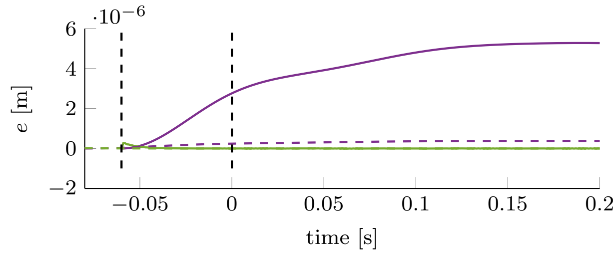
Summary The approximate inversion techniques are based on an approximate infinite horizon design with finite preview.

4.2 Stable inversion

Approach The techniques presented in the previous section are all based on approximations of the unstable part of the inverse system. In contrast, stable inversion regards the unstable part as a non-causal operation and generates signal u based on infinite preview. In practice, the boundaries are finite and introduce boundary errors. The stable approach can be found in, for example, Van Zundert et al. (2016).



(a) Norm-optimal feedforward, (- - -), (——) generates a non-zero input u at the begin of the task to compensate for boundary effects. The effect is larger for 60 samples preview (——) than for 80 samples preview (- - -). Stable inversion generates identical u with 60 samples preview (——) and 80 samples preview (- - -) for $t \geq -0.06$. With 80 samples preview u is non-zero for $-0.08 \leq t < -0.06$ u.



(b) The infinite design of stable inversion (- - -), (——) introduces significant boundary errors on a finite interval, whereas these are much smaller for norm-optimal feedforward (- - -), (——).

Fig. 5. The finite design norm-optimal feedforward generates an input that compensates for boundary effects and thereby outperforms stable inversion on a finite horizon since the latter is an infinite design.

Application to benchmark system To illustrate the influence of preview on the performance of the stable inversion approach, the approach is applied to $H = G$ with r the reference trajectory of Fig. 4 in the time intervals $[-0.06, 0.2]$ and $[-0.08, 0.2]$, i.e., the pre-actuation is restricted to either 60 or 80 samples and the post-actuation to 0 samples. The results shown in Fig. 5 show a considerable performance improvement when increasing the pre-actuation from 60 to 80 samples. For infinite pre-actuation and post-actuation, the results become exact.

Summary Stable inversion is an infinite time design that is exact on an infinite time horizon and has infinite preview. A finite time horizon introduces boundary errors.

4.3 Norm-optimal feedforward/ILC

In this section, norm-optimal inversion techniques based on norm-optimal ILC are considered.

Approach Within ILC there are two main classes. The first class is frequency domain ILC in which a learning filter $L \approx (SG)^{-1}$ is constructed. The filter is typically implemented using ZPETC or ZMETC, see section 4.1.

The second class is norm-optimal ILC in which weighted 2-norms of e_{j+1} and f_{j+1} are minimized. Here, subscript j denotes the current trial and $j+1$ the next trial. For this class, a common solution method is lifted ILC. The method is based on describing input-output relations in lifted/supervector notation (Bristow et al., 2006), where systems are described by $N \times N$ matrices, with N the task length. The use of $N \times N$ matrix calculations results in extensive computation times growing as $\mathcal{O}(N^3)$. An alternative is to use Riccati equations to find the optimal solution (Van Zundert et al., 2016). The approach yields exactly the same optimal solution, but the computation time only grows as $\mathcal{O}(N)$.

The ILC approach can be found in Theorem 6 of Van Zundert et al. (2016). Feedforward can be seen as a special case of ILC with only one trial and hence no input change weight $w_{\Delta f}$. In particular, if H is bi-proper and $w_f, w_{\Delta f} = 0$, the problem reduces to the well-known LQ tracking problem (Athans and Falb, 1966) with criterion

$$\sum_{k=1}^{N-1} (e[k])^\top Q(e[k]) + (u[k])^\top R(u[k]). \qquad (2)$$

Application to benchmark system The norm-optimal feedforward approach is applied to the benchmark system $H = G$ under the same conditions as in section 4.2, i.e., with reduced pre-actuation and post-actuation. The results are shown in Fig. 5. The approach outperforms stable inversion since it takes the boundary effects into account using the linear time-varying (LTV) character of the solution. This behavior can be observed at the start of u in Fig. 5(a).

Summary Norm-optimal ILC/feedforward is a finite time design and has infinite preview (equal to the task length). The approach is optimal in terms of minimizing $\|e\|_2$ if $w_f = 0$ in ILC or if $R = 0$ in feedforward.

4.4 Preview control

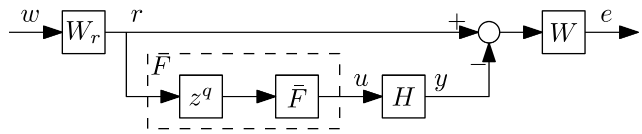
In preview control the inverse system is optimized for a specific infinite time objective, with pre-defined preview.

Approach A general formulation of preview control is shown in Fig. 6(a) where F is decomposed into $F = \bar{F}z^q$, with preview $q \in \mathbb{N}$. Note that an input weighting W_r is added compared to Fig. 1(c). For fixed q, $\bar{F}$ follows from

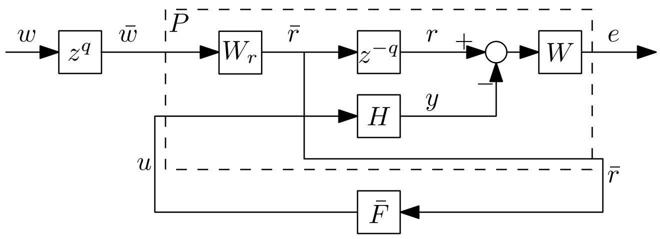
$$\bar{F} = \arg\min_{\bar{F}} \|W\left(z^{-q} - H\bar{F}\right)W_r\|_x, \qquad (3)$$

where x is a certain norm. The problem cast into the general plant formulation is shown in Fig. 6(b). Note that the pre-multiplication with the a-causal part z^q is not part of generalized plant P to ensure $P \in \mathcal{RH}_\infty$. For the special case $q = 0$, the approach in Wen and Potsaid (2004) is recovered.

First, optimal ILC synthesis is presented in which the induced (worst-case) 2-norm of the error e is minimized by using $\mathcal{H}_\infty$ preview control, i.e., $x = \infty$ in (3). The input is set to $w = r$, i.e., $W_r = 1$. $\mathcal{H}_\infty$-synthesis on P minimizes $\|W\left(z^{-q} - H\bar{F}\right)\|_\infty = \|W\left(1 - HF\right)\|_\infty$ which is the ILC convergence criterion if $W = 1$, see also section 2.3.



(a) In preview control the objective is minimization of a certain norm on the transfer $w \to e$.



(b) General plant formulation.

Fig. 6. In preview control the preview q is fixed and $\bar{F}$ is optimized to minimize a certain norm on the transfer $\bar{w} \to e$ in the standard plant formulation.

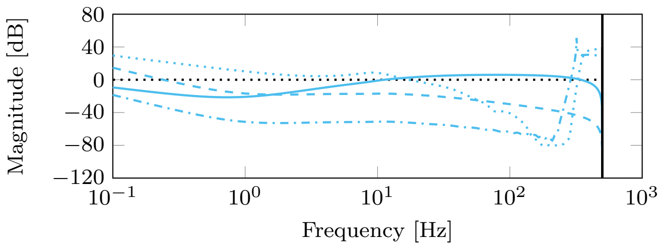


Fig. 7. $W(1-HF)$ for $\mathcal{H}_\infty$ preview control for $q = 0$ (——), $q = 20$ (- - -), $q = 50$ (-·-·-), and $q = 100$ (·····). For larger q, $|W(1-HF)|$ is smaller.

Second, optimal inverse model feedforward synthesis is presented in which $\|e\|_2$ is minimized for a specific reference r by using $\mathcal{H}_2$ preview control, i.e., $x = 2$ in (3). Input w is white noise of unity intensity and W_r is the power spectrum of r such that $\mathcal{H}_2$-synthesis on P minimizes $\|e\|_2$ for the spectrum of $r = W_r w$.

Application to benchmark system For $\mathcal{H}_\infty$-preview control in a feedfoward setting, the results for a range of preview values q are shown in Fig. 7. More preview q introduces more design freedom and hence $|W(1-HF)|_\infty$ decreases.

The filters HF for $\mathcal{H}_2$-preview control in a feedforward setting with input weighting

$$W_r(z) = \left(\frac{2.024(z+0.0330)}{z-0.957}\right)^4 \qquad (4)$$

are shown in Fig. 8. For larger q, HF is closer to unity.

Summary Preview control is an infinite time design with finite pre-defined preview. $\mathcal{H}_\infty$ and $\mathcal{H}_2$ preview control address the control goal in ILC and feedforward, respectively.

5. A CONTROL GOAL PERSPECTIVE

A qualitative overview of the inversion techniques of the previous section is provided in Table 1. In this section, the control goal is added to the inversion techniques of section 4 and the results are validated on the benchmark system of section 3.

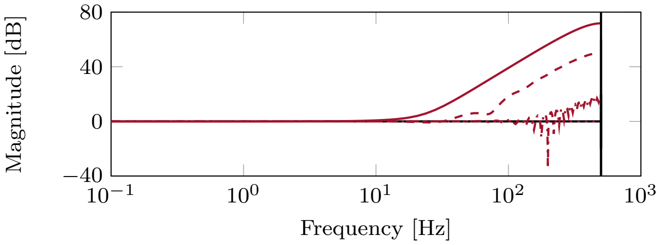


Fig. 8. HF for $\mathcal{H}_2$ preview control for $q = 1$ (——), $q = 20$ (- - -), $q = 50$ (-·-·), and $q = 100$ (·······).

Table 1. Overview of inversion techniques.

Technique	Design	Preview	Aim
NPZ-Ignore	infinite	finite	H^{-1} approximation
ZPETC	infinite	finite	H^{-1} approximation
ZMETC	infinite	finite	H^{-1} approximation
stable inversion	infinite	infinite	H^{-1} exact
norm-optimal	finite	infinite	$\min \|e\|_2$
$\mathcal{H}_\infty$ preview	infinite	finite	$\min \|W(1-HF)\|_\infty$
$\mathcal{H}_2$ preview	infinite	finite	$\min \|e\|_2$

Table 2. Design and performance in a feedforward setting.

Technique	Settings	$\|e\|_2$
NPZ-Ignore	-	0.0153
ZPETC	-	0.0050
ZMETC	-	0.0317
stable inversion	-	9.8782×10^{-10}
norm-optimal	$Q = 1$; $R = 10^{-18}$	4.3453×10^{-8}
$\mathcal{H}_\infty$ preview control	$q = 100$	1.6074×10^{-5}
$\mathcal{H}_2$ preview control	W_r: (4); $q = 100$	3.0667×10^{-8}

5.1 Application to feedforward

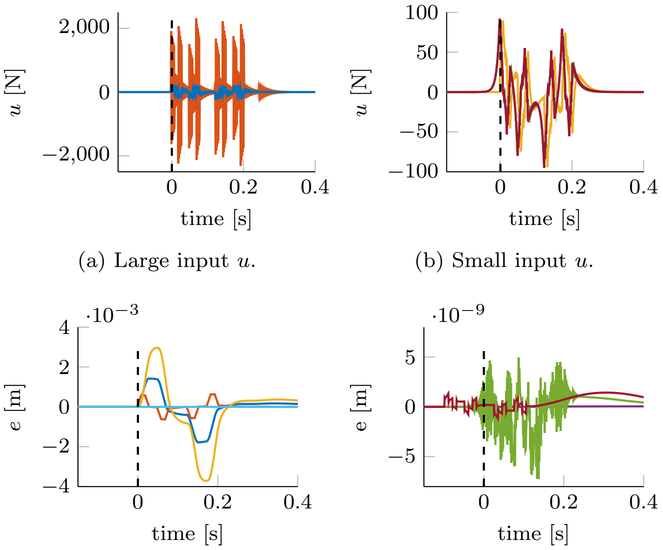
Table 2 summarizes the results of the techniques in a feedforward setting on the benchmark system of section 3. Note that for norm-optimal feedforward, $R > 0$ in (2) is required to avoid singularity, with $Q \gg R$ for small $\|e\|_2$.

The signals u, e are shown in Fig. 9 for the most relevant part of the time axis. Fig. 9(a) shows that the generated inputs of NPZ-Ignore and ZPETC are not very well suited for practical application, which is in line with the analysis in section 6 of Butterworth et al. (2012). Also the errors are considerably large as shown in Table 2 and Fig. 9.

Fig. 9(b) shows that the inputs of stable inversion, norm-optimal feedforward, $\mathcal{H}_\infty$ preview control, and $\mathcal{H}_2$ preview control are similar, whereas that of ZMETC is different. Table 2 and Fig. 9 show a moderately large error for $\mathcal{H}_\infty$ preview control and a considerably larger error for ZMETC. Stable inversion, norm-optimal feedforward, and $\mathcal{H}_2$ preview control achieve the lowest $\|e\|_2$, which is to be expected since these techniques are aimed at minimizing $\|e\|_2$, see also Table 1. The desired preview in preview control depends on the location of the nonminimum-phase zeros (Middleton et al., 2004).

5.2 Application to ILC

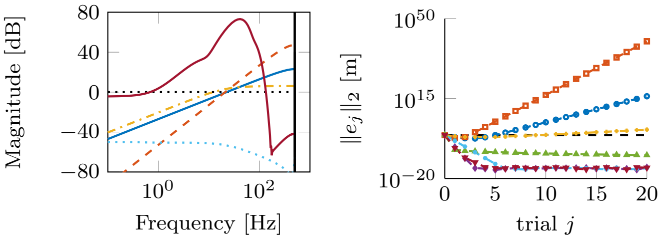
Guaranteed convergence In an ILC setting, there is guaranteed monotonic convergence in error norm $\|e_j\|_2$ over the trials if $\|W(1-HF)\|_\infty = \|1-GSF\|_\infty < 1$, see also section 2.3. It might be possible that the condition cannot be satisfied at all, but if the condition can be satisfied, $\mathcal{H}_\infty$



(a) Large input u. (b) Small input u.

(c) Large error e. (d) Small error e.

Fig. 9. Input u and error e for NPZ-Ignore (——), ZPETC (——), ZMETC (——), stable inversion (——), norm-optimal feedforward (——), $\mathcal{H}_\infty$ preview control (——), and $\mathcal{H}_2$ preview control (——).



(a) ILC criterion only guarantees convergence for $\mathcal{H}_\infty$ preview control.

(b) Convergence of $\|e\|-2$: the approximate inverse techniques diverge, all others converge.

Fig. 10. ILC convergence for NPZ-Ignore (——), ZPETC (——), ZMETC (——), stable inversion (——), norm-optimal feedforward (——), $\mathcal{H}_\infty$ preview control (——), and $\mathcal{H}_2$ preview control (——).

preview control guarantees convergence since it minimizes $\|W(1-HF)\|_\infty$, see section 4.4. Fig. 10(a) shows the Bode magnitude $|W(1-HF)|$ for techniques with explicit design of F. The number of preview samples q in $\mathcal{H}_\infty$ preview control ($q = 50$ in Fig. 10) directly influences the convergence speed. Note that since the benchmark system is SISO, $\|W(1-HF)\|_\infty = \max_\omega |W(e^{j\omega h})(1 - H(e^{j\omega h})F(e^{j\omega h}))|$.

Monotonic convergence in $\|e_j\|_2$ can be guaranteed for stable inversion (exact on an infinite horizon), $\mathcal{H}_\infty$ preview control (minimizes the criterion), and norm-optimal ILC (for proper weight selection). For NPZ-Ignore, ZPETC, ZMETC, and $\mathcal{H}_2$ preview control an additional robustness filter $W \neq 1$, i.e., $W(1-HF)$ for some possibly noncausal W, is required to enforce convergence, at the cost of performance. Note that the convergence condition is only sufficient, i.e., not satisfying the condition does *not* imply that the technique does not converge.

Application to the benchmark system The convergence on the benchmark system of section 3 is investigated. Fig. 10(b) shows $\|e_j\|$ over 21 trials. The results show that there is indeed convergence for $\mathcal{H}_\infty$ preview control and norm-optimal ILC, and also for stable inversion despite the boundary errors. For the approximate inverse techniques NPZ-Ignore, ZPETC, and ZMETC there is no convergence.

A key observation is that the convergence condition in Fig. 10(a) holds for arbitrary reference trajectories r and is therefore only sufficient. In particular, $\mathcal{H}_2$ preview control shows that by explicit design for r high performance can be achieved, while the convergence condition seems to suggest otherwise.

6. CONCLUSION AND OUTLOOK

Inversion techniques are essential for achieving high performance in motion systems, either through inverse model feedforward or learning control. In this paper the criteria for inverse model feedforward and ILC are posed and several inversion techniques are investigated, developed, and compared on a nonminimum-phase benchmark system, resulting in the following guidelines.

For inverse model feedforward, norm-optimal feedforward (section 4.3) is preferred due to boundary effects. If boundary effects are not critical, $\mathcal{H}_2$ preview control (section 4.4) is recommended as infinite time design.

For ILC, filter synthesis via $\mathcal{H}_\infty$ preview control (section 4.4) is strongly recommended as it explicitly optimizes the convergence criterion. For non-optimal filter design, stable inversion (section 4.2) is experienced to yield better results than the approximate inverse techniques NPZ-Ignore, ZPETC, and ZMETC (section 4.1). Importantly, the approximate inverse techniques typically require an additional robustness filter at the cost of performance.

Ongoing research focuses on different system classes such as linear time-varying (LTV), linear periodically time-varying (LPTV), linear parameter varying (LPV), position-dependent, and data-driven methods. Results will be reported elsewhere.

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